

Application of adaptive Sage-Husa and AUKF filtering algorithm In Initial Alignment of SINS

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Abstract. When the system model and noise statistical characteristics are known, the conventional Kalman filtering algorithm is suitable. When the noise statistics are unknown and large initial misalignment angles results in system nonlinear, the application of liner error model and Kalman filtering algorithm is very been subjected to restriction. This text, two kinds of adaptive Sage-Husa and adaptive UKF filtering algorithm be simplified respectively. In adaptive Sage-Husa filtering algorithm, automatic on-line estimation and correction for the noise parameters, the state of the system and the state estimate covariance by the observed data. The algorithm improve the convergence speed and alignment accuracy effectively. In adaptive UKF filtering algorithm, AUKF algorithm can automatically balance the weight ratio of state and observation information in filtering to adjust the covariance of state vector and observation vector in real-time, thereby improving system performance. Experimental results showed that the use of adaptive Sage-Husa and UKF filtering algorithm can obtain better alignment accuracy and capacity of resisting disturbance.

1. Introduction

Initial alignment provides initial attitude matrix for SINS calculation. Its rapid and accuracy will directly influence navigating of SINS accuracy. It is one of SINS key techniques. Initial alignment is divided into coarse alignment and fine alignment. Make use of coarse alignment gain small initial misalignment angles, establishes the liner state equation and the liner observation equation. Under small initial misalignment angles and the known statistics noise, fine alignment will use traditional Kalman filtering algorithm.

The observation equation usually uses speed error as to observation value. Main fault is that the system observation properties are uncertain. When initial observation angles are large, the system noise will be nonlinear. For two faults, initial alignment method is studied by adaptive Sage-Husa and UKF filtering algorithm.

In adaptive Sage-Husa filtering algorithm, automatic on-line estimation and correction for the noise parameters, the state of the system and the state estimate covariance by the observed data. Using forgetting factor can limit memory length of the filter, which could enhance the effect the newly observed data acts on the present estimation. Thus, enable the system to achieve the best filtering effect.

When Kalman filtering algorithm estimates to status of large initial misalignment angle nonlinear system, the system need to be linearized. As EKF method, it influences the accuracy of system to some extent. Based on the U transform, UKF uses the Kalman linear filtering framework. The nonlinear processing is carried out by the deterministic sampling.

In this paper, an adaptive UKF filtering method is used to estimate fine alignment. Not only ensures a good alignment accuracy, but also relax the constraints of large initial misalignment angle.

2. Kalman filtering algorithm for SINS

2.1 Kalman filtering algorithm

SINS is the gyro and accelerometer directly mounted on the carrier. Attitude matrix transform the acceleration and angular velocity of inertial measurement devices from the carrier coordinate system(b) to the navigation coordinate system. The selected navigation coordinate (n) and east-north-up geographic coordinate system(t) are consistent. Attitude matrix, mechanical derivation and systematic error model refer to literature [1].

The state equation of the initial alignment using Kalman filtering algorithm is based on the error equation of SINS. Prerequisite is that the misalignment angle obtained by coarse alignment is a small angle and time consumed by the alignment process is short.

Initial alignment of SINS state equation and observation equation as follows :

$$\dot{X} = AX + W = \begin{bmatrix} A_1 & A_2 \\ 0_{5 \times 5} & 0_{5 \times 5} \end{bmatrix} X + W \quad (1)$$

$$Z = HX + V$$

System state and system noise as follows:

$$X = [\delta v_E \quad \delta v_N \quad \phi_E \quad \phi_N \quad \phi_U \quad \nabla_x \quad \nabla_y \quad \varepsilon_x \quad \varepsilon_y \quad \varepsilon_z]^T$$

$$W = [w_{ax} \quad w_{ay} \quad w_{gx} \quad w_{gy} \quad w_{gz}]^T$$

$$A_1 = \begin{bmatrix} 0 & 2\omega \sin L & 0 & -g & 0 \\ -2\omega \sin L & 0 & g & 0 & 0 \\ 0 & -\frac{1}{R} & 0 & \omega \sin L & -\omega \cos L \\ \frac{1}{R} & 0 & -\omega \sin L & 0 & 0 \\ \frac{\tan L}{R} & 0 & \omega \cos L & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{11} & C_{12} & C_{13} \\ 0 & 0 & C_{21} & C_{22} & C_{23} \\ 0 & 0 & C_{31} & C_{32} & C_{33} \end{bmatrix}$$

