

Research on the Regularity of Distance Distribution between Adjacent Mines

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Abstract: based upon the influence of mine position distribution error, adjacent mine space cannot fulfill the task of actual combat. this paper builds the model of adjacent mine space distribution regularity under two conditions of the same voyage and different voyages. In combination of examples, a series of results can be obtained as follows. Under the condition that the standard deviation of mine position comprehensive distribution and the planning value of mine distance are certain, the probability density distribution of adjacent mine space is comparatively concentrate; the mean value and standard deviation of adjacent mine space distribution will increase when the comprehensive standard deviation of mine position increases and will increase comparatively more quickly when mine laying is conducted in different voyages . The relevant results have some certain military application value.

I Introduction

Adjacent mine space generally refers to the distance between two adjacent mines. When making the plan of minelaying , the laying position of each mine is designated and the space of adjacent mines is definite .But in the practical process of actual operation, due to the comprehensive influence of minelaying platform error , minelaying trajectory error, etc. ,there will be deviation and probability between the practical space of adjacent mines and the expected space .which will be more obvious especially when the minelaying platform error and minelaying position error are relatively bigger ,such as the air minelaying and the self-guided minelaying etc , so we need to make some research on the regularity of random distribution of adjacent mine space to offer scientific basis for reasonably determining expected space between adjacent mines.

II The probability density function of adjacent mine space

In convenience of building the model, we use lower corner on the left as the original point, X axis is paralleled to the front side of obstacle and right direction is positive ; Y axis is vertical to X axis and upper direction is positive , building mine position coordinate system and denoting the expected No.i mine position $\bar{M}_i(\bar{x}_i, \bar{y}_i)$, real position $M_i(x_i, y_i)$.

Supposing mine laying platform error and minelaying trajectory error also conform to circle probability normal distribution, and the standard deviation are σ_p^2 , σ_m^2 respectively, general error is $\sigma_z = \sqrt{\sigma_m^2 + \sigma_p^2}$.

Distribution probability model of mine space will be established on two circumstances of laying adjacent mines in one voyage or different.

1. Under the circumstance of laying in the same voyage

when two adjacent mines are laid in the same voyage, platform error can be considered as system error of mine position distribution ,the distribution in x direction or y direction is relevant, denoting two mines' position M_1 and M_2 respectively, and the joint distribution coordinate in x axis is

$$f_x(x_1, x_2) = \frac{1}{2\pi\sigma_z^2|R|} \exp\left\{-\frac{1}{2}[(\mathbf{x} - \bar{\mathbf{x}})R^{-1}(\mathbf{x} - \bar{\mathbf{x}})^T]\right\}$$

Besides $\mathbf{x} - \bar{\mathbf{x}} = (x_1 - \bar{x}_1, x_2 - \bar{x}_2)$; $R = \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}$; $r = \frac{\sigma_p^2}{\sigma_z^2}$

Supposing $u = x_1 - x_2$; Then $f(u) = \frac{1}{\sqrt{2\pi}\sqrt{2(1-r)}\sigma_z} \exp\left\{-\frac{[u + \bar{x}_1 - \bar{x}_2]^2}{4(1-r)\sigma_z^2}\right\}$;

In the same way, supposing $w = y_1 - y_2$, Then $f(w) = \frac{1}{\sqrt{2\pi}\sqrt{2(1-r)}\sigma_z} \exp\left\{-\frac{[w + \bar{y}_1 - \bar{y}_2]^2}{4(1-r)\sigma_z^2}\right\}$

Supposing $d = \sqrt{w^2 + u^2}$; $w = h \cos \theta$; $u = h \sin \theta$; $\bar{x}_1 - \bar{x}_2 = h_0 \sin \theta_0$; $\bar{y}_1 - \bar{y}_2 = h_0 \cos \theta_0$

Deduce the probability density function with $f(u)$ and $f(w)$,

$$f_t(d) = \frac{d}{2\sigma_r^2} \exp\left\{-\frac{d^2 + h_0^2}{4\sigma_r^2}\right\} I_0\left(\frac{h_0 d}{2\sigma_r^2}\right),$$

Besides, $I_0(z)$ is zero Bessel function

$$I_0(z) = \frac{1}{2\pi} \int_0^{2\pi} \exp[z \cos(\theta - \theta_0)] d\theta$$

$$\sigma_r^2 = (1-r)\sigma_z^2$$

2. Under the circumstance of laying adjacent mines in the different voyages

When two adjacent mines are laid in the different voyage, its distribution in x direction or y direction is irrelevant, $r = 0$. As long as the plug into (1) can get the corresponding adjacent mine distance probability density function,

$$f_b(d) = \frac{d}{2\sigma_z^2} \exp\left\{-\frac{d^2 + h_0^2}{4\sigma_z^2}\right\} I_0\left(\frac{h_0 d}{2\sigma_z^2}\right)$$

III application example and analysis

Probability density function of adjacent mine space can be used to analyze probability density distribution under specific condition, to calculate the expected value and standard deviation of adjacent mine space as well as the probability of adjacent mine space exceeding specified value.

