Blind Doppler Rate Estimation for PSK Signal

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Abstract. We propose a novel blind Doppler rate estimator for phase shift keying (PSK) transmissions. The issue may arise in mobile radio links when the received signal experiences non-negligible time-varying Doppler distortion, as in low earth-orbit satellite systems and terrestrial mobile cellular radio systems. The algorithm first removes the phase modulation without training sequence and then utilizes spectral characteristics of the transformed signal, which ensures its simplicity of implementation. Iterative section is introduced to help enhancing the adaptability and accuracy. The proposed method is almost insensitive to Doppler shift and performs well over a wide range of Doppler rates. Comparisons with existing methods are discussed.

1. Introduction

Phase shift keying has been widely employed in mobile radio transmissions, such as satellite communications. However, in the situations where there is relative motion between the transmitter and receiver, significant time-varying Doppler distortion may be encountered. This occurs, for instance, in communication systems based on a constellation of non-geostationary low-earth-orbit (LEO) satellites [1] and in millimeter-wave mobile communications for traffic assistance and control [2]. In these situations, Doppler shift and Doppler rate estimation at the receiver is the precondition for the correct demodulation of the received signal. On the other hand, Doppler information can be used to compute relevant parameters of the satellite orbit [1] and is important in spectral analysis of targets motion [3].

The problem of estimating the Doppler shift is well covered in the technical literature and several efficient algorithms already exist (see [4] and references therein). On the other hand, Doppler rate estimation has received relatively less attention. Existing algorithms [5]~[11] addressing the problem can be classified into two categories: data-aided (DA) [5]~[7], where estimation is based on known sequences and non-data-aided [8] ~ [11] (e.g. blind).

The DA methods exploit the information contained in pilot symbols, and are normally employed to attain good performance with small burst overhead. However, the information of pilot data may not be available under certain circumstances such as in non-cooperative communication. In fact, even when available, the insertion of a known data sequence would impact on the transmission efficiency.

A number of existing blind techniques are based on the maximum-likelihood (ML) principle [8]~[9]. The ML estimator, however, involves the optimization of a multivariable cost function and requires large amounts of computation. In the pursuit of real-time operation, several suboptimal solutions are proposed. These include the least-squares technique by Baggenstoss and Kay [10] and the iterative procedure developed in [11] for joint estimation of phase, frequency and frequency-rate of an unmodulated carrier. However, estimations of the two schemes are easily plagued by large outliers, which make them unsuitable for coded transmission systems such as the situation concerned in this manuscript. Other deficiencies of existing methods include their sensitivity to the value of Doppler shift and being unable to estimate Doppler rate alone rather than jointly with Doppler shift and phase of the signal.

A novel estimator for Doppler rate is proposed in this paper. Apart from being blind, the algorithm also requires little prior knowledge about the carrier frequency and the Doppler shift, which helps

overcoming the sensitivity to the Doppler shift. Simulation works demonstrate that the proposed algorithm performs well over large range of Doppler rates and Doppler shifts. Comparisons with former algorithms further confirm its effectiveness and robustness.

2. Derivation and implementation

2.1 Modeling of signal

The case of a LEO communication satellite in M-ary phase shift keying transmission with additive white Gaussian noise (AWGN) is considered. Filtering is evenly split between transmitter and receiver and the overall channel response is Nyquist. Timing is ideal but the received signal is affected by time-varying Doppler distortion. Filtering the received waveform in a matched filter yields:

$$e_r(t) = \exp\{j[2\pi f_I t + \pi\alpha_I t^2 + \varphi(t) + \varphi_0]\} + n(t) \quad (1)$$

where $f_I = f_c + \nu - f_L$ is the equivalent carrier frequency after intermediate frequency (IF) process, with f_c being the original carrier frequency, ν the Doppler frequency shift (which is unknown), f_L the frequency of local oscillator, α_I is the Doppler rate, $\varphi(t)$ is the modulation phase and for order M , $\varphi(t) \in \{\frac{2\pi}{M}k, k=0,1,\dots,M-1\}$, φ_0 is the initial phase, and the observation length is T . $n(t)$ denotes noise item that is complex-valued independent and identically distributed Gaussian random variable with zero mean and variance $\sigma^2 = (SNR)^{-1}$.