In A2 matrix, C_b^n is the attitude matrix. In SINS initial alignment process, the system is changing, therefore, the coefficient matrix A is a time-varying matrix.

$$C_b^n = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$Z = \begin{bmatrix} \delta v_E \\ \delta v_N \end{bmatrix}, \quad H = \begin{bmatrix} I_{2 \times 2} \\ 0_{2 \times 8} \end{bmatrix}^T$$

Where: δv_E and δv_N are the speed errors respectively for the east and north; ε is the gyro drift error; ϕ_E and ϕ_N are two horizontal misalignment angles, ϕ_U is the azimuth misalignment. heading angle error; ∇ is the accelerometer zero offset errors; ω is the Earth's rotation angular velocity; L is the geographical latitude.

Where: X is the state vector of the system; A is the state matrix of the system; Z is the observation vector of the system; H is the observation matrix of the system; W is the state noise vector of the system; V is the observation noise vector of the system; Kalman filter requests that W and V are assumed to be zero mean value of Gaussian white noise sequences.

Due to a static base alignment, and alignment time is short, diagonal elements of attitude matrix C_b^n are approximately equals 1, the other elements similar to 0.

Taking level velocity error as an external observation value in the process of initial alignment, the system's observation equation is:

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} X + V$$

2.2 Linear Stochastic discrete Kalman filter principle

Without thought control action, linear discrete state equation and observation equation[2] is:

$$\begin{aligned} X_k &= \Phi_{k,k-1} X_{k-1} + \Gamma_{k,k-1} W_{k-1} \\ Z_k &= H_k X_k + V_k \end{aligned} \quad (2)$$

X_k is the state vector for k moment, also required is estimated vector; Z_k is the observation vector for k moment; W_k is the system noise for k moment; V_k is observation noise for k moment; $\Phi_{k,k-1}$ is transfer matrix; Γ_k is the system noise matrix; H_k is observation matrix for the k moment. The system noise W_k and observation noise V_k are not related.

Statistic characteristics of system noise and observation noise as follows:

$$\begin{aligned} E[W_k] &= 0, E[W_k W_j^T] = Q_k \delta_{kj} \\ E[V_k] &= 0, E[V_k V_j^T] = R_k \delta_{kj} \\ E[W_k V_j^T] &= 0 \end{aligned} \quad (3)$$

Q_k is the system noise covariance matrix; P_k is the status covariance matrix; R_k is the observation noise covariance matrix; δ_{kj} is Kronecker- δ function. When the X_k, Z_k, W_k, V_k are estimated to meet the constraints of the formula (3), Kalman filter is the optimal filter.

The standard Kalman filtering algorithm[3], as follows:

$$\begin{aligned} \hat{X}_{k,k-1} &= \Phi_{k,k-1} \hat{X}_{k-1} \\ P_{k,k-1} &= \Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^T + \Gamma_{k,k-1} Q_{k-1} \Gamma_{k,k-1}^T \\ K_k &= P_{k,k-1} H_k^T [H_k P_{k,k-1} H_k^T + R_k]^{-1} \\ \hat{X}_k &= \hat{X}_{k,k-1} + K_k [Z_k - H_k \hat{X}_{k,k-1}] \\ P_k &= [I - K_k H_k] P_{k,k-1} [I - K_k H_k]^T + K_k R_k K_k^T \end{aligned} \quad (4)$$

According to the status equation of system, status $X_{k,k-1}$ at current k moment can be estimated by last status X_{k-1} of system. $X_{k,k-1}$ is the result of one step status estimate. X_{k-1} is last optimal estimate status. According to the optimal estimate covariance P_{k-1} of last status, current status covariance $P_{k,k-1}$ is renewed accordingly. It is one step status estimate covariance.

It is former two estimate formulas of five formulas of Kalman filter.

Current optimal estimate status X_k is obtained by estimate value $X_{k,k-1}$ and observation value Z_k . Among, K_k is Kalman Gain.

In order to make continuous Kalman filter run until the system process end, the optimal estimate status covariance P_k will be obtained in X_k status. Thus, autoregressive process can keep on operating.

Under the condition that the system noise covariance matrix Q and the observation noise covariance matrix R , only needs to set the starting value X_0, P_0 . According to observation value Z_k at k moment, we can calculate recursively optimal status estimate value X_k at k moment.

