Functional Analysis on Superposition of Harmonic Sources in AC Arc-furnaces

Pu Deng[1,a], Feng-yuan Wang[1,a], Xiao-bo Liu[2,b], Xian-song Pi[1,a], Hong-li Li[3,c]

[1]*Power Grid Planning and Research Center, Guizhou Power Grid Corporation, Guiyang 550002, Guizhou Province, China*

[2]*Guizhou University, Guiyang 550025, Guizhou Province, China*

[3]*Duyun Electric Power Bureau, Duyun 558000, Guizhou Province, China*

[a]*email2410406553@qq.com,* [b]*email799797284@qq.com*

ABSTRACT: To solve the problem of harmonic level evaluation for AC arc-furnace supply area, this paper analyses the synchronous measuring data of harmonic current in AC arc-furnaces with functional analysis, verifies harmonic current released from AC arc-furnaces with independence and orthogonality approximately between each other, proves superposition of harmonic current in accordance with root of sum of squares, derives the formula for evaluating harmonic voltage level of PCC at which AC arc-furnaces were connected. The formula is verified Actually, it shows that multi-arc-furnaces harmonic current can be largely counteracted each other, If these AC arc-furnaces were connected to a PCC in which short-circuit capacity is enough for that the superposed harmonic current of AC arc-furnaces can be electromagnetic compatible, then power quality of the PCC can reach the qualified level.

KEYWORD: AC arc-furnace; superposition of harmonic current; orthogonal; complex-valued co-correlativity coefficient; evaluation of harmonic level.

1 INSTRUCTION

Ac arc furnace is a major source of harmonic. Before the planning and construction, can't to harmonic level assessment, cannot undertake corresponding load planning.

Before AC arc furnace plans to be inserted into the power grid and directly access to the power grid without power quality control under the premise of meeting the grid power quality requirements, good social and economic benefits will be achieved by evaluating AC arc furnace harmonic consumptive ability, making full use of short circuit capacity of PCC on harmonic electromagnetic compatibility, and reducing control cost by reasonable planning and design.

The difficulties of the pre-evaluation assumption is superposition of harmonic current of multi-arc-furnaces in order to realize

The superposition of harmonic current formula is recommended by the utility grid harmonic standard (GB/T 14549-93) and the International conference on power grid (CIGRE) 36-05 working group. Also, one superposition of harmonic current of ac arc furnace is researched at home and abroad[1-4], put forward some analysis methods, but still cannot form superposition analytical formulas or algorithm, consisting of many sets of ac arc furnace arc furnace for harmonic evaluation is still suspended and unanswered questions.

This article[5-10] analyses the synchronous measuring data of harmonic current in AC arc-furnaces with functional analysis and statistical characteristics of harmonic vector, verifies independence and orthogonality among AC arc-furnaces. According to the orthogonality of harmonic vector, it derives superposition formula of multi-AC arc-furnaces and then deduces the formula for evaluating harmonic level of multi-AC arc furnaces inserting into the grid power[11-14]. Then checks AC arc furnace area of a large metallurgical company, and proves the correctness of evaluation formula of AC arc furnace harmonic.

2 THE INDEPENDENCE OF THE AC ARC FURNACE HARMONIC CURRENT AND ORTHOGONALITY

2.1 *The synchronous measurement of AC electric arc furnace harmonic current*

At the high-voltage side of electric furnace transformer, synchronously measuring four signals ,the in-phase harmonic current of A and B two AC arc furnace, the in-phase superimposed harmonic current on power supply lines and the in-phase busbar voltage,

as shown in fig.1. The whole measurement process covers the AC arc furnace smelting cycle, we get more than 1400 times harmonic data. Formation sequence in chronological order, every moment of the measurement data is synchronized, measurement time interval is 10 seconds, each moment data contains every harmonic amplitude and phase of the measured signal.

