2FSK-LFM Compound Signal Parameter Estimation Based on Joint FRFT-ML Method

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***Abstract*—This paper proposes a novel method to estimate 2FSK-LFM compound signal based on joint FRFT-ML. The representation of the compound signal in FRFT domain is derived and the chirp rate and the code length of the compound signal are estimated. Joint with ML method, a high accurate local search approach is proposed and the carrier frequencies of the compound signal are estimated. Simulation results show when SNR>5dB, our algorithm can achieve a high accurate performance and distinguish the different carrier frequencies of the compound signal.**

Keywords-2FSK-LFM compound signal; parameter estimation; fractional fourier transform; maximum likelihood

I. Introduction

FSK (frequency shift keying) signal and LFM (linear frequency modulation) signal are two kinds of frequency modulation signals which have been used in communication, radar and detection areas and their estimation methods have been discussed by many researchers [1-2].

In order to improve the anti-interception capability or the detection performance, many kinds of FSK-LFM compound signals which are LPI (low probability interception) signals have been employed in communication and radar fields [3-5]. Among them, 2FSK-LFM signal can overcome the range or velocity measurement flaws of MFSK and LFMCW (LFM continuous wave) signals and can achieve high accurate range and velocity measurements [5]. For 2FSK-LFM signal, the common LFMCW signal estimation method [6] cannot handle it.

Therefore, this paper proposes a joint FRFT-ML (fractional Fourier transform-maximum likelihood) approach to estimate the main parameters of this type of signal. The representation of the compound signal in FRFT domain using Ozaktas algorithm is derived and a high accurate local search ML approach which can distinguish small frequency shift is proposed. Simulation results show that the proposed algorithm works well when SNR>5.

This paper is organized as follows. In section II, the model of 2FSK-LFM signal is given. In section III, the chirp rate and the code length estimations are proposed. In section IV, the carrier frequency estimation is discussed. In section V the simulation analysis is given, and section VI is the conclusion.

II. Signal Model

2FSK-LFM compound signal combines 2FSK signal and LFMCW signal together. For each LFM signal, its carrier frequency is modulated by each carrier frequency of 2FSK alternately, which means the carrier frequencies of 2FSK signal are modulated alternately. The noised signal model can be expressed as

$$x(t)=\sum_{m=1}^{M}s_m(t)+n(t) \tag{1}$$

where $s_m(t)=A_0e^{j\pi\mu_0[t+\tau_0-(m-1)T_0]^2+2j\pi f_m[t+\tau_0-(m-1)T_0]+j\theta_0}$, $t\in[0,T)$, $f_m=\begin{cases}f_1,\mathrm{mod}(m,2)=1\\ f_2,\mathrm{mod}(m,2)=0\end{cases}$.

In (1), $n(t)$ is Gaussian white noise and T is the signal total length. $\sum_{m=1}^{M}s_m(t)$ is the signal part and it can be regarded as a sum of M LFM signals which have alternative carrier frequencies. A_0 and θ_0 are amplitude and initial phase, respectively. Then, assume $A=A_0e^{j\theta_0}$. μ_0 denotes chirp rate and τ_0 is time offset. T_0 is code length which equals the sweep length of LFMCW signal. f_m is carrier frequency. When m is odd, $f_m=f_1$ while when m equals even, $f_m=f_2$. In order to achieve high accurate range and velocity measurements simultaneously, the frequency shift between f_1 and f_2 is small [5].

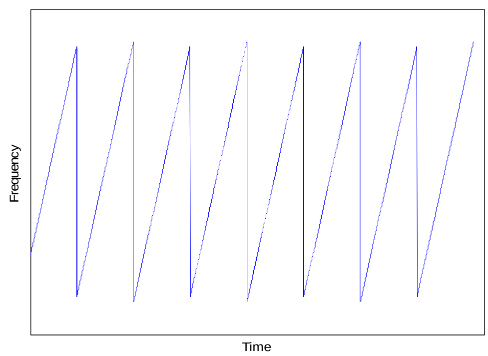

FIGURE I. 2FSK-LFM SIGNAL TIME-FREQUENCY REPRESENTATION

Figure 1 shows the time-frequency representation of 2FSK-LFM signal without noise. The gap of adjacent LFM signals carrier frequencies is narrow. Our objective is to estimate the chirp rate, code length and two carrier frequencies of the signal.

