Improved Tracking Differentiator and Fuzzy PID for Electric Loading System

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Abstract: Electric loading system can be used in aircrafts, vehicles and ships for loading test. It has the advantages of simple structure, high universality, low noise and fast tracking speed in loading test. But it has nonlinear factors and surplus torque. These are the important factors that affect dynamic quality of electric loading system. To solve these problems, the paper constructed the mathematical model of the electric loading system. Improved tracking differentiator and fuzzy PID controller were designed. Then a simulation experiment was carried out. It is proved that this method can greatly eliminate the influence of nonlinear factors and surplus torque, and enhance precision of dynamic loading test.

Keywords: electric loading system, improved tracking differentiator, fuzzy PID controller

1 Introduction

Electric loading system can reasonably simulate the load of object under laboratory conditions. The performance of electric loading system is directly related to the reliability of the loading test [1].

Engineering application practice shows that the control problems of the electric loading system are mainly manifested in nonlinear characteristics of the system and the inhibition of surplus torque [2,3]. There are many nonlinear factors in electric loading system, such as clearance, the change of loading motor’s armature resistance and armature inductance over time, the damping coefficient of connecting mechanism and the change of the drive’s amplification coefficient along with working condition etc [4]. When electric loading system apply load for the loaded object, it need to follow the movement of the loaded object at the same time. The movement of the loaded object for loading system is a kind of great disturbance, and the torque caused by the disturbance is called surplus torque. It is inevitable as long as the movement of the loaded object [5].

To solve above problems, improved tracking differentiator and fuzzy PID controller were designed based on feed-forward compensation of the loaded object’s angle, and simulation experiment was carried out.

2 Mathematical Model of Electric Loading System

Electric loading system is mainly composed of three parts: loading motor, torque sensor and coupling flange, loaded object. Mathematical model of electric loading system is included as follows: mathematical model of the loading motor, mathematical model of the torque sensor, mathematical model of the loaded object’s system.

Loading motor is used by permanent magnet synchronous motor (PMSM). Voltage equation, electromagnetic torque equation and mechanical dynamics equation of PMSM were calculated so that the state space equation in the d-q coordinate system was gained [6]. The control method of $I_d = 0$ was adopted. The model of the loading motor was gained as Fig. 1 according to the state space equation.
Among them, $U_q$ is the input voltage, $T_L$ is the load torque, $R_s$ is the stator resistance. $L$ is the winding self-induction. $p_m$ is the number of the motor’s pole, $M$ is the winding inductance. $I_f$ is the field current. $J$ is the rotational inertia of the motor. $B$ is the friction coefficient of angular velocity. $\theta$ is the angle of the rotor.

The torque sensor is approximately equivalent to a proportional component:

$$T_{out} = T_A \cdot (\theta_1 - \theta_2) .$$  \hspace{1cm} (1)

Among them, $T_{out}$ is the actual torque, $T_A$ is the coefficient of torque sensor, $\theta_1$ is the angle of the loading motor, $\theta_2$ is the angle of the loaded object.

The system of the loaded object used permanent magnet brushless DC motor as actuator. The model of loaded object is shown in Fig. 2.

Among them, $U(s)$ is the input voltage, $E(s)$ is the feedback voltage, $e_{K}$ is the coefficient of counter electromotive force, $L_a$ is the inductance of armature circuit, $R_a$ is the total resistance of armature circuit, $K_p$ is armature current, $K_t$ is the coefficient of motor torque, $T_e$ is the electromagnetic torque, $T_i$ is the load torque, $J$ is the rotational inertia of the motor, $f$ is the coefficient of motor friction, $\omega(s)$ is the velocity of the motor’s angle.

3 The Design of the Improved Tracking Differentiator and Fuzzy PID

The conventional PID is suitable for system which is easy to realize static loading stable, but there are many nonlinear factors and surplus torque in the dynamic loading of the electric loading system. Therefore, the conventional PID is incompetent [7]. But the high precision of dynamic loading test and strong robustness are usually needed. Improved tracking differentiator (ITD) can weaken nonlinear factors and enhance precision of loading test. Fuzzy PID has the advantage of strong robustness, so we combined tracking differentiator and Fuzzy PID controller to solve these
The structure of ITD and fuzzy PID controller is shown in Fig. 3. It mainly composes of an improved tracking differentiator and a fuzzy PID controller. Tracking error \( E \) and the error change \( EC \) is the output of improved tracking differentiator. The controller parameters are decided online through fuzzy reasoning by the fuzzy PID controller based on \( E \) and \( EC \).

![Fig. 3 ITD and fuzzy PID controller](image)

### 3.1 Improved Tracking Differentiator (ITD)

Although the discrete tracking differentiator (TD) has good performance, no flutter phenomenon, but the algorithm is a little complex. The improved tracking differentiator (ITD) not only has no flutter, good performance, but also has simple algorithm. The specific form is as follows [9,10]:

\[
\begin{align*}
    x_1 &= x_2 \\
    x_2 &= -\alpha_1 R^2[(\beta(x_1 - v)^{\nu/q}) + x_1 - v] - \alpha_2 R^2[(\beta x_2 / R)^{\nu/q} - x_2 / R]
\end{align*}
\]  

(2)

Among them, \( \alpha_1 > 0, \quad \alpha_2 > 0, \quad \beta \geq 1(\beta \) is a constant), \( R > 0, \quad p > q > 0, \quad p \) and \( q \) are odd, \( v(t) \) is the arbitrary input signal.