2.2 Estimation algorithm

We begin by carrying out exponentiation on the received signal, after which $e_r(t)$ is transformed as:

$$e(t) = \exp\{j[2\pi(Mf_I)t + \pi(M\alpha_I)t^2 + M\varphi_0]\} + n(t)' \quad (2)$$

where $n(t)'$ is the compound noise item with newly generated cross-terms included. For $M=2$, which indicates the situation of BPSK, we have:

$$n(t)' = 2\exp\{j[2\pi f_I t + \pi\alpha_I t^2 + \varphi(t) + \varphi_0]\}n(t) + n^2(t) \quad (3)$$

Notice that now $\varphi(t)$ the item vanishes which indicates the removal of phase modulation in the proposed algorithm.

Let α denotes $M\alpha_I$, then we have:

$$e(t) = \exp\{j[2\pi(Mf_I)t + \pi\alpha t^2 + M\varphi_0]\} + n(t)' \quad (4)$$

Thus, $e(t)$ may be viewed as an unmodulated carrier affected by Doppler distortion and embedded in non-Gaussian noise.

Presuming high SNR condition, we temporarily ignores $n(t)'$ the component for the sake of simplicity and thus, $e(t)$ can be regarded as a typical linear frequency modulation (LFM) signal.

We refer the Fourier Transform (FT) of $e(t)$ as $E(j\Omega)$, according to [12], a well-known conclusion is that :

$$|E(j\Omega)| \propto \sqrt{\frac{1}{|\alpha|}} \quad (5)$$

where ' $|\cdot|$ ' denotes absolute value. The relationship is tenable under the presumption that the Bandwidth Time Product (TBP) of the signal is far greater than 1.

Next, two assistant signals are composed as:

$$\begin{aligned} c_1(t) &= \exp(-j\pi\alpha_{c_1}t^2) \\ c_2(t) &= \exp(-j\pi\alpha_{c_2}t^2) \end{aligned} \quad (6)$$

with

$$\alpha_{c_2} = \alpha_{c_1} + \Delta\alpha_c \quad (\Delta\alpha_c > 0) \quad (7)$$

Multiplying $e(t)$ with $c_1(t)$ and $c_2(t)$ respectively, we obtain:

$$\begin{aligned} e_1(t) &= \exp\{j[2\pi(Mf_l)t + \pi(\alpha - \alpha_{c_1})t^2 + M\varphi_0]\} \\ e_2(t) &= \exp\{j[2\pi(Mf_l)t + \pi(\alpha - \alpha_{c_2})t^2 + M\varphi_0]\} \end{aligned} \quad (8)$$

Set $\alpha - \alpha_{c_1} = \Delta\alpha_1$, $\alpha - \alpha_{c_2} = \Delta\alpha_2$ and the FT of $e_1(t)$ and $e_2(t)$ as $E_1(j\Omega)$ and $E_2(j\Omega)$.

Then according to the conclusion mentioned in Eq. (5), we have:

$$\frac{|E_1(j\Omega)|}{|E_2(j\Omega)|} \approx \sqrt{\frac{|\Delta\alpha_2|}{|\Delta\alpha_1|}} \quad (9)$$

Combining Eq.(7), we get:

$$\Delta\hat{\alpha}_2 = \begin{cases} \frac{|E_1(j\Omega)|^2 \cdot \Delta\alpha_c}{|E_2(j\Omega)|^2 - |E_1(j\Omega)|^2}, & \text{for } \Delta\alpha_1 \Delta\alpha_2 > 0 \\ \frac{-|E_1(j\Omega)|^2 \cdot \Delta\alpha_c}{|E_2(j\Omega)|^2 + |E_1(j\Omega)|^2}, & \text{for } \Delta\alpha_1 \Delta\alpha_2 < 0 \end{cases} \quad (10)$$

And finally, the estimation of α , denoted by $\hat{\alpha}$, can be obtained with:

$$\hat{\alpha} = \alpha_{c_2} + \Delta\hat{\alpha}_2 \quad (11)$$

And consequently, the estimation of α_l should be:

$$\hat{\alpha}_l = \hat{\alpha} / M \quad (12)$$

2.3 Improvements on the construction of assistant signals

The construction of assistant signals is crucial in the proposed scheme. There are two main problems to be resolved before the algorithm can work out.

The first one is that we are unable to decide whether $\Delta\alpha_1$ and $\Delta\alpha_2$ are of the same sign when deciding which formula to adopt in Eq.(10).

To remove this vagueness, we set one more trial point denoted as α_{c_3} .