III. CHIRP RATE AND CODE LENGTH ESTIMATION

Because of Heisenberg uncertainty principle, windowed time-frequency analysis cannot give accurate estimations of the frequencies with small gap and code length simultaneously. However, FRFT can overcome this flaw since it is an overall time-frequency analysis and its multi-resolution property is suitable for the signal with LFM part.

The definition of FRFT is

$$S_\alpha(u) = \begin{cases} B_\alpha \int_{-\infty}^{\infty} e^{j\pi(u^2\cot\alpha - 2ut\csc\alpha + t^2\cot\alpha)} s(t)dt & \alpha \neq n\pi \\ s(t) & \alpha = 2n\pi \\ s(-t) & \alpha = (2n+1)\pi \end{cases} \tag{2}$$

where α is the rotation angle, $B_\alpha = \sqrt{1 - \mathrm{j}\cot\alpha}$ and n is an integer. Ozaktas algorithm is widely used to realize FRFT [7]. However, it requires that the signal length should equal to the observation time length and the observation time should be symmetric around the 0 time. Unfortunately, our signal cannot satisfy these requirements and the representation of 2FSK-LFM signal in FRFT domain using Ozaktas algorithm should be derived first.

Consider one LFM signal in (1), we have

$$s_m(t) = Ae^{j\pi\mu_0[t+\tau_0-(m-1)T_0]^2 + 2j\pi f_m[t+\tau_0-(m-1)T_0]} \left\{\varepsilon\left[t+\tau_0-(m-1)T_0\right] - \varepsilon\left[t+\tau_0-mT_0\right]\right\} \quad t\in\left[0,T\right] \tag{3}$$

Make (3) be symmetric around the 0 time and yield

$$s_m(t-t') = Ae^{j\pi\mu_0[t-t'+\tau_0-(m-1)T_0]^2 + 2j\pi f'_m[t-t'+\tau_0-(m-1)T_0]} \left[\varepsilon(t+T_0/2) - \varepsilon(t-T_0/2)\right] \quad t\in\left[\tau_0 - mT_0 + T_0/2, \tau_0 - mT_0 + T_0/2 + T\right] \tag{4}$$

where $f'_m = f_m + \mu_0 T/2$ and $t' = \tau_0 - mT_0 + T_0/2 + T/2$. Substitute (4) into (2) and consider the time domain translation property of FRFT [8], we can obtain the FRFT of (3)

$$S_m(u) = C\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{j\pi(\mu+\cot\alpha)t^2 + 2j\pi[\mu_0\tau_0 - \mu_0 t' - \mu_0(m-1)T_0 + f'_m - (u+t'\cos\alpha)\csc\alpha]}dt \tag{5}$$

where $C = AB_\alpha e^{j\pi(u+t'\cos\alpha)^2\cot\alpha + j\pi\mu[\tau_0 - t' - (m-1)T_0]^2 + 2j\pi f'_m[\tau_0 - t' - (m-1)T_0]}$.

when $\mu_0 = -\cot\alpha_0$, (5) can be written as

$$\left|S_m(u)\right| = \left|CT_0 Sa\left\{\pi\left[\mu_0\tau_0 - \mu_0(m-1)T_0 + f'_m - u\csc\alpha_0\right]\right\}\right| \tag{6}$$

When $u = \left[\mu_0\tau_0 - \mu_0(m-1)T_0 + f'_m\right]/\csc\alpha_0$, (6) will achieve its maximum. Consider dimensional normalization, assume the sampling frequency is f_s and the coordinate of FRFT domain corresponding to the peak of (6) is u_m, we have

$$u_m = \sqrt{T/f_s}(\mu_0\tau_0 - \mu_0(m-1)T_0 + f_m + \mu_0 T/2)/\csc\alpha_0 \tag{7}$$

