Influence research of fabrication errors on modal amplitude in a dual-mass silicon micro-gyroscope

Hu Di¹, ²,a, Yang Bo¹,²,b* and Dai Bo¹,²,c

¹School of Instrument Science & Engineering, Southeast University, Nanjing 210096, China
²Key Laboratory of Micro-Inertial Instrument and Advanced Navigation Technology, Ministry of Education, Nanjing 210096, China

adi92hu@outlook.com, byangbo20022002@163.com, cumtdaibo@126.com

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Abstract: Influence of fabrication errors on modal amplitude in a dual-mass silicon micro-gyroscope is present in this paper. Firstly, the structure and operation principle a dual-mass silicon micro-gyroscope is introduced. Then the equation of amplitude ratio between left and right proof mass is deduced. Finally, the simulation analysis of fabrication errors on modal amplitude is implemented. The simulation results show the amplitude ratio is decreased to -1.2741 when the left proof mass error is 5% in anti-phase driving mode. The amplitude ratio is increased to -0.800 when the left stiffness error is 5% in anti-phase sensing mode.

Introduction

Silicon micro-gyroscopes are sensors using MEMS technology. Because of their features such as small size, intelligence and suitable for mass production, silicon micro-gyroscopes have been widely applied in military, industrial and other fields. The biggest difference between silicon micro-gyroscopes and traditional gyroscopes is that silicon micro-gyroscopes are fabricated by micromachining process. Although the precision of the micromachining process is quite high, there are inevitable errors during processing. Two proof masses and stiffnesses in the left and right structures will be unequal if fabrication error occurs in the dual-mass micro-gyroscope [1]. The unequal proof masses will change the gyroscope mechanical properties. Therefore, it is important to analyze the impact of fabrication errors on mechanical properties in dual-mass micro-gyroscopes. In this paper, we present the analysis and simulation of fabrication errors on modal amplitude in a decoupled dual-mass silicon micro-gyroscope.

Principle of decoupled dual-mass silicon micro-gyroscope

A decoupled dual-mass silicon micro-gyroscope is shown in Fig.1. This gyroscope is composed of two identical single-mass gyroscopes. There are two sets of driving frames on the left and right side of the gyroscope. Driving frames are connected to proof masses by sensing decoupling springs. There are two sets of driving sensing frames between two proof masses and two sets of sensing frames on the top and bottom of the proof masses. Sensing frames are connected to proof masses by driving decoupling springs. On the top and bottom of the gyroscope, there are two lever mechanisms. The lever mechanism is used to reduce the in-phase sensing modal and amplify the anti-phase sensing modal. When driving voltage is applied on driving electrodes, the driving frames will be excited to
resonant vibration by the driving combs. The vibration will be translated to proof masses by sensing decoupling springs, whereas driving decoupling springs in driving direction is flexible, leading to a drive decoupling with respect to the sensing frames. If there is an external angular rate along the Z-axis, the proof masses are driven into a vibrating motion by the Coriolis forces. This motion is translated to the sensing frames via driving decoupling springs, whereas sensing decoupling springs in driving direction is flexible, leading to a sense decoupling with respect to the driving frames.

(a) Schematic drawing of a decoupled dual-mass silicon micro-gyroscope
(b) decoupling principle of the gyroscope

Fig.1 The structure scheme and decoupling principle of decoupled dual-mass silicon micro-gyroscope

**Influence of fabrication errors on modal amplitude**

During processing, the grinding and thinning processes will make unequal in the thickness of micro gyroscope, which results in the errors of proof masses and stiffnesses in the left and right structures [2]. The unequal proof masses will affect the scale factors during driving and sensing vibration [3]. As shown in Fig.1, assuming that the proof masses are $m_a$ and $m_b$, the stiffnesses of two springs are $k_a$ and $k_b$, the coupling stiffness is $k_c$, the applied harmonic forces are $F_1(t) = F_1 \sin(\omega t)$ and $F_2(t) = F_2 \sin(\omega t)$.

\[
\begin{bmatrix}
F_1(t) \\
F_2(t)
\end{bmatrix} =
\begin{bmatrix}
K_a & K_c \\
K_c & K_b
\end{bmatrix}
\begin{bmatrix}
x_a \\
x_b
\end{bmatrix} +
\begin{bmatrix}
k_a + k_c & -k_c \\
-k_c & k_b + k_c
\end{bmatrix}
\begin{bmatrix}
x_a \\
x_a
\end{bmatrix} =
\begin{bmatrix}
-F_1(t) \\
F_2(t)
\end{bmatrix}
\]

(1)