Because the error estimate value is closely related with the filter gain, so Kalman filter parameters(P_0, Q, R) reasonable degree will important influence the filtering estimation performance. When the filter parameters and the true values chosen consistent estimation, filtering effect is good.

3. Application of two adaptive Kalman filtering algorithm

3.1 Adaptive Sage-Husa Kalman filtering algorithm

In the actual alignment, because of the influence of the simplified mathematical model and the alignment of environmental uncertainties, statistical parameters of system noise and measurement noise are often difficult to give the precise. For this, usually the adoption Kalman filters is used. Sage and Husa proposed adaptive filtering algorithm, system noise Q and measurement noise R [4] can be

computed on-line. Actually, when the Q and the R didn't know, Sage-Husa filters can not be acquired at the same time[5]. Only when the Q have been already known R can be estimated, or when the R have been already known Q can be estimated. Generally think that the SINS system noise has a stability and usually only measurement noise will be estimated. This paper, based on the traditional adaptive Sage-Husa algorithm[7-8], a simplified adaptive alignment filtering algorithm is proposed for improve the real-time filtering.

The simplified adaptive filtering formula[6], as follows:

$$\begin{aligned}
 \hat{X}_{k,k-1} &= \Phi_{k,k-1} \hat{X}_{k-1} \\
 P_{k,k-1} &= \Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^T + \Gamma_{k,k-1} Q_{k-1} \Gamma_{k,k-1}^T \\
 K_k &= P_{k,k-1} H_k^T [H_k P_{k,k-1} H_k^T + R_k]^{-1} \\
 \hat{X}_k &= \hat{X}_{k,k-1} + K_k (Z_k - H_k \hat{X}_{k,k-1}) \\
 P_k &= [I - K_k H_k] P_{k,k-1} [I - K_k H_k]^T + K_k R_k K_k^T \\
 d_k &= (1-b)/(1-b^{k+1}) \\
 \varepsilon_k &= Z_k - H_k \hat{X}_{k,k-1} \\
 \hat{R}_k &= (1-d_k) \hat{R}_{k-1} + d_k [(I - H_k K_{k-1}) \varepsilon_k \varepsilon_k^T (I - H_k K_{k-1})^T + H_k P_{k,k-1} H_k^T]
 \end{aligned} \tag{5}$$

In the formula, ε_k is a new information series. b is the forgetting factor, usually the value of 0.95 ~ 0.99. The forgetting factor can limit filter memory length, improve the effect of new observational data. That recent data play a major role in estimation, and make the old data gradually forgotten.

The initial conditions, as: $\hat{X}_{0,0} = \hat{X}_0$, $P_{0,0} = P_0$, $\hat{Q}_0 = Q_0$, $\hat{R}_0 = R_0$

By alternating the formula(5), we can calculate the statistical characteristics estimation value of noise and state.

3.2 Adaptive UKF Kalman filtering algorithm

When initial misalignment angle is is larger, traditional little error linear equation already not ability accurate description system characteristic. In order to improve the effect of non-linear filtering, Julier proposed using unscented Kalman filter (Unscented Kalman Filter, UKF) method to estimate the nonlinear filtering problem. This algorithm still includes in Kalman filtering algorithm's frame.

This method processes Unscented Transform (U tranformation) on the state equation, estimate the filtering result after the UT, in turn reduce the estimation error[7]. For the one step estimate equation, UKF use the assured sample strategy directly handle non-linear deliver of system mean value and covariance error.

The design step of UKF [8] is described as below.

Initialization:

$$\left. \begin{aligned} \hat{x}_0 &= E[x_0] \\ P_0 &= E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \end{aligned} \right\} \tag{6}$$

Calculating Sigma Point and Corresponding Weight:

$$\left. \begin{aligned} X_{0,k-1} &= \hat{x}_{k-1} \\ X_{i,k-1} &= \hat{x}_{k-1} + \sqrt{(n+\lambda)P_{k-1}}, (i=1,2,L,n) \\ X_{i,k-1} &= \hat{x}_{k-1} - \sqrt{(n+\lambda)P_{k-1}}, (i=n+1,n+2,L,2n) \\ w_0^n &= \lambda/(n+\lambda) \\ w_0^c &= \lambda/(n+\lambda) + (1-\xi^2 + \eta) \\ w_i^n &= w_i^c = 0.5/(n+\lambda) (i=1,2,...,2n) \\ \lambda &= \xi^2(n+\kappa) - n \end{aligned} \right\} \tag{7}$$

n indicates the dimension of the state vector; $\lambda = \xi^2(n+\kappa) - n$ is a scaling factor; ξ can determine the value range of sampling points, usually taken between a small positive number between 0-1, such as

$1e-3$; κ is a constant, generally 0 or 3- n ; η depends on distribution of state variables, achieving its optimal when subjecting to gaussian distribution.