From (6) and (7), the compound signal will generate several peaks in FRFT domain whose number equals to the number of LFM signals in the compound signal. The coordinate of $m-1^{th}$ peak in FRFT domain is greater than the m^{th} one and these coordinates will be affected by the code length and the carrier frequency according to (7). From the above, 2FSK-LFM signal parameter estimation based on Ozaktas algorithm can be written as

$$\begin{cases} \hat{\mu}_0 = -\frac{T}{f_s}\cot\alpha_0 \\ \hat{f}_m = u_m\csc\alpha_0\sqrt{\frac{f_s}{T}} - \hat{\mu}_0\tau_0 + \hat{\mu}_0(m-1)T_0 - \hat{\mu}_0\frac{T}{2} \\ \hat{f}_{m-1} = u_{m-1}\csc\alpha_0\sqrt{\frac{f_s}{T}} - \hat{\mu}_0\tau_0 + \hat{\mu}_0(m-2)T_0 - \hat{\mu}_0\frac{T}{2} \end{cases} \tag{8}$$

where $u_m < u_{m-1}$ .From (6), the peak amplitude will be affected by the corresponding LFM signal time length. Thus, to guarantee accuracy, the first and the last peaks which sometimes mean the smaller peak should not be used when we calculate (8). Based on the carrier frequency property $\hat{f}_m = \hat{f}_{m-2}$ and (8), we can obtain the code length estimation

$$\hat{T}_0 = \csc\alpha\sqrt{f_s/T}(u_{m-2} - u_m)/2\hat{\mu}_0 \tag{9}$$

Substitute (9) into (8) and the unknown parameters in (8) become $\hat{f}_m$, $\hat{f}_{m-1}$ and τ_0. When τ_0 is known [3], other parameters can be estimated by (8). However, for the non-cooperative situation, τ_0 is always unknown and (8) becomes an underdetermined system of equation. To deal this problem, next section will give an approach.

IV. CARRIER FREQUENCY ESTIMATION

For the non-cooperative situation, Section III has already estimated the chirp rate $\hat{\mu}_0$ and code length $\hat{T}_0$ of compound signal using (8) and (9). Normally, the frequency shift of adjacent carrier frequency is small. For the purpose of improving estimation performance, de-chirp method is employed and signal data within two code length is selected.

Structure LFM signal as

$$s_{rec}(t) = e^{-j\pi\hat{\mu}_0 t^2}, \; t \in [0, 2\hat{T}_0) \tag{10}$$

Consider the compound signal data in the first estimated code length. If $\tau_0 > 0$, these data exist two LFM signals. For the first and second LFM signal, assume their carrier frequencies as f_1 and f_2, respectively.

$$x_{1Ori}(t) = Ae^{j\pi\mu_0(t+\tau_0)^2 + 2j\pi f_1(t+\tau_0)} + n(t), \; t \in [0, \hat{T}_0 - \tau_0) \tag{11}$$

$$x_{2Ori}(t) = Ae^{j\pi\mu_0(t+\tau_0-\hat{T}_0)^2 + 2j\pi f_2(t+\tau_0-\hat{T}_0)} + n(t), \; t \in [\hat{T}_0 - \tau_0, \hat{T}_0) \tag{12}$$

Use (10) and the same length data of (1) to do de-chirp process. Here, we consider the data in first code length and when $\hat{\mu}_0 \to \mu_0$, we have

$$x_1(t) = x_{1Ori}(t)s_{rec}(t) = A_1 e^{2j\pi f_1' t + 2j\pi f_1 \tau_0} + n'(t), \; t \in [0, \hat{T}_0 - \tau_0) \tag{13}$$

$$x_2(t) = x_{2Ori}(t)s_{rec}(t) = A_2 e^{2j\pi f_2' t + 2j\pi\mu_0(\tau_0 - T_0)^2 + 2j\pi f_2(\tau_0 - T_0)} + n'(t)$$
$$t \in [\hat{T}_0 - \tau_0, \hat{T}_0) \tag{14}$$

where $f_1' = f_1 + \mu_0\tau_0$, $f_2' = f_2 + \hat{\mu}_0\tau_0 - \hat{\mu}_0 T_0$, $A_1 = Ae^{2j\pi\mu_0\tau_0^2}$, $A_2 = Ae^{-j\pi\mu_0(\tau_0 - T_0)^2}$. At this time, the frequency shift between f_1' and f_2' is increased as compared with the original frequency shift.