### 3.2 The design of fuzzy PID controller

The realization of the self-adaptive PID parameters is to find out fuzzy relationship among three parameters of PID, tracking error \( E \) and the error change \( EC \). The fuzzy PID controller constantly tests \( E \) and \( EC \) in the process of running. It can modify three parameters online according to the principle of fuzzy control, and it can satisfy different \( E \) and \( EC \) for the different requirements of PID controller parameters [11]. The changes of \( K_p, K_i \) and \( K_d \) to the influence on the system’s quality is related to system's stability, stable speed, overshoot and steady state accuracy and so on. The role of the three parameters in different time and the interconnected relationship between each other must be considered on setting PID parameters [12].

The work of this paper focuses on the design of fuzzy controller. Its input variable is \( E \) and \( EC \), and its output variable is \( K_p, K_i \) and \( K_d \). The steps of algorithm design are as follows: Firstly, blur \( E, EC, K_p, K_i \) and \( K_d \). Secondly, determine their respective membership degree of fuzzy subsets. Thirdly, use fuzzy calibration model of \( K_p, K_i \) and \( K_d \) to express the process of parameter calibration. Finally, use fuzzy reasoning to calculate fuzzy matrix correction table of \( K_p, K_i \) and \( K_d \). Specific steps are as follows:

1. The membership functions of the variables
   All variables were defined on the domain of fuzzy set as follows:
   
   \[
   E, EC, K_p, K_i, K_d = \{-3, -2, -1, 0, 1, 2, 3\}. \tag{3}
   \]

   And set up its fuzzy subset:
   
   \[
   E, EC, K_p, K_i, K_d = \{NB, NM, NS, ZO, PS, PM, PB\}. \tag{4}
   \]

   Take membership functions of \( E, EC, K_p, K_i \) and \( K_d \) for the triangular function. The fuzzy
variables of the error E and error change rate EC is shown in table 1:

<table>
<thead>
<tr>
<th>E/EC</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>PB</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>PM</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>PS</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ZO</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NS</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NB</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The fuzzy variables of $K_p$, $K_i$, and $K_d$ is shown in table 2:

<table>
<thead>
<tr>
<th>$K_p$ / $K_i$ / $K_d$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>PB</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>PM</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>PS</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ZO</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NS</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NB</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(2) Set up the table of fuzzy control rules

Parameter setting rules is the core of the controller, the main basis of fuzzy control rules is the principle of self-adaptive PID parameter. Table 3 was compiled as follows:

(3) Fuzzy reasoning algorithm and the decision matrix

After obtaining the adjustment rules of the $K_p$, $K_i$, and $K_d$, three control tables of the $K_p$, $K_i$, and $K_d$ would be calculated according to the fuzzy theory. Because the process of calculating three control tables are exactly the same, we just calculated control table of $K_p$ as follows:

(a) Synthesis reasoning algorithm
If tracking error and the error change respectively take E and EC respectively, control changes of fuzzy controller is given by the fuzzy reasoning rules:

$$K_p = (E \times EC) \circ R.$$  \hspace{1cm} (5)

(b) To calculate the fuzzy relation R
\[ R_k = (E_i \times EC_j)^T \times K_{pk}. \]  
(6)

\[ R = R_{\text{YR}i}Y_{\text{AYR}j}. \]  
(7)

Among them, \( i=1,2,3\cdots L \cdots m; j=1,2,3\cdots L \cdots n; k=1,2,3\cdots L \cdots m \times n; \)

Following formula is obtained by the control rule table and the fuzzy rule table:

\[
R = R_{\text{YR}i}Y_{\text{AYR}j} = \begin{pmatrix}
0 & 0 & 0 & 0.2 & 0.6 & 1 \\
0 & 0 & 0 & 0.2 & 0.6 & 1 \\
0 & 0 & 0 & 0.5 & 0.6 & 1 \\
1 & 0.6 & 0.2 & 0 & 0 & 0 \\
1 & 0.6 & 0.2 & 0 & 0 & 0 \\
1 & 0.6 & 0.2 & 0 & 0 & 0
\end{pmatrix}. \]  
(8)

(c) The fuzzy sets of PID control parameter \( K_p \)

Because \( K_p = (E \times EC) \circ R \), then

\[
K_{p1} = \begin{bmatrix}
0 & 0 & 0 & 0.2 & 0.5 & 1 \\
\end{bmatrix}
\]

\[
K_{p2} = \begin{bmatrix}
0 & 0 & 0 & 0.5 & 0.6 & 1 \\
\end{bmatrix}
\]

\[
\vdots
\]

\[
K_{p48} = \begin{bmatrix}
1 & 0.6 & 0.5 & 0.2 & 0.5 & 0 & 0 \\
\end{bmatrix}
\]

\[
K_{p49} = \begin{bmatrix}
1 & 0.6 & 0.5 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

At this point, the fuzzy subset of \( K_p \) has obtained, and then we made a fuzzy decision.