Updating Time:

$$\begin{aligned}\chi_{i,k|k-1} &= f(\chi_{i,k-1}) \\ \hat{\mathbf{x}}_k^- &= \sum_{i=0}^{2n} w_i^{(n)} \chi_{i,k|k-1} \\ \mathbf{P}_k^- &= \sum_{i=0}^{2n} w_i^{(c)} [\chi_{i,k|k-1} - \hat{\mathbf{x}}_k^-][\chi_{i,k|k-1} - \hat{\mathbf{x}}_k^-]^T + \mathbf{Q}_k \\ \mathbf{Z}_{i,k|k-1} &= \mathbf{H} \times \chi_{i,k|k-1} \\ \hat{\mathbf{Z}}_k^- &= \sum_{i=0}^{2n} w_i^{(n)} \mathbf{Z}_{i,k|k-1}\end{aligned}\quad (8)$$

\mathbf{Q}_k is the variance of the system noise \mathbf{w} at moment k .

Updating Measurement:

$$\begin{aligned}\mathbf{P}_{\mathbf{Z}_k, \mathbf{Z}_k} &= \sum_{i=0}^{2n} w_i^c [\mathbf{Z}_{i,k|k-1} - \hat{\mathbf{Z}}_k^-][\mathbf{Z}_{i,k|k-1} - \hat{\mathbf{Z}}_k^-]^T + \mathbf{R}_k \\ \mathbf{P}_{\mathbf{x}_k, \mathbf{Z}_k} &= \sum_{i=0}^{2n} w_i^c [\chi_{i,k|k-1} - \hat{\mathbf{x}}_k^-][\mathbf{Z}_{i,k|k-1} - \hat{\mathbf{Z}}_k^-]^T \\ \mathbf{K}_k &= \mathbf{P}_{\mathbf{x}_k, \mathbf{Z}_k} \times \mathbf{P}_{\mathbf{Z}_k, \mathbf{Z}_k}^{-1} \\ \hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{Z}_k - \hat{\mathbf{Z}}_k^-) \\ \mathbf{P}_k &= \mathbf{P}_k^- - \mathbf{K}_k \times \mathbf{P}_{\mathbf{Z}_k, \mathbf{Z}_k} \times \mathbf{K}_k^T\end{aligned}\quad (9)$$

\mathbf{Z}_k is the measurement vector at moment k ; \mathbf{R}_k is the variance of the measurement at moment k .

In practice, it's discovered that UKF is more sensitive to the choice of the initial value. An error in the initial value will directly affect the outcome of filtering result. Moreover, even if the initial value is reasonable, the existence of various disturbance error and uncertainty of statistical noise models will also affect the accuracy of UKF filtering. This paper, based on the principles of adaptive algorithm, put forward a brand new filter estimation method to achieve fine alignment.

Adaptive UKF algorithm and traditional UKF has basically the same structure[9], we just need to change the Eq.(10):

$$\begin{aligned}\mathbf{P}_{\mathbf{Z}_k, \mathbf{Z}_k} &= \frac{1}{a_k} \sum_{i=0}^{2n} w_i^c [\mathbf{Z}_{i,k|k-1} - \hat{\mathbf{Z}}_k^-][\mathbf{Z}_{i,k|k-1} - \hat{\mathbf{Z}}_k^-]^T + \mathbf{R}_k \\ \mathbf{P}_{\mathbf{x}_k, \mathbf{Z}_k} &= \frac{1}{a_k} \sum_{i=0}^{2n} w_i^c [\chi_{i,k|k-1} - \hat{\mathbf{x}}_k^-][\mathbf{Z}_{i,k|k-1} - \hat{\mathbf{Z}}_k^-]^T \\ \mathbf{P}_k &= \frac{1}{a_k} \mathbf{P}_k^- - \mathbf{K}_k \times \mathbf{P}_{\mathbf{Z}_k, \mathbf{Z}_k} \times \mathbf{K}_k^T\end{aligned}\quad (10)$$