Supposing $Z_{11}(\omega) = k_{11} - \omega^2 m_a$, $Z_{12}(\omega) = k_{12}$, $Z_{21}(\omega) = k_{21}$, $Z_{22}(\omega) = k_{22} - \omega^2 m_b$, and assuming $\vec{Z}(\omega)$ is the matrix-vector of $Z$, $\vec{u}$ is displacement amplitude vector, $\vec{F}$ is excitation amplitude [5]. The equation of amplitude vector is

\[
\vec{u} = \vec{Z}(\omega)^{-1} \vec{F}
\]

(2)

The inverse matrix of $\vec{Z}(\omega)$ is
\[ \ddot{Z}(\omega)^{-1} = \frac{1}{Z_{11}(\omega)Z_{22}(\omega) - Z_{12}(\omega)^2} \begin{bmatrix} Z_{11}(\omega) & -Z_{12}(\omega) \\ -Z_{21}(\omega) & Z_{22}(\omega) \end{bmatrix} \]  

(3)

Substitute equation (3) into equation (2), the amplitude ratio of two proof masses is

\[ \frac{u_2}{u_1} = \frac{Z_{21}(\omega)F_1 + Z_{11}(\omega)F_2}{Z_{22}(\omega)F_1 + Z_{12}(\omega)F_2} \]  

(4)

For dual-mass micro-gyroscopes, the driving and sensing modal are operated in anti-phase, so we only research the amplitude ratio in anti-phase modal.

Simulation analysis of fabrication errors on modal amplitude

The design parameters of dual-mass micro-gyroscope in anti-phase driving mode are shown in Table.1. We assume all of stiffnesses are equal, fabrication error only occurs in the left proof mass \( m_a \). Fig.3 (a) shows the amplitude ratio when the proof mass error ranges between -5% and +5%. The simulation results show the amplitude ratio is -1 when both proof masses are equal. When there is an error of 1% in the left mass, the amplitude ratio is decreased to -1.1027. The amplitude ratio reaches -1.5928 in the left proof mass error of 5%. Similarly, Fig.3 (b) shows the amplitude ratio when the stiffness error ranges between -5% and +5%. The simulation results show the amplitude ratio is -1 when both stiffnesses are equal. When the left stiffness error is 1%, the amplitude ratio is increased to -0.911. The amplitude ratio reaches -0.6368 in the left stiffness error of 5%.

Table.1 structure parameters of anti-phase driving mode

| parameters \( k_{11} \) \( k_{22} \) \( k_{12} \) \( m_a \) \( m_b \) |
|---|---|---|---|---|
| value | 284.33N/m | 284.33N/m | 15.23N/m | \( 0.536 \times 10^{-6} \)kg | \( 0.536 \times 10^{-6} \)kg |

![Fig.3](image)

(a) mass fabrication error  (b) stiffness fabrication error

The design parameters of dual-mass micro-gyroscope in anti-phase sensing mode are shown in Table.2. We assume all of stiffnesses are equal, fabrication error only occurs in the left proof mass \( m_a \). Fig.4 (a) shows the amplitude ratio when the proof mass error ranges between -5% and +5%. The simulation results show the amplitude ratio is -1 when both proof masses are equal. When there is an error of 1% in the left mass, the amplitude ratio is decreased to -1.051. The amplitude ratio reaches -1.2741 in the left proof mass error of 5%. Similarly, Fig.4 (b) shows the amplitude ratio when the stiffness error ranges between -5% and +5%. The simulation results show the amplitude ratio is -1 when both stiffnesses are equal. When the left stiffness error is 1%, the amplitude ratio is increased to -0.956. The amplitude ratio reaches -0.800 in the left stiffness error of 5%.
<table>
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<th>parameters</th>
<th>$k_{11}$</th>
<th>$k_{22}$</th>
<th>$k_{12}$</th>
<th>$m_a$</th>
<th>$m_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>302.74N/m</td>
<td>302.74N/m</td>
<td>33.64N/m</td>
<td>$0.6 \times 10^{-6}$kg</td>
<td>$0.6 \times 10^{-6}$kg</td>
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</tbody>
</table>

**Conclusion**

In this paper, the structure and operation principle a dual-mass decoupled silicon micro-gyroscope is present. Then the equation of amplitude ratio between left and right proof mass is deduced. Finally, the simulation analysis of fabrication errors on modal amplitude is implemented. The simulation results show the amplitude ratio is decreased to -1.2741 when the left proof mass error is 5% in anti-phase driving mode. The amplitude ratio is increased to -0.800 when the left stiffness error is 5% in anti-phase sensing mode.

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**References**