Through the above fuzzy reasoning, the fuzzy set was obtained, but it must use a certain value to control the controlled object in practice. The method of maximum membership degree was took for fuzzy judgment according to the actual situation. Finally the table 4 was got.

Table 4 The table of the fuzzy adjustment about \( K_p / K_i / K_d \)

<table>
<thead>
<tr>
<th>( K_p ) / ( K_i ) / ( K_d )</th>
<th>( E )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>3/1/-3</td>
<td>3/1/-2</td>
<td>3/-1/-2</td>
<td>3/-1/-1</td>
<td>2/-1/-1</td>
<td>1/-1/-1</td>
<td>0/1/0</td>
<td>0/1/0</td>
</tr>
<tr>
<td>-2</td>
<td>3/1/-3</td>
<td>3/-3/-2</td>
<td>3/-1/-1</td>
<td>1.5/-1/-1</td>
<td>0/1/0</td>
<td>0/1/0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>2/0/2</td>
<td>2/-2/2</td>
<td>2/-2/-1</td>
<td>1/-2/0</td>
<td>0/-2/0</td>
<td>-1/0/-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>3/0/1</td>
<td>1/-3/0</td>
<td>-1/-3/0</td>
<td>-2/0/0</td>
<td>-2/0/0</td>
<td>-2/0/0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3/1/0</td>
<td>0/-3/0</td>
<td>-3/0/0</td>
<td>-3/-3/0</td>
<td>-3/-3/0</td>
<td>-3/3/0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0/-3/0</td>
<td>-1/-3/0</td>
<td>-3/0/0</td>
<td>-3/-3/0</td>
<td>-3/-3/0</td>
<td>-3/-3/0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0/-3/0</td>
<td>-1/-3/0</td>
<td>-3/0/0</td>
<td>-3/-3/0</td>
<td>-3/-3/0</td>
<td>-3/-3/0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(d) Control algorithm of the fuzzy PID

Define parameter adjustment formula of \( K_p, K_i \) and \( K_d \) as follows:

\[
\begin{cases}
K_p = K_p + \{E, EC\}K_p = K_p + \cap K_p \\
K_i = K_i + \{E, EC\}K_i = K_i + \cap K_i \\
K_d = K_d + \{E, EC\}K_d = K_d + \cap K_d
\end{cases}
\]  
(10)

Through constantly detecting response value of system online, and to calculate the deviation and the deviation change rate in real time, then blur them to obtain \( E \) and \( EC \), three parameters adjustment of the \( K_p, K_i \) and \( K_d \) can be obtained by querying fuzzy adjustment matrix, the adjustment of controller parameters is completed.
4 simulation experiment of electric loading system

The transfer function of the system’s output torque can be expressed as follows:
\[ T_{out}(s) = G_1(s) \cdot T_{ref}(s) + G_2(s) \cdot \theta(s) \]  \hspace{1cm} (11)

Among them, \( T_{out} \) is the actual torque of the system, \( T_{ref} \) is the expected torque, \( \theta \) is the angle of loaded object system. When \( T_{ref}(s) = 0 \), the change of \( T_{out} \) caused by \( \theta(s) \) is called the surplus torque.

The angle feed-forward compensation of the loaded object was introduced based on compensation strategy of structure invariance principle[13]. Then, PID controller in the torque loop was adopted. Fig. 4 is shown as follows.

According to dynamic performance index, the electric loading system should meet the 90% or more about inhibition of the surplus torque within 2 Hz. The loaded object did the sinusoidal movement of 5 degree and 3 Hz frequency, the electric loading system input sinusoidal signal of 10Nm amplitude, 2Hz frequency. The result of the simulation is shown in the Fig.5.

It can see be seen in the Fig.5, the electric loading system is not stable before 0.2s because of nonlinear factors, and the torque error is about 4 Nm after the system is stable. It is too difficult to meet the performance index. To solve this problem, improved tracking differentiator and fuzzy PID controller in the torque loop was adopted. Fig.6 is shown as follows.
The simulation experiment was carried out like before. The loaded object did the sinusoidal movement of 5 degree and 3 Hz frequency, the electric loading system input sinusoidal signal of 10Nm amplitude, 2Hz frequency. The result of the simulation is shown in the Fig.7. The electric loading system is stable after 0.08s. The influence of nonlinear factors is greatly eliminated, and the torque error is about 0.3 Nm after the system is stable.

Fig.7 The expected torque and the actual torque of improved tracking differentiator and fuzzy PID in the torque loop

5 conclusion

It can be seen from the result of the simulation experiment, with the introduction of angle feed-forward compensation of the loaded object, improved differential tracker and fuzzy PID controller in the torque loop, the actual torque can better track the expected torque. It can eliminate more than 95% surplus torque and greatly reduce the influence of nonlinear factors. It indeed can enhance precision of dynamic loading test and meet the dynamic performance index.

References