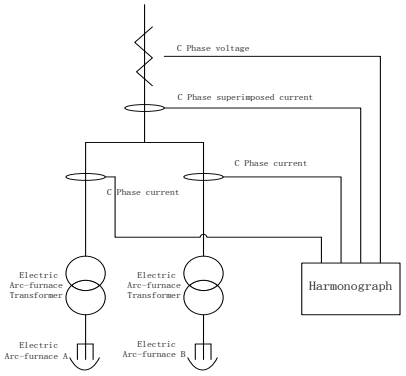

Fig. 1 The wiring diagram of synchronous measuring harmonic wave at AC arc-furnaces

2.2 *Functional definition on synchronous harmonic current measuring sequence of AC arc furnace*

Definition the extensive function $f_{hA}(t)$, that is complex time series by measuring h times harmonic current of AC arc furnace A when t = 1, 2,...n.

$$f_{hA}(t) = i_{hA}(t)\cos\varphi_{hA}(t) + j \cdot i_{hA}(t)\sin\varphi_{hA}(t). \qquad (1)$$

(t = 1, 2,... n measuring moment; h = harmonic frequency)

$f_{hA}(t)$ is a sequence of complex numbers, according the measurement time to rank, $i_{hA}(t)$ is h times harmonic current amplitude of arc furnace A, and φhA(t) means the lag Angle on h times harmonic current and h times harmonic busbar voltage of furnace A at t moment.

Similarly, $f_{hB}(t)$ is a sequence of complex numbers that is measured from the h times harmonic current of furnace A.

$$f_{hB}(t) = i_{hB}(t)\cos\varphi_{hB}(t) + j \cdot i_{h}B(t)\sin\varphi_{hB}(t). \qquad (2)$$

(t = 1, 2,… n measuring moment; h = harmonic frequency)

2.3 *The independence of the communication between the arc furnace harmonic current*

The definition of regularization correlation coefficient:

$$\rho(x, y) = \frac{E([(x - E(x))(\overline{y - E(y)})])}{\sqrt{E[(x - E(x))^2]E(y - E(y)^2)}}. \qquad (3)$$
$$x = f_{hA}(t), y = f_{hB}(t)$$

(x,y) is plural, conjugated with □ (*y,x*), the module is the same, respectively, the *x*, *y* are functional $f_{hA}(t)$ and $f_{hB}(t)$.

Calculated by formula (1), co-correlativity coefficient of harmonic current measurement sequence value of AC arc furnace are shown in table 1.

Due to harmonics release from AC arc furnace are mainly distributed at 2 times odd harmonic[1] or and lower times, therefore, just list the odd harmonic correlation coefficient on 2 and 3~19 in table.1.

The real component symbols of co-correlativity coefficient is positive sign, it shows positive correlation, means that the harmonic direction and increase of amplitude on arc furnace A are same as B; If the symbol is negative, the opposite is true. module of Regulation co-correlativity coefficient in the range of 0 ~ 1, it means completely related that co-correlativity coefficient is 1, co-correlativity coefficient is 0 means that the harmonic of two electric arc furnaces are unrelated and independent of each other.

Data in table 1 indicates that it is mostly weak negative correlation among each other to every harmonic between two electric arc furnace, similar to independence, and has character that cancel each other out. Independent to each other also means that there is no the time synchronous relationship that ac arc furnace release harmonic which can be measured separately and independently.

Tab.1 Complex correlation coefficient between harmonic current released by two arc-furnaces

Harmonic order	Two same type Silicon manganese smelting furnace		Two same type Yellow phosphorus furnaces		A silicon manganese, A ferrosilicon furnace	
	ρ	|ρ|	ρ	|ρ|	ρ	|ρ|
2	-0.04 -0.12i	0.13	0.13 -0.17i	0.18	-0.07 +0.06i	0.10
3	-0.02 -0.04i	0.04	-0.04 -0.04i	0.05	0.02 +0.07i	0.07
5	-0.08 +0.03i	0.09	-0.01 -0.13i	0.13	0.06 +0.04i	0.07
7	-0.06 +0.03i	0.07	0.03 +0.01i	0.03	0.00 -0.01i	0.01
9	-0.08 +0.03i	0.08	-0.06 -0.02i	0.07	-0.00 +0.01i	0.01
11	-0.06 -0.00i	0.06	0.09 +0.05i	0.1	0.03 +0.01i	0.03
13	-0.05 -0.01i	0.05	-0.11 +0.03i	0.11	-0.01 -0.00i	0.01
15	-0.01 +0.01i	0.01	-0.11 -0.15i	0.19	-0.03 +0.00i	0.03
17	0.08 -0.02i	0.08	-0.06 +0.02i	0.07	-0.02 -0.02i	0.03
19	0.02 -0.02i	0.03	0.04 -0.00i	0.04	-0.02 +0.01i	0.02

2.4 *Orthogonality on AC arc-furnaces harmonic current*

The literature[15] with the framework of classical probability, proves that the orthogonality of the random variables and independence are equivalent. But the random complex sequence, measured from AC arc furnace harmonic source, cannot be incorporated into the classical probability. Therefore, it's necessary to make the specialized research about the orthogonality of AC arc-furnaces harmonic current.