a_k is the adaptive factor, the initial value is taken as 1, value range $0 \leq a_k \leq 1$. If the value is reasonable, it is possible to balance the ratio between the prediction and measurement information in the system model. a_k is constructed as in Eq.(11):

$$\begin{aligned}a_k &= \begin{cases} 1 & \text{tr}(\mathbf{V}_k \mathbf{V}_k^T) \leq \text{tr}(\mathbf{P}) \\ \frac{\text{tr}(\mathbf{P})}{\text{tr}(\mathbf{V}_k \mathbf{V}_k^T)} & \text{tr}(\mathbf{V}_k \mathbf{V}_k^T) > \text{tr}(\mathbf{P}) \end{cases} \\ \mathbf{P} &= \sum_{i=0}^{2n} w_i^c [\mathbf{Z}_{i,k|k-1} - \hat{\mathbf{Z}}_k^-][\mathbf{Z}_{i,k|k-1} - \hat{\mathbf{Z}}_k^-]^T\end{aligned}\quad (11)$$

The above improved UKF algorithm[9] shows that when there exists bias in initial value selection or abnormal disturbance in system model, a_k will be less than 1, which means the weight of forecast information in system model is as low as possible in the final filtered result; when an obvious abnormality in the prediction model is noticed which should be discarded, a_k will approach 0., To conclude, the value of \hat{x}_k^- can be adaptively adjusted by dynamic parameter a_k based on the predicted residual V_k and measurement information \hat{Z}_k^- .

4. Experiment analysis

4.1 Adaptive Sage-Husa Kalman filter simulation

Adaptive Sage-Husa simulation initial conditions is as follows: the initial value \mathbf{X}_0 are assigned to zero; the initial misalignment angles Φ_E, Φ_N, Φ_U are taken as 1° ; gyro drift is taken as $0.001^\circ/\text{h}$, accelerometers taken as $5 \times 10^{-5} g$, velocity errors taken as 0.1 m/s , inertial navigation system location of north latitude 43.25° . then:

$$\mathbf{X}_0 = \text{diag}\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

$$\mathbf{P}_0 = \text{diag}\{(0.1 \text{ m/s})^2, (0.1 \text{ m/s})^2, (1^\circ)^2, (1^\circ)^2, (1^\circ)^2, (5 \times 10^{-5} g)^2, (5 \times 10^{-5} g)^2, (0.001^\circ/\text{h})^2, (0.001^\circ/\text{h})^2, (0.001^\circ/\text{h})^2\}$$

$$\mathbf{Q} = \text{diag}\{(5 \times 10^{-5} g)^2, (5 \times 10^{-5} g)^2, (0.001^\circ/\text{h})^2, (0.001^\circ/\text{h})^2, (0.001^\circ/\text{h})^2, 0, 0, 0, 0, 0\}$$

$$\mathbf{R} = \text{diag}\{(0.1 \text{ m/s})^2, (0.1 \text{ m/s})^2\}$$

The simulation comparison results, as shown in Figure 1-3.

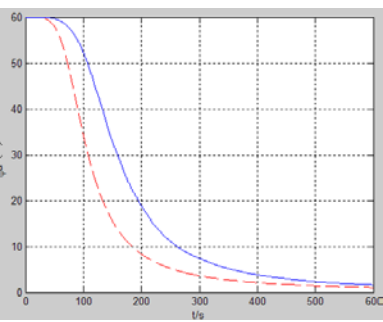
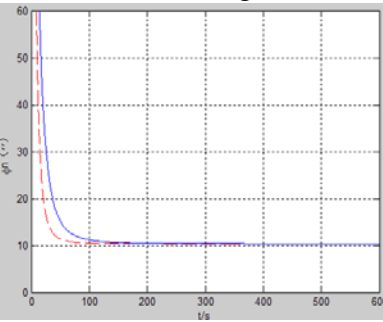
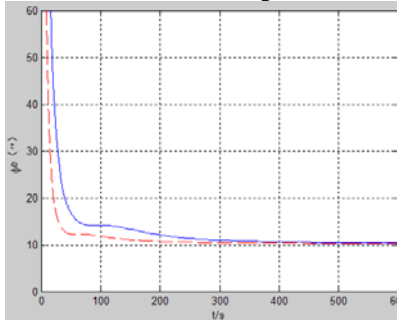