1. The probability density curve of adjacent mine space distribution

From the model built in this paper, the probability density of adjacent mine space distribution is closely related to the standard deviation σ_z of the mine position comprehensive distribution and the planning value of adjacent mine space d_0 . For the two mines laid in the same voyage, the probability density of adjacent mine space distribution is also related to the relevant coefficient r . Supposing $\sigma_z = 500$ m, $d_0 = 200$ m, $r = 0.5$, Fig. 2 provides the corresponding probability density curves of adjacent mine space distribution under two different conditions.

As is demonstrated in Fig. 1, if σ_z and d_0 are certain, the two mines laid in the same voyage or different voyages, the corresponding probability density curves of adjacent mine space are different. The former distribution density is relatively concentrate while the latter distribution curve varies comparatively gently, which may illustrate that the standard deviation of adjacent mine space distribution under the latter condition would be bigger.

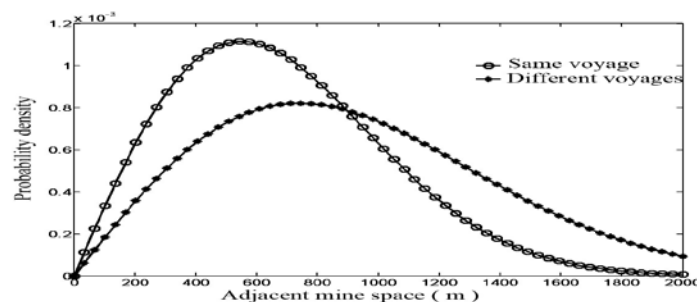


Fig1. The corresponding probability density curves of adjacent mine space

2. The expected value and standard deviation of adjacent mine space

By applying the probability density function of adjacent mine space and method of probability theory, it can be calculated as follows,

$$Ed = \int_0^{\infty} xf_d(x)dx ; Ed^2 = \int_0^{\infty} x^2 f_d(x)dx ; \sigma_d = \sqrt{Ed^2 - (Ed)^2} ;$$

$f_d(x)$ in (3) and (4) represents $f_t(d)$ or $f_b(d)$. From the above 3 formulas, the expected value Ed and standard deviation σ_d of adjacent mine space can be obtained.

Supposing $d_0 = 250$ m , $r = 0.3$, the variable curves showing how the expected value Ed and the standard deviation σ_d change with the standard deviation σ_z of mine position comprehensive distribution are given in Fig. 1 and Fig.2

From Fig. 2 and other calculation examples, under the influence arising from the comprehensive error of mine position distribution, the expected value of adjacent mine space is greater than the planning value d_0 and increasing while σ_z is increasing. The expected value of space increases more quickly especially for the two mines laid in different voyages.

From Fig. 3, when σ_z is comparatively small, the standard deviation σ_d of is slightly bigger than σ_z . Only σ_z increases, the two tend to be identical, and then the phenomenon of σ_d being smaller than σ_z . Meanwhile, the standard deviation of adjacent two mine space distribution laid in different voyages should be greater than that in the same voyage, as is compatible to the circumstance demonstrated in Fig. 2.

