# Anti-submarine Patrol Aircraft's Responding-antisubmarine Effectiveness Model Research by Using Magnetic Finder

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**Abstract:** According to the characteristic of responding-antisubmarine, this paper analyzes submarine maneuvering model and anti-submarine patrol route model combining anti-submarine patrol aircraft and using the method of magnetic finder searching under the condition of responding-antisubmarine, establishes the effectiveness model of anti-submarine patrol aircraft by using magnetic finder to check submarine so as to lay a foundation for magnetic finder optimization.

## I. INTRODUCTION

Responding-antisubmarine refers to combat operations that anti-submarine patrols at the specified position on standby and when get the submarine activity information, flies to the sea area finding sub and searches submarine. Its characteristic is that we know the position of the submarine at a certain moment before search. Antisubmarine patrol aircraft flight speed is fast, so when implement responding-antisubmarine, we can greatly reduce the delay time [1]. At the same time, magnetic finder is not affected by acoustic environmental impact, continuous search, high positioning accuracy, etc.; therefore anti-submarine patrol aircraft can immediately put search assignments after reaching the responding waters with a few minutes magnetic compensation flight by using magnetic finder. This narrows the scope of submarine exercise, thus improves the probability of submarine finding. So responding-antisubmarine is the main search campaign pattern of anti-submarine patrol aircraft using magnetic finder [2].

## **II. SUBMARINE MANEUVER MODEL**

## A. Submarine's initial position

For other forces such as surface ships, sonobuoy, there is a great deal of uncertainty for the first time contact's submarine location data. It is generally believed the submarine's initial position is of two-dimensional Gaussian distribution, and distribution center is the initial position of the point [3]. Set the initial position of distribution center in the origin of coordinates, and the expected value is 0. The initial position point  $(x_0, y_0)$  meets independent identical distribution in X, Y directions. If the mean square error is  $\sigma_x = \sigma_y = \sigma_0$ , the joint probability density function of the initial position point  $(x_0, y_0)$  is.

$$f(x_0, y_0) = \frac{1}{2\pi\sigma_0^2} \cdot \exp\left(-\frac{x_0^2 + y_0^2}{2\sigma_0^2}\right)$$
(1)

# B. The movement model of the submarine

When patrol arrives at sea and begins to search, set the position spread of the submarine movement to obey two-dimensional Gaussian distribution, the location point  $(x_1, y_1)$  after exercise meets the same independent identically distribution [4], the mean square error is  $\sigma_x = \sigma_y = \sigma$ . Set  $x_1 = R \cos \theta$ ,  $y_1 = R \sin \theta$ , the joint density function is:

$$\varphi(R,\theta) = \frac{R}{2\pi\sigma_1^2} \cdot \exp\left(-\frac{R^2}{\sigma_1^2}\right)$$
(2)

In it:  $R > 0, \theta \in [0, 2\pi]$ . R is the radius of spreading circle after submarine campaign.

To get  $\varphi(R,\theta)$ 's edge density function about  $R \pi \theta$ , the probability density function of  $R \pi \theta$  is:

$$\varphi(R) = \int_{0}^{2\pi} \varphi(R,\theta) d\theta = \frac{R}{\sigma_{1}^{2}} \cdot \exp\left(-\frac{R^{2}}{2\sigma_{1}^{2}}\right) \qquad R \in [0,+\infty)$$

$$\varphi(\theta) = \int_{0}^{+\infty} \varphi(R,\theta) dR = \frac{1}{2\pi} \qquad \theta \in [0,2\pi]$$
(3)

Set the speed of the submarine as  $V_q$ , then  $R = V_q \cdot t_0$ , plug  $\sigma_1 = \sigma_V \cdot t_0$  in Formula (3) and get:

$$\varphi(V_q t_0) = \frac{V_q t_0}{(\sigma_V \cdot t_0)^2} \cdot \exp\left(-\frac{(V_q t_0)}{2(\sigma_V \cdot t_0)^2}\right)$$
(5)

In it, t0 is a constant, so the probability density function of submarine speed  $V_q$  is:

$$f(V_q) = \int_0^{2\pi} \varphi(R,\theta) d\theta = \frac{V_q}{\sigma_V^2} \cdot \exp\left(-\frac{V_q^2}{2\sigma_V^2}\right)$$
(6)

Take the submarine's underwater economic speed  $V_{sc}$  as the mathematical expectation of submarine distribution function, obtain:

$$\sigma_{V} = \sqrt{\frac{2}{\pi}} \cdot V_{sc}$$
(7)

#### III. ANTI-SUBMARINE PATROL MACHINE SEARCH MODEL

A. Anti-submarine patrol machine search trajectories

While searching, as shown in Figure 1, when patrol arrives possible encounter point A0 in the hope to direct the motor to submarine position, the spread radius of the circle is:

 $R_0 = V_q \cdot t + r \quad (8)$ 

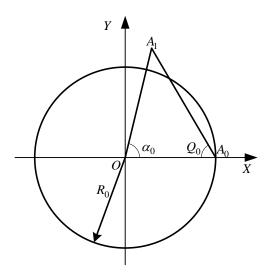


Figure 1 Meet condition analysis

In it: t is the time used for patrol aircraft flight fly to meet with submarine distribution circle; r is the position error of submarines.