Assume the noised signal constructed by (13) and (14) is

$$x_{T_0}(t) = [x_1(t), x_2(t)] \tag{15}$$

whose data points number is N. Rewrite $x_{T_0}(t)$ as $\mathbf{x} = \left[x_{T_0}(1), x_{T_0}(2), \ldots, x_{T_0}(N)\right]^T$ and assume the frequency hopping point as K. Thus, the first LFM signal data length is K and the τ_0 estimation has been transformed to the K estimation.

Combine (13) and (14), we have

$$\mathbf{x} = \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} = \begin{bmatrix} A_1\mathbf{s_1} \\ A_2\mathbf{s_2} \end{bmatrix} + \begin{bmatrix} \mathbf{n_1} \\ \mathbf{n_2} \end{bmatrix} \tag{16}$$

where $\mathbf{s_1} = \left[e^{2j\pi f_1'(N-K)T_s}, \ldots, e^{2j\pi f_1'(N-1)T_s}\right]^T$, $\mathbf{s_2} = \left[1, \ldots, e^{2j\pi f_2'(N-K-1)T_s}\right]^T$, $\mathbf{n_1} = \left[n'(1), \ldots, n'(K)\right]^T$, $\mathbf{n_2} = \left[n'(K+1), \ldots, n'(N)\right]^T$ and T_s is the sampling interval. In (16), $\mathbf{n_1}$ and $\mathbf{n_2}$ are Gaussian white noise.

The likelihood function of (16) is

$$L(A_1, A_2, f_1', f_2', K) = \frac{1}{(\sqrt{2\pi}\sigma)^M} e^{-\frac{1}{2\sigma^2}\|\mathbf{x}-\mathbf{s}\|^2}$$
$$= \frac{1}{(\sqrt{2\pi}\sigma)^M} e^{-\frac{1}{2\sigma^2}(\|\mathbf{x_1} - A_1\mathbf{s_1}\|^2 + \|\mathbf{x_2} - A_2\mathbf{s_2}\|^2)} \tag{17}$$

To estimate the parameters in (17) is equivalent to minimizing the objective function $\|\mathbf{x_1} - A_1\mathbf{s_1}\|^2 + \|\mathbf{x_2} - A_2\mathbf{s_2}\|^2$. Assume $\varphi_1(A_1, f_1', K) = \|\mathbf{x_1} - A_1\mathbf{s_1}\|^2$ and $\varphi_2(A_2, f_2', K) = \|\mathbf{x}_2 - A_2\mathbf{s}_2\|^2$. For $\varphi_1(A_1, f_1', K)$, we have

$$\partial\varphi_1 / \partial f_1' = -jT_s(\mathbf{D_1 s_1 s_1^H} - \mathbf{s_1 s_1^H D_1}) / K \tag{18}$$

where $\mathbf{D_1} = diag(N-K, \ldots, N-1)$. When (18) equals to 0, it can be written as $\partial\|\mathbf{s_1^H x_1}\|^2 / (K\partial f_1') = 0$. Thus, the estimation of f_1' is

$$\hat{f}_1' = \arg_{f_1'}\left[\max(\|\mathbf{s_1^H x_1}\|^2 / K)\right] \tag{19}$$

Similarly, the estimation of f_2' is

$$\hat{f}_2' = \arg_{f_2'}\left\{\max\left[\|\mathbf{s_2^H x_2}\|^2 / (N-K)\right]\right\} \tag{20}$$

Minimizing objective function is equivalent to maximizing $\varphi(\hat{f}_1', \hat{f}_2') = \bar{\varphi}_1(\hat{f}_1') + \bar{\varphi}_2(\hat{f}_2')$, where $\bar{\varphi}_1(\hat{f}_1') = \|\mathbf{s_1^H x_1}\|^2 / K$ and $\bar{\varphi}_2(\hat{f}_2') = \|\mathbf{s_2^H x_2}\|^2 / (N-K)$. Therefore, K estimation is

$$\hat{K} = \arg_K\left\{\max\left[\varphi(\hat{f}_1', \hat{f}_2')\right]\right\} \tag{21}$$

According to (19), (20) and (21), an iterative approach can estimate the carrier frequencies and frequency hopping transition point. Normally, global search will be used in the greedy method and its computational load is heavy. Furthermore, when K is closed to the edge of selected data window (the selected data with one code length), ML method cannot estimate the two carrier frequencies [9].