Fourier transform is orthogonal transformation[16], and illustrates that the different time of harmonic current are orthogonal to each other, the key is that wheather the two AC arc furnace harmonic current measurement sequence is orthogonal.

Functional $f_{hA}(t)$ and $f_{hB}(t)$ are all belong to the plural in n unitary space, inner product of $f_{hA}(t)$ and $f_{hB}(t)$ are as follows:

$$\begin{aligned}\left(f_{hA}(t)\bullet f_{hB}(t)\right)&=\int_t f_{hA}(t)\times\overline{f_{hB}(t)}dt\\&=\Delta t\times\sum_{t=1}^{n}\left(f_{hA}(t)\times\overline{f_{hB}(t)}\right)\end{aligned}\quad . \tag{4}$$

$\int_t f_{hA}(t)\times\overline{f_{hB}(t)}\,dt$ is the Lebesgue integral, $\overline{f_{hB}(t)}$ is conjugate with $f_{hB}(t)$, □ t is harmonic measuring time interval. Noticed that Inner product space which $\left(f_{hA}(t)\cdot f_{hB}(t)\right)$ belongs to is Hilbert space.

The norm of functional $f_{hA}(t)$ and $f_{hB}(t)$:

$$\begin{aligned}\left|f_{hA}(t)\right|&=\sqrt{\int_t f_{hA}(t)\times\overline{f_{hA}(t)}\,dt}\\&=\sqrt{\Delta t\times\sum_{t=1}^{n}\left(i_{hA}(t)\right)^2}\end{aligned} \tag{5}$$

$$\left|f_{hB}(t)\right|=\sqrt{\Delta t\times\sum_{t=1}^{n}\left(i_{hB}(t)\right)^2}\quad . \tag{6}$$

In Hilbert space, the angle of $f_{hA}(t)$ and $f_{hB}(t)$:

$$\begin{aligned}cos\phi&=\left(\left|\left(f_{hA}(t)\bullet f_{hB}(t)\right)\right|\right)\Big/\left(\left|f_{hA}(t)\right|\times\left|f_{hB}(t)\right|\right)\\&=\left(\left|\sum_{t=1}^{n}\left(f_{hA}(t)\times\overline{f_{hB}(t)}\right)\right|\right)\Big/\left(\sqrt{\sum_{t=1}^{n}\left(i_{hA}(t)\right)^2}\times\sqrt{\sum_{t=1}^{n}\left(i_{hB}(t)\right)^2}\right)\end{aligned} \tag{7}$$

By formula (7), Calculating the Angle of plural measuring sequence of harmonic current between two AC arc-furnaces the, as shown in table 2.

As table 2 shows, angle of complex sequence, on the same number harmonic current of two AC arc furnace, is approximation 90 degrees and pairwise orthogonal.

3 THE SUPERPOSITION OF HARMONIC CURRENT AND HARMONIC LEVEL EVALUATION ON AC ARC FURNACE

3.1 *Superposition formula of AC arc furnace harmonic current content*

If the complex sequences of every times harmonics on two AC arc furnaces is orthogonal, it can be derived that the statistical formula superposition of two ac arc furnace harmonic current.

Assume that superposition of two ac arc furnace harmonic current is functional fh(t).

$$f_h(t)=f_{hA}(t)+f_{hB}(t)\quad . \tag{8}$$

(t=1, 2…n Measuring time; h= Harmonic frequency)

Such as $f_{hA}(t)$ and $f_{hB}(t)$ are mutual orthogonal function, according to the Pythagorean theorem[17]:

$$\left|f_h(t)\right|^2=\left|f_{hA}(t)\right|^2+\left|f_{hB}(t)\right|^2 . \tag{9}$$

Simplify as:

$$\sum_{t=1}^{n}i_h^2(t)=\sum_{t=1}^{n}i_{hA}^2(t)+\sum_{t=1}^{n}i_{hB}^2(t)\quad . \tag{10}$$

Further simplify:

$$\sum_{t=1}^{n}\sum_{h=2}^{\infty}i_h^2(t)=\sum_{t=1}^{n}\sum_{h=2}^{\infty}i_{hA}^2(t)+\sum_{t=1}^{n}\sum_{h=2}^{\infty}i_{hB}^2(t)\,. \tag{11}$$