When patrol get to point  $A_0$ , if there is no found of submarines, assume patrol searches toward line  $A_0A_1$ , then the time patrol travels from point  $A_0$  to point  $A_1$  is set to  $\Delta t_0$  [5], solving trigonometric function:

$$\Delta t_{0} = \frac{2 \cdot R_{0} \cdot (V_{q} + V_{fs} \cdot \cos X_{0})}{V_{fs}^{2} - V_{q}^{2}}$$
(9)  
$$\alpha = \frac{V_{fs} - V_{q} - V_{fs} \cdot V_{q} \cdot X - V_{fs} \cdot (X)}{V_{fs} + V_{q} + V_{fs} \cdot V_{q} \cdot X}$$
(10)

From formula (9), it can be seen that patrol have meeting point with constant speed direct submarines when  $V_q + V_{fs} \cdot \cos X_0 \ge 0$ , when  $X_0 = 180^\circ - \arccos(V_q/V_{fs})$ ,  $\Delta t_0 = 0$ , that is patrol maintains the fixed Angle of search  $180^\circ - \arccos(V_q/V_{fs})$ , and meets every possible location point of the target submarines, the search efficiency is the highest [6].

Set the meeting point's trajectory equation as y = f(x) and set differential equation:

$$\tan\left(180^{\circ} - \arccos\frac{V_q}{V_{fs}}\right) = \frac{\left(f'(x) - \frac{y}{x}\right)}{\left(1 + f'(x) \cdot \frac{y}{x}\right)}$$
(11)

Solute equation (11) and make the results the trajectory equation of polar form:

$$\rho(\varphi) = R_0 \cdot \exp(k\varphi) \quad (12)$$

 $k = \tan \left( \arcsin \frac{v_q}{V_{fs}} \right), \varphi \in [0, 2\pi], \text{ the collection of all possible meeting points of patrol aircraft and submarine is spiral line.}$ 

#### B. The feasibility analysis of spiral line search

From the above analysis, in spiral line search process, every bit on the spiral line is a possible meeting point, thus responding search efficiency with spiral line model is the highest, and the probability of finding the submarine is the largest. But there is a problem in the process of actual execution: while using spiral line search, the plane will be in a state of turning, and must make the pledge that we shall meet with the initial point angle with fixed value in the process of turning. This way is difficult to achieve in the process of actual execution [7]. Therefore, adopt the method of "straight instead of music", and replace the approximate spiral line with straight line.

The principle is that take losing contact points as poles, and divide spiral into several equal parts at a certain angle, so the attachment between spiral line points is linear search route.

#### IV. RESPONDING-ANTISUBMARINE FUNCTION CALCULATING MODEL

#### A. Submarine position coordinates calculating

According to submarine maneuvering model, the initial position point  $(x_{q0}, y_{q0})$  of submarine is located in the origin (0, 0). According to the course and speed probability density function, extract heading Hq0 and speed Vq, submarine will do constant speed direct movement. Then, at any time t, submarine coordinate is:

$$\begin{cases} x_{qi} = V_q \cdot t \cdot \cos H_{q0} \\ y_{qi} = V_q \cdot t \cdot \sin H_{q0} \end{cases}$$
(13)

#### B. Patrol location coordinates calculating

According to the analysis of section 3.2, adopt straight but curve method, and calculate the coordinates of each route. AB segment in Figure 2 is one of the paragraphs. We know point A's coordinate (x0, y0), get the coordinates of point B (x1, y1).

By cosine theorem:

 $AB^{2} = OA^{2} + OB^{2} - 2OA \cdot OB \cdot \cos e \quad (14)$ 

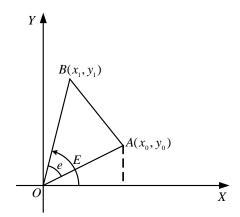


Figure 2 Patrol location coordinates calculating

At any time point, the coordinate is derived as follows:

 $\begin{cases} x_i = x_0 + k \cdot (x_1 - x_0) \\ y_i = y_0 + k \cdot (y_1 - y_0) \end{cases}$ (15)

C. Responding-antisubmarine detection probability decoding

Analyze section 4.1 and 4.2, get any time patrol coordinate  $(x_{fi}, y_{fi})$  and submarines coordinate  $(x_{qi}, y_{qi})$ , and calculate the distance between the two coordinates  $d_{fq}$ , get  $d_{fq} = \sqrt{(x_{fi} - x_{qi})^2 + (y_{fi} - y_{qi})^2}$ , and set the magnetic search finder's width is W. If  $d_{fq} \le \frac{W}{2}$ , we think that magnetic finder has found submarines.

#### V. CONCLUSION

Through the analysis of dynamic model of submarines and anti-submarine patrol search route model, the paper establishes a magnetic finder responding-antisubmarine efficiency calculating model. On the basis of this model, set up parameters and conditions and do simulation calculation. It is safe to get the effectiveness of the anti-submarine patrol aircraft with magnetic finder responding-antisubmarine under various factors.

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