

The Phase Detection Algorithm of Weak Signals Based on Coupled Chaotic-oscillators

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Abstract. Traditional phase detection methods of weak signals have some problems such as high complexity and low accuracy. To cope with these problems,, this paper proposes one novel parallel detection algorithm based on multi-coupled oscillators. Firstly, the chaotic detection model and double coupled oscillator model are studied through EM algorithm. And then, the nonlinear dynamics are studied in-depth, and the noise immunity and stability analyzes of the algorithm are proved in paper. Finally, the weak harmonic power carrier signals simulation results are demonstrated for the effectiveness of the algorithm.

1. Introduction

Known as one of the most important theories in the 20th century, Chaotic algorithm has been widely used in many fields such as signal detection and estimation, neural network control and so on [1-4]. It is a complex mathematical process for using nonlinear chaotic oscillator to the process of weak signal detection. Especially, the key technology of the algorithm is detection of weak signal's initial phase [5].

At present, domestic and foreign scholars have made a lot of research work on the method of coupled oscillators. But, there are still some problems. The double-coupled oscillator is introduced as detection model in literature [6] and [7], but the studies lack the in-depth analysis of system dynamics and robustness.

2. System Model

2.1 Principle of chaotic system model

The common model of chaotic dynamical systems is composed of Duffing oscillator, Lorenz oscillator, Hemon oscillator and so on [9,10]. Respectively, there is most fully research and application about classical Duffing oscillator. And hence, the classical Duffing system is introduced as detection model:

$$\ddot{x}(t) + k\dot{x}(t) - x(t) + x^3(t) = A(t) + n(t), \quad (1)$$

where $A(t) = \cos(\omega t)$ is the cycle driving force, k is the damping ratio, $u = -x + x^3$ is the non-linear restoring force, and $n(t)$ is the white noise. The combined driving force could be rewritten as follows:

$$\begin{aligned} A(t) &= \lambda_d \cos(\omega t) + a \cos(\omega t + \phi) \\ &= \lambda(t) \cos(\omega t + \theta(t)) \end{aligned}, \quad (2)$$

where λ_d is the critical threshold of system, a is the amplitude of to-be-detected signals. At this time, the overall amplitude of excitation signal can be described as:

$$\lambda = \sqrt{\lambda_d^2 + 2\lambda_d \cos(\phi) + a^2}. \quad (3)$$

The overall phase of excitation signal can be described as:

$$\theta = \arctg \frac{a \sin(\phi)}{\lambda_d + a \cos(\phi)}. \quad (4)$$

The specific relationship between system states and the phase of to-be-detected signals can be written as follows:

$$\pi - \arccos(a/2f_d) \leq \phi \leq \pi + \arccos(a/2f_d). \quad (5)$$

When the initial phase of to-be-detected signals could satisfy the mapping function above, the system continues to be in chaotic state. It does not occur an transition change from chaotic state to large-scale periodic state. On the contrary, system state changes into large-scale periodic state, encouraged by the excitation signal.

2.2 Analysis of coupled oscillators model

The double coupled oscillator system model is written as follows:

$$\begin{cases} \ddot{x}(t) + k\dot{x}(t) + \varepsilon - u = \lambda \cos(\omega t) \\ \ddot{u}(t) + k\dot{u}(t) + \varepsilon - u = \lambda \cos[(\omega + \Delta\omega)t + \varphi] \end{cases} \quad (6)$$

where the coupling term is $\varepsilon = ck(x(t) - u(t))$.

The system model is one second-order stochastic differential equation and the exact analytical solution is not available by traditional methods. In addition, the double coupled system is rather sensitive to the initial value of to-be-detected signals. If the numerical solution accuracy of differential equations is not suitable, the characteristics of chaotic system could not be described accurately. Therefore the EM algorithm (Euler-Maruyama) is introduced, and the system model could be rewritten as follows:

$$\begin{cases} x_{k+1} = hy_k \\ y_{k+1} = (-ky_k + x_k - x_k^3 + \lambda \cos(\omega t_k))h + \sqrt{2D} \cdot \sqrt{h}\sigma_k \end{cases} \quad (7)$$

where h is the integration step, σ_k is the Gaussian random sequence with zero-mean value and unit variance.