$\sqrt{\sum_{h=2}^{\infty}i_h{}^2(t)}$, $\sqrt{\sum_{h=2}^{\infty}i_{hA}{}^2(t)}$ and $\sqrt{\sum_{h=2}^{\infty}i_{hB}{}^2(t)}$ respectively are superposition of harmonic current content at t time, the harmonic current content of the furnace A and B, respectively denoted by $I_H(t)$、$I_{HA}(t)$ and $I_{HB}(t)$, formula (3) also can be shown as:

$$\sum_{t=1}^{n}\left(I_H(t)\right)^2=\sum_{t=1}^{n}\left(I_{HA}(t)\right)^2+\sum_{t=1}^{n}\left(I_{HB}(t)\right)^2\quad . \tag{12}$$

In statistical sense, $I_H=\sqrt{I_{HA}^2+I_{HB}^2}$.

This suggests that if the harmonic current is orthogonal to each other, the synthesis harmonic current is the superposition that according to the law of the square root of square sum on the harmonic source current. This means that the harmonic superposition is not arithmetic sum for several approximate orthogonal AC arc furnace, with the increase of number of AC arc furnace, the increase of the superposition of harmonic current is not large, in a larger extent, the harmonic has been cancel each other out that released by AC arc furnace

.

Tab.2 Interaction angle ϕ between measuring serials of same order harmonic current in two AC arc-furnaces

harmonic order	Two same type Silicon manganese smelting furnace	Two same type Yellow phosphorus furnaces	A silicon manganese, A ferrosilicon furnace
2	99.7 °	100.2 °	98.1 °
3	93.0 °	79.7 °	82.2 °
5	94.3 °	82.8 °	93.2 °
7	93.3 °	81.4 °	92.0 °
9	94.8 °	86.6 °	94.7 °
11	93.7 °	87.0 °	89.7 °
13	92.9 °	85.6 °	91.0 °
15	90.8 °	94.9 °	88.3 °
17	85.5 °	85.1 °	96.3 °
19	88.7 °	84.1 °	91.5 °

3.2 *The definition and the superposition product of harmonic current times*

This paper introduces some new definitions to derive the harmonic evaluation formula on AC arc furnace.

Defination of product of harmonic current time is:

$$J_H = \sqrt{\sum_{h=2}^{\infty} (h \cdot I_h)^2} \quad . \tag{13}$$

h is the harmonic frequency, I_h is h times harmonic current.

If harmonic current sequence of AC arc furnace is strictly orthogonal, according to formula (3),it can be proved that it also conforms to the law of the square root of sum of squares that the product of harmonic current time on superposition for 1~m AC arc furnaces:

$$J_H = \sqrt{\sum_{i}^{m} J_{Hi}^2} \quad . \tag{14}$$

The J_{Hi} is the product of harmonic current time on i-th AC arc furnace.

It must be pointed out that the formula (4) only has the statistical significance, and considering that the AC arc furnace harmonic current is approximate orthogonal rather than strictly orthogonal, the deviation error of formula (4) is very important.

Tab.3 The bias of the formula (4)

Product of harmonic current times	Two same type silicon manganese furnace	Two type with yellow phosphorus furnace	A silicon manganese, a ferrosilicon furnace
JHA	52.57	13.82	52.58
JHB	57.62	11.26	76.66
$\sqrt{J_{HA}^2 + J_{HB}^2}$	80.00	17.82	92.96
Superimposed current JH(A+B)	80.71	19.83	92.01
The relative error	-0.9%	-10.01%	1.03%

According to the measuring data of whole production cycle in three different fields, through electric arc furnace A,B and the harmonic current measured values of superposition on two arc furnaces, calculating the 95% probability value of the harmonic current times product. The error of the formula (4) is in the range of - 10.01%~1.03%,it can be used to evaluate calculation, shown in table 3.

3.3 *The definition on distortion rate of harmonic current times*

The definition is :

$$THD_J = J_H / I_1 \times 100\% \quad . \tag{15}$$

I_1 is rated power frequency current of electric furnace transformer.

THD_J can be got by statistic that taking out every harmonic measured values from harmonic meter at every moment, table 4 there are some measurements on AC arc furnace THD_J (take 95% probability value of the measured).

The types and production technology of AC arc furnace are different, THD_J is different, THD_J is characteristic value that can accurately appraise the harmonic released by nonlinear load.

3.4 *Evaluation formula of AC arc-furnace harmonic voltage in power supply area*

To make an assumption that 1~m AC arc-furnaces were connected to a PCC, Harmonic impedance of PCC is inductive,its h times harmonic impedance:

$$X_h = 2\pi h L = h X_1 . \tag{16}$$

X_1 is power frequency short-circuit reactance of PCC.