Figure. 1 Three trial points and two cases

As Figure. 1 has depicted, the trial points are equidistant, which means:

$$\alpha_{c_3} = \alpha_{c_2} + \Delta\alpha_c \quad (13)$$

And similarly, there is $E_3(j\Omega)$ corresponding to α_{c_3} .

Two different situations are considered:

In **Case 1**, if the truth-value locates at the left side of α_{c_2} , then consequently $|\Delta\alpha_3|$ would be the maximum among $|\Delta\alpha_i| (i=1,2,3)$, hence $|E_3(j\Omega)|$ would in turn be the minimum among $|E_i(j\Omega)| (i=1,2,3)$.

In **Case 2**, $|E_1(j\Omega)|$ would be the minimum one instead.

By comparing $|E_i(j\Omega)| (i=1,2,3)$, we can make judgment on the possibilities. For **Case 1**, we transfer Eq. (10) and (11) as:

$$\Delta\alpha_3 = \frac{|E_2(j\Omega)|^2 \cdot \Delta\alpha_c}{|E_3(j\Omega)|^2 - |E_2(j\Omega)|^2} \quad (14)$$

$$\hat{\alpha} = \alpha_{c_3} + \Delta\hat{\alpha}_3 \quad (15)$$

For **Case 2**, the upper formula of Eq. (10) and Eq. (11) will be adopted.

The second problem is that without the prior knowledge on the truth-value, trial points selected may be inappropriately distant from it.

When this happens, $|\Delta\alpha_i| (i=1,2,3)$ will consequently be large, which according to Eq. (5), also indicates reduced magnitudes of $|E_i(j\Omega)| (i=1,2,3)$, and hence causes the estimation to be vulnerable to noise and worsens the result.

Figure. 2 depicts the Normalized Root Mean Square Error (NRMSE) of the estimation under different pairs of trial points. It can be seen that for either $\alpha=2.5\times 10^{-6}$ or 5×10^{-6} (normalized to R_s^2 , where R_s is the symbol rate), trial points distant from the truth-value may lead to worse estimation accuracy, which is consistent with the theoretical analysis.

Figure. 2 NRMSE under different trial points

To overcome the weakness, an iterative section is introduced as follow:

Step 1: Select the first group of trial points (denoted as $\alpha_{c_i}^1 (i=1,2,3)$) moderately according to experience. Avoid being excessively small or large. And set $\Delta\alpha_c$ ensuring the validity that $\Delta\alpha_c T^2 = P \gg 1$, in the meanwhile, let $\hat{\alpha}_0 = \alpha_{c_1}^1$.

Step 2: Operate the k th ($k=1,2,\dots$) estimation, obtain $\hat{\alpha}_k$, and calculate $\left| \frac{\hat{\alpha}_k}{\hat{\alpha}_{k-1}} - 1 \right|$.

Step 3:

If $\left| \frac{\hat{\alpha}_k}{\hat{\alpha}_{k-1}} - 1 \right| \leq \varepsilon$, then let $\hat{\alpha} = \hat{\alpha}_k$;

If $\left| \frac{\hat{\alpha}_k}{\hat{\alpha}_{k-1}} - 1 \right| > \varepsilon$, then let

$$\begin{aligned} \alpha_{c_1}^{k+1} &= \hat{\alpha}_k \\ \alpha_{c_2}^{k+1} &= \alpha_{c_1}^{k+1} + \Delta\alpha_c \\ \alpha_{c_3}^{k+1} &= \alpha_{c_1}^{k+1} + 2\Delta\alpha_c \\ k &= k+1 \end{aligned} \tag{16}$$

and return to **Step 2**, where ε is a pre-set threshold in accordance to the expected accuracy.

3 Simulation and discussions

Simulations have been carried out to testify the effectiveness of the proposed method in terms of NRMSE. Influences by crucial factors of the algorithm are also discussed.

Example 1 Discussion on the selection of P .