To overcome the above flaws, we give a new search method. Calculate Fourier transform of (15) and yield

$$X_{T_0}(f) = A_1'(T_0-\tau_0)Sa\left[\pi(f_1'-f)\right]e^{j\pi(f_1'-f)(T_0-\tau_0)} + A_2'\tau_0 Sa\left[\pi(f_2'-f)\right]e^{j\pi(f_2'-f)(T_0-\frac{\tau_0}{2})} + N'(f) \tag{22}$$

where $A_1' = A_1 e^{2j\pi f_1 \tau_0}$ and $A_2' = A_2 e^{2j\pi\mu_0(\tau_0-T_0)^2+2j\pi f_2(\tau_0-T_0)}$. When $f \approx f_1'$ and $f \approx f_2'$, $\left|X_{T_0}(f_1')\right| \approx A(T_0-\tau_0)$ and $\left|X_{T_0}(f_2')\right| \approx A\tau_0$, which means $\left|X_{T_0}(f)\right|$ will bring two peaks. Because the de-chirp process, the two peaks will not be overlapped. The amplitudes of the two peaks will be affected by K. When K is at the middle of the data window, ML estimation will meet its optimal results. According to the amplitudes of the two peaks in Fourier domain, we can shift the data window and let K be close to the middle of the window.

Set $R_1 = \left|X_{T_0}(f_1')\right| / \left|X_{T_0}(f_2')\right|$ and $R_2 = \left|X_{T_0}(f_2')\right| / \left|X_{T_0}(f_1')\right|$. When $R_1 \ge 1$ and $R_2 \le 1$, $\tau_0 \le T_0/2$. Otherwise, when $R_1 < 1$ and $R_2 > 1$, $\tau_0 > T_0/2$. Consider the noise, the shift principle and coarse frequency estimation are as follows.

(a). When $R_2 \le 0.1$, the data window should be shifted backward $N/2$ points and estimate the two carrier frequencies in Fourier domain. When $0.1 < R_2 < 0.5$, the data window should be shifted backward $N/4$ points and estimate the two carrier frequencies.

(b). When $R_1 \le 0.25$, the first LFM signal data in the data window is small. Shift the data window backward $3N/4$ points and the window will cover the second and the third LFM signals. This is the reason (10) covers two code length. After shift process, the estimated frequencies become $f_1' \approx f_1 + \hat{\mu}_0\tau_0 - 2T_0\hat{\mu}_0$ and $f_2' \approx f_2 + \hat{\mu}_0\tau_0 - T_0\hat{\mu}_0$.

(c). If R_1 and R_2 are not satisfied the above condition, there will be no shift process.

After shift process and coarse estimation, the estimated carrier frequencies can be written as $\hat{f}_1^{(0)}$ and $\hat{f}_2^{(0)}$. Substitute them into (21) and yield $\hat{K}^{(0)}$. Now, $\hat{K}^{(0)}$ is not at the edge of data window and the iterative convergence rate is enhanced. Furthermore, instead of global search, the ML frequency search can be done near the coarse estimated carrier frequencies, which reduces the computational load.

Fix $K = \hat{K}^{(0)}$ and we have $\hat{f}_1^{(1)} = \arg_{f_1'}\left[\max(\left\|\mathbf{s}_1^{\mathbf{H}}\mathbf{x}_1\right\|^2 / \hat{K}^{(0)})\right]$ and $\hat{f}_2^{(1)} = \arg_{f_2'}\left[\max(\left\|\mathbf{s}_2^{\mathbf{H}}\mathbf{x}_2\right\|^2 / N - \hat{K}^{(0)})\right]$ from (19) and (20). Then, according to (21), $\hat{K}^{(1)} = \arg_K\left\{\max\left[\varphi(\hat{f}_1^{(1)}, \hat{f}_2^{(1)})\right]\right\}$. Repeat the above process. When $\hat{K}^{(i)} = \hat{K}^{(i-1)}$, terminate the iterative process and the final estimation results are $\hat{f}_1^{(i)}$, $\hat{f}_2^{(i)}$ and $\hat{K}^{(i)}$. Finally, the τ_0 estimation is $\hat{\tau}_0 = \hat{K}^{(i)} / f_s - T_{shift}$, where T_{shift} is the shift length in time domain.

