A Novel Piecewise Linear Recursive Convolution Approach for Lorentz media Using the ADE- FDTD Method

Bing-kang Chen^{*}

School of Informatics, Linyi University, Linyi, 276000, China

bingkangchen@163.com

Keywords: Piecewise linear recursive convolution(PLRC). Auxiliary differential equation(ADE). Finite-difference time-domain(FDTD) method. Lorentz media.

Abstract: In order to study the reflection of electromagnetic wave in Lorentz media, A novel piecewise linear recursive convolution(PLRC) method based on the auxiliary differential equation (ADE) of finite-difference time-domain(FDTD) technique is used to obtain the formulation of 2-D TM wave propagation in lossy Lorentz media. In 1-D case, the reflected coefficients calculated by the PLRC ADE-FDTD method and exact theoretical result are excellent agreement. This expresses that the 2-D formulas of electromagnetic wave propagation in lossy Lorentz media are right. Furthermore, Plane wave reflected by Lorentz media layer is calculated and simulated. Results display that reflected effect is evident.

Introduction

Broadband analysis is needed in many practical engineering problems, i.e., s parameters, antenna feed impedance and the radar scattering cross section calculation. It is important to seek a high-efficient broadband algorithm in theory and in engineering. When broadband electromagnetic wave propagates in dispersive media, dispersive property is very important.

Finite-difference time-domain (FDTD) method is a widely used electromagnetic computational method, which was proposed by Yee[1] in 1966. Using this method, time-domain numerical result of Maxwell's equation could be calculated directly, and the result can embody time-domain property of electromagnetic field[2]. The existing frequency-dependent FDTD methods have been studied in many literatures[3-5] which can be summarized as: 1) methods implement a discrete convolution of the dispersion relation[3]; 2) methods implement the auxiliary differential equation(ADE)[4]; 3) Z-transform methods[5]. In which, ADE method is an effective method to obtain the electric intensity vector($\vec{E}$) and auxiliary in dispersive media.

In the second part, 2-D TM wave propagation formula in lossy Lorentz media is derived by using a novel PLRC ADE-FDTD method; In the third part, the derived formula is tested and the reflection of electromagnetic wave impinging on infinite high Lorentz media is simulated.

Mathematical formulation

Assuming the media are Lossy, linear and isotropy. For simplicity, relative permittivity changing with frequency is only discussed (relative magnetic permeability changing with frequency can be disposed as the same method). Maxwell's equation in Lorentz media are given by

$$\nabla \times \vec{H} = \epsilon_0 \epsilon_\infty \frac{\partial \vec{E}}{\partial t} + s \vec{E} + \epsilon_0 \sum_{s=1}^N \vec{J}_s \quad (1)$$

$$\nabla \times \vec{E} = -m_0 \frac{\partial \vec{H}}{\partial t} \quad (2)$$

$$\vec{J}_s = c_s(t) * \frac{\partial \vec{E}(t)}{\partial t} \quad (3)$$

Where ϵ_0 and μ_0 are the electric permittivity and the magnetic permeability of the free space, ϵ_∞ is the infinite frequency relative permittivity, $\vec{H}$ is the magnetic field, $\vec{E}$ is the electric field, $\vec{J}_s$ is the polarized current density, S is the conductivity, $c_s(t)$ used here is the susceptibility time function of Lorentz media.

We define $\vec{T}_s$ as a complex function and its real part is the polarized current density $\vec{J}$. We assume

$$\vec{T}_s = W_s e^{Q_s t} * \frac{\partial \vec{E}}{\partial t} \quad (4)$$

where W_s and Q_s can be both real and complex numbers

Using the piecewise linear recursive convolution method and a central difference scheme to calculate the derivative of the electric field in time, we can obtain

$$\vec{T}_s^{(n+a)\Delta t} = e^{Q_s a \Delta t} \vec{T}_s^{n\Delta t} - \frac{W_s}{Q_s} \frac{\vec{E}(n+1)\Delta t - \vec{E}^{n\Delta t}}{\Delta t} (1 - e^{Q_s a \Delta t}) \quad (5)$$

Taking $a = 1/2$ and substituting equation (5) into equation (3), and discretizing it, the following expression can be obtained

$$\vec{E}^{(n+1)\Delta t} = \frac{C1}{C2} \vec{E}^{n\Delta t} - \frac{1}{C2} \vec{\Phi}^{n\Delta t} + \frac{1}{C2} \nabla \times H^{(n+1/2)\Delta t} \quad (6)$$

$$\vec{\Phi}^{n\Delta t} = \epsilon_0 \sum_{s=1}^N \text{Re} \left\{ e^{Q_s \frac{\Delta t}{2}} \vec{T}_s^{n\Delta t} \right\} \quad (7)$$

$$C1 = \frac{\epsilon_0 \epsilon_\infty}{\Delta t} - \frac{\epsilon_0}{\Delta t} \sum_{s=1}^N \text{Re} \{ Z_s \} - \frac{S}{2} \quad (8)$$

$$C2 = \frac{\epsilon_0 \epsilon_\infty}{\Delta t} - \frac{\epsilon_0}{\Delta t} \sum_{s=1}^N \text{Re} \{ Z_s \} + \frac{S}{2} \quad (9)$$

$$\text{Where } Z_s = \frac{W_s}{Q_s} (1 - e^{Q_s \frac{\Delta t}{2}}).$$

Thus, In lossy, Lorentz media, for 2-D TM wave propagation in z-direction, the relationship of the components H_x, H_y, E_z can be discripted as follow