3. Characteristic analysis of Coupled System Model

3.1 Analysis of system dynamics

When the initial phase of cycle driving force built in system changes, the bifurcation value of double coupled oscillator varies from the pristine one.

To study the system fractal characteristics, simulation experiments are designed. The main parameters are set as follows: the frequency of cycle driving force built in system is $\omega = 1 \text{ rad/s}$, and the integration step is $h = 1/100$, the range of initial phase is from $-\pi/4$ to $\pi/6$. The results compared with two different models between initial phase and bifurcation values are shown in Tab. 1.

Table 1 bifurcation values of single and coupled model with different initial phase

Initial phase	$-3\pi/12$	$-2\pi/12$	$-1\pi/12$
Single oscillator	0.82489	0.82514	0.82421
Coupled oscillator	0.82561	0.82534	0.82476
Initial phase	0	$1\pi/12$	$2\pi/12$
Single oscillator	0.82560	0.82614	0.82456
Coupled oscillator	0.82550	0.82632	0.82514

From Tab. 1, we can get a conclusion that the initial phase of cycle driving force built in system is influential to the system fractal characteristics. Therefore, in the actual signal detection experiments, the phase of excitation signal should be determined according to the initial phase of to-be-detected signals. However, compared with classic single oscillator, the bifurcation value of coupled oscillator system represents more stable, the affection generated by the preset phase is much smaller. These reasons are conducive for the actual signals detection.

3.2 Analysis of system state stability

To study the system state stability, simulation experiments are designed. The main parameters are set similar to the former chapter. Afterwards, with the false alarm and missed alarm probability as detection index, the results under chaotic state and large-scale periodic state are studied by simulation experiments.

Both single and double coupled oscillator systems are adjusted to achieve the critical chaotic

state, and the false alarm probability under the background noise in different intensity. The strength range of to-be-detected signals is from 0.001 to 0.01, the background noise generated by the noise generator with different intensities is injected into the test system. There are 100 times of Monte Carlo simulation experiments, each time 10000 valid data is selected for the system state stability judging. The false alarm and missed alarm probabilities obtained through experiments are shown in Tab. 2.

Table. 2 Detection performance under noise of different strength

	Noise (W)	Single oscillator	Coupled oscillator
	0.001	0.002	0
false alarm (critical state)	0.002	0.004	0.001
	0.005	0.022	0.012
	0.01	0.051	0.032
missed alarm (periodic state)	0.001	0.001	0
	0.002	0.003	0
	0.005	0.017	0.007
	0.01	0.043	0.026

From Tab. 1, we can get a conclusion that with the strength of background noise increasing, the false alarm and missed alarm probabilities of single and coupled oscillator systems gradually become larger. However, the false alarm and missed alarm probabilities of two coupled oscillators are always lower than single oscillator. The results indicate that the proposed model is more stable, more conducive to the accurate detection and synchronize of weak signals' initial phase under strong noise background.

3.3 Analysis of noise immunity

The noise immunity of system is most important to the phase states discrimination. Therefore simulation experiments are designed to study the capability of both single and double coupled oscillator systems.

Both single and double coupled oscillator systems are adjusted to achieve the critical chaotic state, compare with the track circumstances at the same noise strength. Te results obtained by simulation are shown in Fig. 2. The main parameters are set similar to the first chapter, and the noise power is $\sigma = 0.001$ W.