Fig. 1: Φ_E misalignment angle

Fig. 2: Φ_N misalignment angle

Fig. 3: Φ_U misalignment angle

Tab.1 Comparison of simulation results between Sage-Husa and conventional Kalman

| misalignment angle Φ | | Kalman | Sage-Husa |
|---------------------------|----------------------|--------|-----------|
| Φ_E | Convergence angle(") | 13.5 | 13.2 |
| | Convergence time(s) | 180 | 40 |
| Φ_N | Convergence angle(") | 13.2 | 13.05 |
| | Convergence time(s) | 70 | 30 |
| Φ_U | Convergence angle(") | 3.8 | 3.6 |
| | Convergence time(s) | 500 | 350 |

On the convergence precision, the adaptive Sage-Husa Kalman filter's and conventional Kalman's is basically the same, or increased slightly. the convergence time is better than the conventional Kalman filter.

4.2 Adaptive UKF Kalman filter simulation

In initial alignment, for acquiring large initial misalignment angle, we use low cost MEMS inertial measurement unit to imitate really.

Adaptive UKF simulation initial conditions is as follows[10]: The initial misalignment angle after a coarse alignment is $[0.05^\circ \ 0.56^\circ \ 1.31^\circ]$; the initial value \mathbf{X}_0 is assigned to zero; gyro drift $0.4^\circ/\text{h}$; the initial bias of accelerometer in three directions are 0.5 mg , random deviations 0.1 mg ; horizontal velocity error of 0.1 m/s ; latitude $L = 43.25^\circ$.

The simulation comparison results, as shown in Figure 4-6.

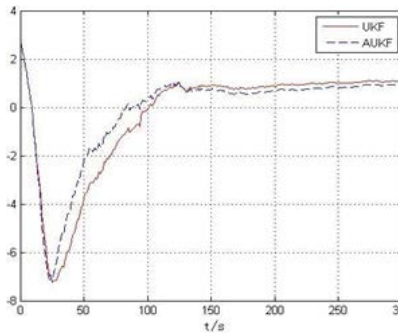


Fig. 4: Φ_E misalignment angle

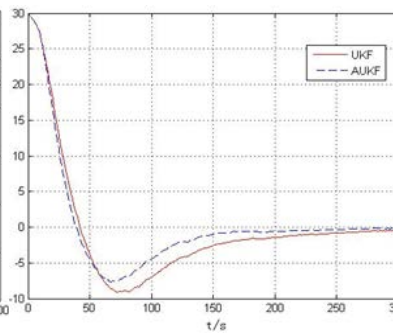


Fig.5: Φ_N misalignment angle

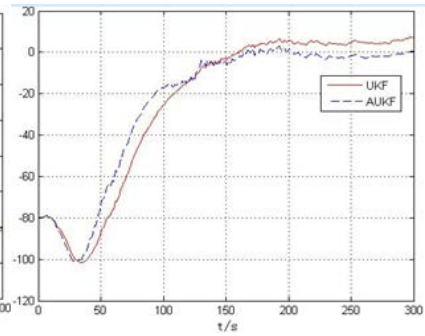


Fig.6: Φ_U misalignment angle

The figures below show :The convergence precision of three misalignment angles in AUKF are respectively 1.0', 0.1' and 1.0', UKF as 1.2', 0.3' and 6.0'. The convergence time of three misalignment angles in AUKF are respectively 110 s, 152 s and 173 s, UKF as 120 s, 180 s and 200 s.

5. Conclusion

Under statistical characteristic of noise in unknown, adaptive Sage-Husa filter can access to the filter steady-state much more fast. The convergence precision is basically the same. The convergence time of adaptive Sage-Husa filter is better than Kalman filter's. The convergence time was shortened by about 140s, 40s and 150s respectively.

Under initial alignment angle was larger, AUKF filter can access to the filter steady-state much more fast. The alignment precision and convergence time are better than UKF filter's. The accuracy was increased by 0.2', 0.2' and 5.0'; The convergence time was shortened by 10s, 28s and 27s respectively.

When the noise statistics are unknown and large initial misalignment angles results in system nonlinear, the adaptive algorithm can adjust and correct the estimate value in real time by auto balancing the weight ratio of state and observation information in the filtering results.

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