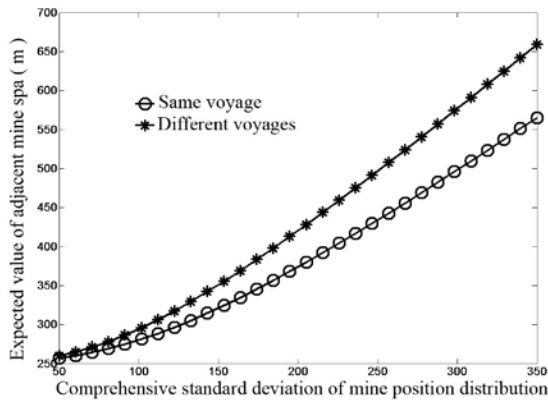


Fig.2 The expected value curves of adjacent mine

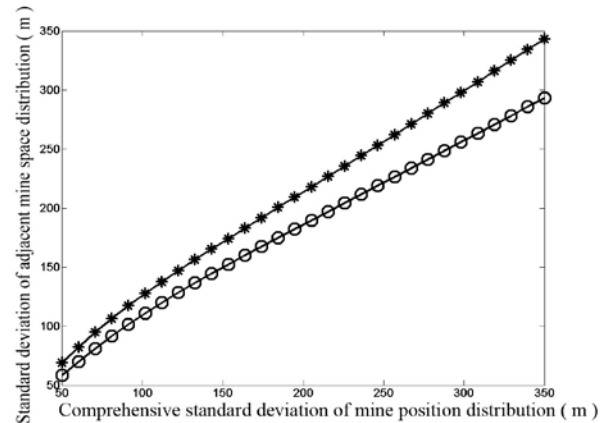


Fig.3 Standard deviation of adjacent mine

3. The probability of the adjacent mine space being greater than the specified value

The probability density function of adjacent mine space under certain conditions can be applied to calculate the probability $P(d)$ of adjacent mine space not smaller than d_{\min} or the specified value, as follows,

$$P(d) = 1 - \int_0^d f_d(x)dx \quad (6)$$

Supposing $\sigma_z = 200$ m, $r = 0.3$, $d_{\min} = 200$ m, the variable curve of the probability $P(d)$ of adjacent mine space being smaller than 200 m changing with its planning value.

The curve demonstrated in Fig. 4 reflects some regularities between the probability $P(d)$ and d_0 . Firstly, $P(d)$ increases while d_0 is increasing. Secondly, under the condition of d_0 being smaller than d_{\min} , the circumstance of actual adjacent mine space being greater than d_{\min} might still appear with lower probability. Thirdly, concerning the influencing degree of d_0 over $P(d)$ for the two conditions that two mines are respectively laid in the same voyage or different voyages, the

former should be bigger than the latter. The main reason for this is due to the relevance of mine position distribution.

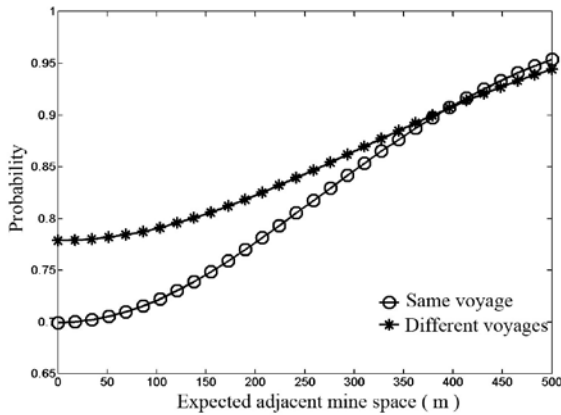


Fig. 4 The corresponding probability $P(d)$ curves of d_0

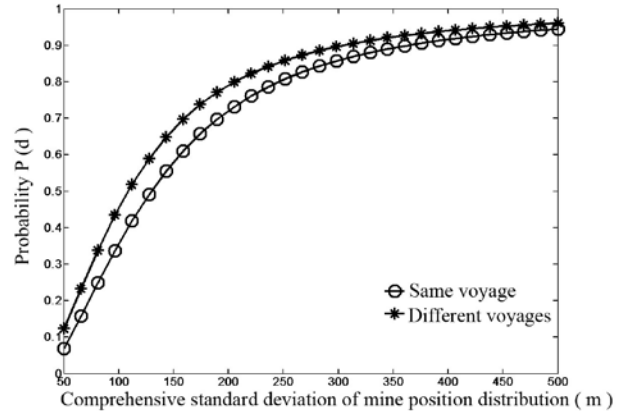


Fig.5 Regularities between the probability $P(d)$ and d_0 .

Supposing $d_0 = 100$ m, $r = 0.3$, $d_{\min} = 200$ m, the variable curve reflecting how the probability $P(d)$ of adjacent mine space being deeper than 200m changes with σ_z is demonstrated in Fig.5

As is demonstrated by the curve in Fig. 5, probability $P(d)$ will increase when σ_z increases; at primary stage it will increase comparatively quickly and then gently. This shows that probability $P(d)$ can be increased by making use of mine position comprehensive distribution error adding certain limitation to the increasing range.

IV Conclusion

In the mine laying operations, randomness of adjacent mine space exists under the influence of various factors. However, there are rules to follow. When conducting the actual operations, the model built in this paper can be used as the basis for optimizing strategies, decreasing the probability of adjacent mine space being smaller than minimum mine laying distance, this relevant conclusion will have major application value in the military scope.

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