Signal Estimation under Specific Direction based on Extrapolation Method

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Abstract. For the problem of space spectrum cannot reserve the original time-domain signal under specific direction after beamforming, a method of primitive signal estimation based on the results of beamforming is proposed. The method proposed is based on the traditional extrapolation, and can be used to estimate the signal of the virtual array sensors. Simulation results of both amplitude and phase show the veracity of the estimated signal.

1. Introduction

Only the direction-of-arrival (DOA) of the signal can be got by the beamforming methods. Conventional beamforming methods can be used to get different DOAs of the signals by summate the time delay of echo in time domain. For the classes of correlation based beamforming methods, such as the MVDR method and the MUSIC method, there is no effective way to obtain the time domain data of specific directions.

The traditional extrapolation is proposed by Papoulis and is used for spectrum estimation of band limited signal [1]. This method can be realized by the calculation of FFT/IFFT iterative, and can be used for stationary process. This But Papoulis extrapolation can only be used for the band limited functions (e.g. sinc function). If the signal is not band limited, the convergence rate is much slow and the calculation is more complex by the Papoulis method. In order to overcome the shortcomings of Papoulis method, M.S.Sabri and W.Steenaaart proposed the extrapolation matrix method in 1978 [2]. This method makes the iterative extrapolation process into a non-iterative extrapolation process, and the extrapolation algorithm can be written as a simple matrix, called extrapolation matrix. In 1981, A.K.Jain and S.Ranganath [3] proved that Papoulis extrapolation iterative method and extrapolation matrix is a special case of the minimum norm least squares criterion. And the convergence characteristic can be improved by the conjugate gradient iterative method. In [4], the original signal in transform domain is processed and applied the samples concept of boundary matching to improve the recovery of the lost samples problem.

A way to get time-domain data based on extrapolation method is proposed in this paper, and analyses and discussions are taken to testify the feasibility the proposed method by simulations.

2. Extrapolation Method

In general, signal sampling in time field and space field have similar properties. The sampling interval of the signal in time field is the reciprocal of the sampling frequency:

$$\tau_T = \frac{1}{f_s} \quad (1)$$

Under the assumption of far-field narrowband signal, the time difference between each two sensors is associated with the source azimuth function:

$$\tau_s = \frac{d \sin(\theta)}{c} \quad (2)$$

For the distance of the element spacing is d and DOA of the signal θ is invariant, τ_s is a constant. If the number of samples is equal to the number of array elements, then the array antenna pattern is equivalent to the Fourier transform domain. And the spatial frequency is $\beta = 2\pi d \sin(\theta) / \lambda$.

Because spatial and temporal equivalence in treatment, so some increase spectral estimation method for the time domain to the airspace can be used to increase the angle of the signal processing array resolution. Because signal sampling in time field and space field have similar properties, signal in time domain under specific directions can be got.

Hypothesis, discrete signal is $x(k), k = 0, \pm 1, \pm 2, \pm 3, \dots$, and it is band-limited. The frequency of the signal is:

$$\begin{cases} x(f) \neq 0, f_1 \leq |f| \leq f_2 \\ x(f) = 0, f_2 < |f| < f_s/2 \end{cases} \quad (3)$$

f_s is the sampling frequency, and satisfy the sampling theorem.

Assume, vector $\mathbf{y} = \{y(k); -N \leq k \leq N\}$, and vector $\mathbf{x} = \{x(k); M \leq k \leq M\}$, $M > N$, and they have the relationships as follows:

$$\mathbf{y} = \mathbf{P}\mathbf{x} \quad (4)$$

$\mathbf{P}$ is $(2N+1) \times M$ dimensional matrix:

$$P_{ik} = \begin{cases} 1; & i = k \quad -N \leq i \leq N \\ 0; & i \neq k \quad k = 0, \pm 1, \dots \end{cases} \quad (5)$$

That vector $\mathbf{y}$ can be seen as resulting from the vector $\mathbf{x}$ through the sample matrix $\mathbf{P}$ after sampling.

Since (4) is an underdetermined equation, there are countless solutions. Therefore, several additional conditions are required. Extrapolation is the method for raw data estimation under the minimum mean square norm criterion of linear converted, which can described as the following constrained optimization problem:

$$\begin{cases} \min \|\hat{\mathbf{x}}\| \\ \mathbf{y} = \mathbf{P}\mathbf{x} \end{cases} \quad (6)$$

According to matrix theory, the optimal solution of such problems is:

$$\hat{\mathbf{x}} = \mathbf{Q}\mathbf{P}^T (\mathbf{P}\mathbf{Q}\mathbf{P}^T)^{-1} \mathbf{y} \quad (7)$$

In practice, the received signal is added with noise, and the receiving signal can be written as $\mathbf{y} = \mathbf{P}(\mathbf{x} + \mathbf{n})$. Using minimum mean square estimate norms, the external vector $\mathbf{x}$ is as follows:

$$\hat{\mathbf{x}} = \mathbf{Q}\mathbf{P}^T (\mathbf{P}\mathbf{Q}\mathbf{P}^T + \gamma \mathbf{I})^{-1} \mathbf{y} + \hat{\mathbf{n}} \quad (8)$$

Where $\mathbf{Q}$ is the filter matrix, $\mathbf{I}$ is the unit matrix, γ equal to σ_n^2 / σ_s^2 , and it is related with the signal to noise ratio. But in practice, γ is variable and unknown, generally it is replaced by a small amounts. It is can be proved that if the power of $x(t)$ is limited, and the sampling interval is very small, $\hat{\mathbf{x}}$ convergence in $\mathbf{x}$.