$$\vec{E}_z^{(n+1)\Delta t} = \frac{C1}{C2} \vec{E}^{n\Delta t} - \frac{1}{C2} \vec{\Phi}^{n\Delta t} + \frac{1}{C2} \left[\frac{H_y^n(i + \frac{1}{2}, j) - H_y^n(i - \frac{1}{2}, j)}{\Delta x} - \frac{H_x^n(i, j + \frac{1}{2}) - H_x^n(i, j - \frac{1}{2})}{\Delta y} \right] \quad (10)$$

$$H_x^{n+\frac{1}{2}}(i, j) = H_x^{n-\frac{1}{2}}(i, j) - \frac{1}{m_0} \frac{\Delta t}{\Delta y} [E_z^n(i, j + \frac{1}{2}) - E_z^n(i, j - \frac{1}{2})] \quad (11)$$

$$H_y^{n+\frac{1}{2}}(i, j) = H_y^{n-\frac{1}{2}}(i, j) + \frac{1}{m_0} \frac{\Delta t}{\Delta x} [E_z^n(i + \frac{1}{2}, j) - E_z^n(i - \frac{1}{2}, j)] \quad (12)$$

Simulation result

For a Lorentz medium the susceptibility function is equal to

$$c_s(t) = \text{Re}\left\{-jg_s e^{(-a_s + jb_s)t}\right\} \quad (13)$$

Where $a_s = d_s$; $b_s = \sqrt{w_{p,s}^2 - d_s^2}$, $g_s = \frac{w_{p,s}^2(e_s - e_\infty)}{b_s}$, $w_{p,s}$ is the resonant frequency, $d_{p,s}$ is the

damping factor and $j = \sqrt{-1}$.

Thus, from equation (4) and (14), we can get

$$W_s = -jg_s \quad (14)$$

$$Q_s = -a_s + jb_s \quad (15)$$

The validity of the proposed algorithm is tested. A wide band pulse of the form $E_s(t) = e^{-(t-t_0)^2/T^2}$ with $t_0 = 0.1$ ns, $T_0 = 6$ ps is normally incident from air onto the Lorentz media. The thickness of Lorentz media is 12mm, $e_s = 3$, $e_\infty = 1.5$, $s = 0.0062$ S/m, $w_{p,1} = 200$ p PHz, $d_{p,1} = 0.1w_{p,1}$, $w_{p,2} = 300$ p PHz, $d_{p,2} = 0.1w_{p,2}$. The reflection coefficient[6] at the air interface between the area 1 and the area 2 theoretically given by

$$\Gamma = \frac{\Gamma_{12} + \Gamma_{23} e^{j2b_2d}}{1 + \Gamma_{12}\Gamma_{23} e^{j2b_2d}} \quad (16)$$

Where d is the thickness of the Lorentz media and b_2 is the phase constant in the Lorentz media.

Using the proposed algorithm in this paper, the numeric reflection coefficient in each frequency can be calculated at the interface of area 1 and area 2, and using equation (16), the theoretical reflection coefficient can also be calculated. The comparison of them shows better agreement of the reflected coefficient between the numeric result and the theoretical result. It shows that the expressions obtained in section 2 are correct.

Using the equations (6)-(12), the reflection of electromagnetic wave impinging on infinite high Lorentz media with 12mm thickness is calculated. The graph of the description as follow

Fig.1. Graph of 2-D description

When plane electromagnetic wave generated at ja plane propagates in y axis direction and still not reach Lorentz media, the waveform after 50 time steps is depicted in Fig.2.

Fig.2. Plane wave after 50 time steps

Fig.3. Plane wave after 140 time steps

In Fig.2, plane electromagnetic wave does not reach the Lorentz media layer, the waveform keeps very well. After 140 time steps, plane electromagnetic wave propagates through the Lorentz media layer, and the waveform changes evidently. Part of them is reflected and the rest transmits and propagate in Lorentz media. The waveform is Fig.3.

Conclusion

Expressions about 2-D TM electromagnetic wave based on a novel PLRC ADE-FDTD in lossy Lorentz media are derived in this paper. Using these expressions, the reflection field in time domain is calculated. Through comparison of reflection coefficient of numeric result and theoretical result, the agreement of them is better. This shows that expressions obtained in this paper are correct. Using the equation derived, the reflection of plane electromagnetic wave impinging on infinite high Lorentz media with 12mm thickness is simulated. The results show that the reflected phenomenon is evident.

Acknowledgements

This work was financially supported by the Linyi university (4613034).

References

- [1] Yee K S. Numerical solution of initial boundary value problem involving Maxwell equations in isotropic media. *IEEE Trans. Antennas Propagat.*, May, 14(1966):302-307
- [2] Liu Zongxin, Chen Yiwang, Xu Xin, et al. Weakly conditionally stable 3-D FDTD method for periodic structures. *High Power Laser and Particle Beams*, 24 (2012): 2687-2692
- [3] R.J. Luebbers and F. Hunsberger. FDTD for Nth-order dispersive media. *IEEE Trans. Antennas Propagat.*, 40(1992):1297-1301
- [4] S C I Kong,, J J Simpson, V Backman. ADE-FDTD scattered-field formulation for dispersive materials. *IEEE Micro. Wire.Comp. Lett.*, 22(2013):4-6
- [5] Bingkang Chen, Xufei Lu. Reflection and Propagation of Electromagnetic Wave in Time-varying Media. *Modern Applied Science*, 5(2011):243-252
- [6] C A Balanis. *Advanced Engineering Electromagnetics*. New York: John Wiley, 1989