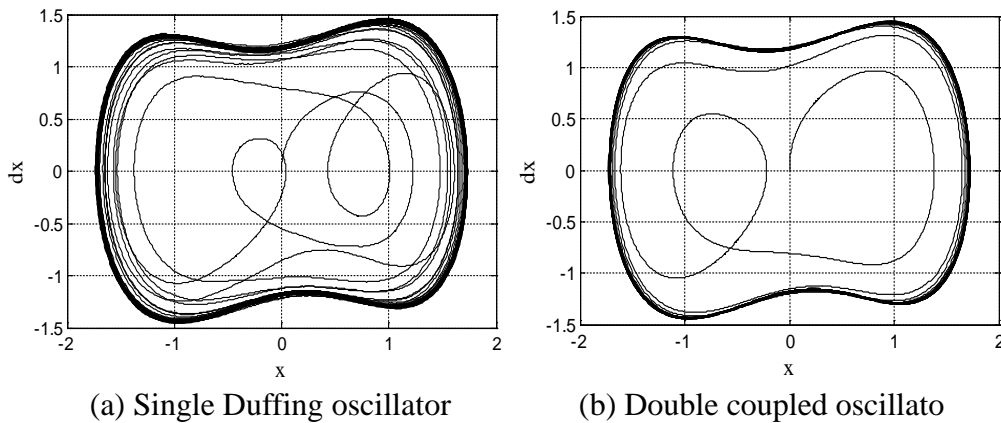


Fig. 2 Track circumstances of single and double coupled oscillator systems

From Fig. 2, we can see that at the same noise strength, compared with classic Duffing oscillator, double coupled oscillator has always maintained one better working condition, and the speed transforming from chaotic state into large-scale periodic state is much faster. The results indicate that the proposed system has much more excellent noise immunity.

4. Simulation Experiments

To study the effectiveness of coupled oscillator model application in phase detection of weak signals. This section uses double coupled oscillators array to study the initial phase detection of weak harmonic power carrier signals.

However, according to the state determination algorithm and the restriction of chaotic oscillators, the algorithm accuracy and phase detection range are determined by the strength of to-be-detected signals. the sensitivity of initial phase could not achieve the accuracy and range of theoretical detection. Therefore multi-oscillator array is designed to make up this deficiency. The phase range of $[-\pi, \pi]$ is divided in different parts such as $\{-k\pi/n, \dots, 0, \dots, k\pi/n\}$, in which $k = 0, 1, \dots, n$.

The multi-oscillator array composed of coupled oscillatot is introduced in experiments. The main parameters are set as follows: the damping ratio of system $k = 0.5$, the integration step is $h = 0.01$ s, the frequency of cycle driving force built in system is $\omega = 1$ rad/s, the simulation time is 1000s. The detection object is 1000 troops of sinusoidal signals with random initial phase. The scope range of initial phase is $[-\pi, \pi]$, and the detection error results are shown in Fig. 3.

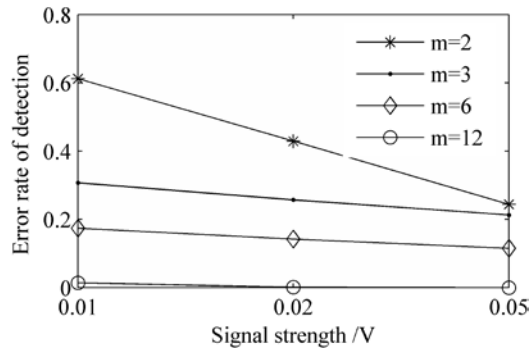


Fig. 3 Detection error results under noise of different strength

From Fig.3, we can get a conclusion that with the enhance of the excitation signal strength, the number of multi-oscillator array required is also reduced. In addition, with the increment of array number, the detection error is also reduced quickly. The experiment results indicate that the proposed parallel detection array based on multi-coupled oscillators is available for the phase detection of weak signals.

5. Summary

Traditional phase detection methods of weak signals have some problems such as high complexity and low accuracy. To cope with the problems, one novel parallel detection algorithm based on multi-coupled oscillators is introduced in paper. Firstly, the chaotic detection model and double coupled oscillator model are studied through EM algorithm. On this basis, the system dynamics, system state stability and noise immunity are analyzed in-depth. Finally, the weak harmonic power carrier signals are employed to verify the effectiveness of proposed algorithm. Experiment results indicate that the robustness of proposed algorithm is better than traditional single oscillator, the anti-noise performance is strong enough for the phase detection of weak signals. The proposed algorithm provides a new way for the phase detection of weak signals, and has one certain theoretical meaning and practicable value.

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