The extrapolation method takes the signal spectrum as the prior knowledge of the observed signal. Take $X(k)$ as the DFT transform of the observe signal. Filter of the band-limited signal means the product of the filter frequency response and $X(k)$. In time domain, it is the convolution of the inverse DFT transform and the $x(k)$.

Here define $a_h(m)$ is the inverse DFT of $|X(k)|^2$, the filter matrix is defined as follows:

$$\mathbf{Q} = \begin{bmatrix} a_h(0) & a_h(M-1) & a_h(M-2) & \dots & a_h(1) \\ a_h(1) & a_h(0) & a_h(M-1) & \dots & a_h(2) \\ a_h(2) & a_h(1) & \vdots & \dots & \vdots \\ \vdots & \vdots & \vdots & \dots & \vdots \\ a_h(M-1) & a_h(M-2) & \vdots & \dots & a_h(0) \end{bmatrix} \quad (9)$$

Array signal beamforming processing is a kind of spatial spectrum estimation method, and the DOA of the source in spatial can be got. The spatial spectrum is discussed as a spatial filter, and it is

used to estimate the primitive signal of the signal. This estimation method is adapted to estimate a single snapshot of the signal, so it is possible to adapt the signal parameters rapidly changing. However, the estimation accuracy of extrapolation is related with the filter matrix Q . The shape of the main directional beamforming is sharper, and the better spatial filtering effect can be got. At low SNR, the estimated spatial filter matrix cannot reflect the characteristic of the signal, so the virtual primitive signal cannot be estimated accurately.

Therefore, if extrapolation is applied to the primitive data estimates, a way to build robust filter matrix is needed. To construct the filter matrix, we need to consider the following three conditions:

The orientation information of the signal can be accurately reflected by spatial spectrum;

1. The energy information of the signal can be accurately reflected by spatial spectrum;
2. The spatial spectrum has a sharp spatial resolution.

In theoretical, the filter matrix Q is $-\infty \times \infty$ dimension. But in reality matrix Q is finite dimensional, generally 10-15 times of the number of the array sensors is choosen. If the dimension is too large, more calculation will cost. Assume that matrix Q is $M \times M$ dimension, the sensors number of the array is N , the steps of the extrapolation method is as follows:

1. Calculate Bartlett spectrum according to the actual data, and the scan angle is M points. Assume the scanning range is $\theta \in [-90^\circ \quad 90^\circ]$, and $u = \sin(\theta), u \in [-1 \quad 1]$, u space is divided into $M - 1$ intervals, then $M - 1$ scan points can be got.

2. Normalize the M -point Bartlett spectrum, and rearrange the midpoint of the sequence and the subsequent move to the front of the sequence, in order to facilitate filter matrix structure.

3. Make inverse DFT calculation for the rearranged sequence based on.

4. Configure the filter matrix Q according to (3).

5. Calculate the virtual array data according to (4).

It is worth noticing that the extrapolation method can be used to estimate the data of virtual array sensors. However, not all the calculated data of the virtual array can be used as the sensors data, because the extrapolation error is larger when the virtual array is longer. Commonly, extrapolation method can be used to estimate the data of sensors extended to about 2 times aperture of the real one.

3. Simulation Analysis

Simulation condition 1: the incident signal is a single frequency sine wave, normalized frequency is 0.03, 1000 sample points, the signal incident from azimuth 0 degree (horizontal direction), no noise. The array is an 18 uniform linear array, and the array element spacing is half wavelength. Both amplitude and phase characteristic simulation analyses are taken to verify the time-domain data output beam characteristics through extrapolation.

Fig. 1(a) is the actual primitive received signal, and Fig. 1 (B) is the estimated signal at the position of the primitive sensor. It can be seen from the simulation result, the amplitude of the recovered signal is the same as the primitive received signal in time-domain. Therefore, extrapolation method can completely estimate the amplitude of the primitive signal.

Fig.2 (a) is the contrast on amplitude between primitive signal and the estimated signal. Fig.2 (b) is the contrast on phase between primitive signal and the estimated signal. By the comparison of both amplitude and phase, the extrapolated signal has high estimation accuracy in real primitive position, and can be used to estimate the primitive signal. The errors of the amplitude and phase of the estimated signals increase with the distance from the sensor to the center of the real array.

Fig.1 single-source, single-frequency signal and the extrapolation signal

Fig.2 Contrast on amplitude and phase between primitive signal and the estimated signal

4. Summary

Extrapolation method can be used to estimate the signal after beaming under specific direction. And the estimated signal has high precision both in amplitude and phase at the real sensor position. But for the sensors at the virtual position, the errors of the estimated signals increase. Generally, the estimated signal of 2-3 times long of the virtual array can be taken for keeping the precision.

References

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