When $R_1 < 1$, the compound signal's carrier frequencies estimation is

$$\hat{f}_1 = \hat{f}_1^{(i)} - \hat{\mu}_0\hat{\tau}_0 + 2\hat{\mu}_0\hat{T}_0,\ \hat{f}_2 = \hat{f}_2^{(i)} - \hat{\mu}_0\hat{\tau}_0 + \hat{\mu}_0\hat{T}_0$$

When $R_2 < 1$, the compound signal's carrier frequencies estimation is

$$\hat{f}_1 = \hat{f}_1^{(i)} - \hat{\mu}_0\hat{\tau}_0,\ \hat{f}_2 = \hat{f}_2^{(i)} - \hat{\mu}_0\hat{\tau}_0 + \hat{\mu}_0\hat{T}_0$$

V. SIMULATION ANALYSIS

Consider 2FSK-LFM signal. The sampling frequency is set to 50MHz and the amplitude is set to 1. The chirp rate is 0.5MHz/us, the code length is 10us and the two carrier frequencies are 7MHz and 7.1MHz. Total time length and time offset are set to 78.2us and 1.8us, respectively. The noise is Gaussian white noise. Choose SNR from -5 to 10dB. 500 times Monte-Carol simulations are conducted for each SNR. Figure II and III show the NRMSE of code length/chirp rate estimation and carrier frequencies estimation, respectively.

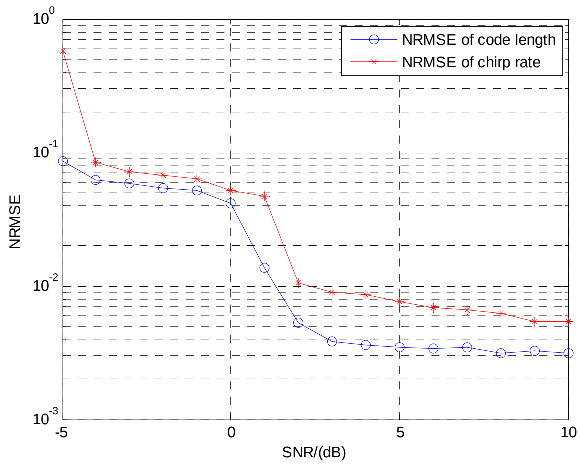

FIGURE II.NRMSE OF CODE LENGTH /CHIRP RATE

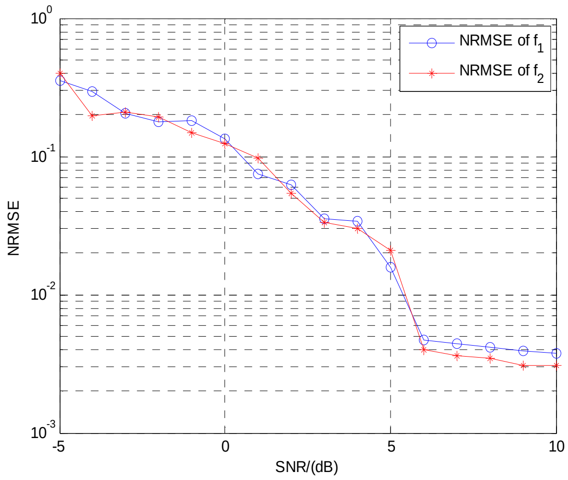

FIGURE III.NRMSE OF CARRIER FREQUENCIES

From Fig. II, because FRFT has a better anti-noise property, code length estimation and chirp rate estimation can achieve a high accurate performance (NRMSE<0.01) when SNR>2dB. In addition, from Fig. III, when SNR>5dB, the carrier frequencies estimation is accurate enough (NRMSE<0.01) and it can distinguish the carrier frequencies with small frequency shift.

VI. CONCLUSION

This paper proposes a novel method to estimate the main parameters of 2FSK-LFM compound signal based on joint FRFT-ML. The compound signal representation in FRFT domain is derived and the improved ML estimation process is elaborated. Simulation results show that our algorithm can achieve a high accuracy when SNR>5.

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