In this simulation, suppose the transmitted signal to be BPSK (Binary Phase Shift Keying) with $R_s = 2 \times 10^4$ Baud and the baseband roll-off factor $\rho = 0.5$; the original Doppler shift and Doppler rate are set as 2×10^{-4} (normalized to R_s) and 7.5×10^{-7} (normalized to R_s^2). $SNR = 25dB$ and ε is fixed to be

0.02. 500 times of Monte Carlo simulations are carried out under each pair of parameters. Figure. 3 illustrates the influence of P on the estimation accuracy under varied observation length T . It can be seen that: On the one hand, for P greater than 40, the estimation accuracy deteriorates along with its increase; this is consistent with our theoretical prediction since under the proposed iterative process, there is $\alpha_{c_1} \approx \alpha$, $\alpha_{c_2} \approx \alpha + \frac{P}{T^2}$, $\alpha_{c_3} \approx \alpha + \frac{2P}{T^2}$ after several times of iteration. Hence, greater P is equivalent to larger distance between trial points and the truth-value which consequently brings worse accuracy as has been explained in **Section II**. On the other hand, for P under certain thresholds, there starts to be dramatic rises of the NRMSE. This occurs because Eq. (5) is tenable only under large TBP, and that of the compensated signals ($e_i(t)$ ($i=1,2,3$)) adopted in the estimating process are proportional to the value of P , after the iteration starts. And since Eq. (5) is the fundamental formula on which the proposed scheme is based, it is not surprising that inappropriately small P will lead to catastrophic results. In conclusion, to achieve satisfying estimation, the suggested value of P will be slightly over 40 in general.

Figure. 3 Influence of P on the estimation

Example 2 Influence of ε the factor.

Estimation results under different ε are demonstrated in Figure. 4, where we set $SNR = 20dB$ and $P=80$. 500 times of Monte Carlo simulations are carried out under each pair of parameters. It is noticed that in general, better estimation results can be expected under smaller values of ε for varied α . This is because the larger ε is, the greater the probability that the iteration ends before it fully converges.

Figure. 4 NRMSE versus ε

Comparisons have been made with estimators proposed by Michele Morelli (MM) in [5], by Giannetti, Luise, and Reggiannini (GLR) in [9] as well as by Baggenstoss and Kay (BK) in [10]. The MM estimator is data-aided while the other two are non-data-aided. The aim is to demonstrate comparative performances versus varying Doppler shift, Doppler rate and SNR.

Example 3 Robustness to Doppler shift.

Figure. 5 illustrates performances of the proposed algorithm in comparison with existing methods, concerning different ν . It can be seen that since only the magnitude property of the Fourier transformed signal is utilized in the proposed algorithm, its estimation results are almost insensitive to varying Doppler shifts. For MM and BK estimators, results that are satisfying under small ν suffer

from sharp deterioration due to the so-called ' 2π discontinuity' which may cause severe unreliability when values of Doppler shifts go beyond a proper interval [9].

This advantage is significant when it is difficult to enforce previous or joint estimation of carrier phase in particular operating conditions, such as in a satellite-mobile scenario where the received signal is impaired by time-varying phase and frequency shift.

Figure. 5 Robustness of the proposed method versus Doppler shift

Example 4 Comparative performances under varying Doppler rates.

The estimating NRMSE of the proposed method, the MM estimator and the BK estimator are demonstrated in Figure. 6. It can be seen that the estimating results of the proposed method and the MM estimator are quite close while the BK estimator performs less satisfying under circumstances of the simulation. Meanwhile, it is obvious that intervals of Doppler rates that can be dealt well with by the three schemes are different (it has been stated in [5] that the acceptable estimation interval of the MM estimator depends on L , which is a pre-set parameter of the algorithm), estimating results of the MM estimator and the BK estimator become out of control for $\alpha > 5.3 \times 10^{-6}$ and $\alpha > 3 \times 10^{-6}$ respectively, and the proposed algorithm provides better adaptability to varying α .

Figure. 6 Comparative performances under varying α

Example 5 Influence of the noise.

Figure. 7 illustrates comparative performances under different SNR environments. From the curves we notice that the proposed method outperforms existing methods under moderate SNR, and its resolution is almost insensitive to the change of the power of noise for SNR within the interval of 10 ~ 40dB. For SNR above 30dB, the data-aided MM scheme surpasses the proposed scheme. However, the main defect of the proposed algorithm lies in that for $SNR < 10dB$, its estimation results become un- acceptable which to certain extent hinders its practicality.

Figure. 7 Performances versus SNR

4 Conclusion

We have proposed a novel algorithm for Doppler rate estimation in PSK transmission. The method is designed for situations where pilot sequence is not available. Discussions on crucial factors of the algorithm have been carried out, besides from comparisons with existing schemes. Simulations show that satisfying accuracy is provided at intermediate/high SNR values. And comparing with existing methods, the new algorithm is more robust against Doppler shifts; besides, the range of Doppler rates that can be coped well with is extended.

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