Voltage total harmonic distortion rate $THDu$ of PCC :

$$\begin{aligned} THD_u &= \sqrt{\sum_{h=2}^{\infty} U_h^{\,2}} \Big/ U_1 = \sqrt{\sum_{h=2}^{\infty} (X_h \cdot I_h)^2} \Big/ U_1 = \\ &\left(X_1 \cdot \sqrt{\sum_{h=2}^{\infty} (h \cdot I_h)^2} \right) \Big/ U_1 = \left(X_1 \cdot J_H \right) \Big/ U_1 = \\ &\left(X_1 \cdot \sqrt{\sum_{i}^{m} J_{Hi}^2} \right) \Big/ U_1 = \sqrt{\sum_{i}^{m} \left(U_1 \cdot I_{1i} \cdot \frac{J_{Hi}}{I_{1i}} \right)^2} \Big/ \left(U_1^2 / X_1 \right) \\ &= \sqrt{\sum_{i}^{m} (S_i \cdot THD_{Ji})^2} \Big/ S \end{aligned} \tag{17}$$

U_1 is the rated power frequency voltage of PCC, U_h is h times harmonic voltages, I_h is h times superimposed harmonic currents, J_H is product of superimposed harmonic times current on the PCC, S is short-circuit capacity of PCC. I_{1i}, THD_{Ji} and S_i , respectively are the rated power frequency current of i-th AC arc-furnace, electric capacity and distortion rate of product of harmonic current times.

The evaluation formula of harmonic voltage level on multiple AC arc-furnaces:

$$THD_u = \sqrt{\sum_i^m (S_i \cdot THD_{Ji})^2} \Big/ S \cdot \quad (18)$$

If THD_J of all arc furnaces be measured by classifying as the model, the production of products and production technology, forming a table can retrieve and access to the data. Evaluating the power quality level of PCC, before the AC arc furnace load connect to the electricity grid. For this, it can achieve electromagnetic compatibility for the harmonic of AC arc furnace area, and have effective use of PCC short-circuit capacity by planning load.

Tab.4 Measuring data of THDJ in AC arc-furnaces

35kV AC arc furnace	THDJ
Guimeng coarse smelting furnace(3.4MVA)	0.937
Guimeng coarse smelting furnace(3.4MVA)	1.027
Guimeng refining furnace(8MVA)	0.338
Crude refining ferrosilicon furnace(5MVA)	0.929
Silicon refining furnace(9MVA)	0.286
Yellow phosphorus(6MVA)	0.119
Yellow phosphorus(6MVA)	0.0975

4 THE MEASUREMENT AND CHECK FOR AC ARC-FURNACE AREA

A large silicon metal, (ferrosilicon, silicon manganese) metallurgy company be supplied power by 220 kV/35 kV self-provided substation, the short-circuit capacity of 35 kV bus is 1072 MVA, and there are 34 AC arc furnaces with 6300kVA access to the 35 kV busbar. The assessment on the total harmonic distortion rate of 35 kV bus voltage in factory :

$$THD_U = \sqrt{34 \times (6.3 \times 1.027)^2} \Big/ 1072 = 3.67\%$$

It can directly connected to the electricity grid without taking control measures.

Measuring when 25 AC arc furnace are running, the measurement value *THDu* (95% probability) of 35 kV bus voltage is 3.2%, review value in formula (18) is:

$$THD_U = \sqrt{25 \times (6.3 \times 1.027)^2} \Big/ 1072.7 = 3.15\%$$

These two data is very close to confirm the validity of the AC arc furnace harmonic evaluation formula, and also show that the AC arc furnace access to one for the rationality of PCC. These two data is very close to confirm the validity of the ac arc furnace harmonic evaluation formula, will also show the rationality that focusing AC arc furnaces access to one special PCC.

CONCLUSIONS

It has the orthogonality and independence between each two of the in-phase harmonic current for several AC arc furnaces.

The harmonic current of AC arc furnace is superposited according to the law on root of sum of squares, with the increase of number of AC arc furnaces, it is not big that the increase of the superposition of harmonic current.

According to harmonic level assessment formula, on large extent, it can lead harmonics offset each other by focusing several AC arc furnaces on an independent power supply area. Then let the arc furnace area access PCC which short-circuit capacity is larger, and make full use of the given ability in EMC(Electro Magnetic Compatibility) in power grid. In this way, can reduce the cost on the governance for AC arc furnace harmonic, even can directly connected to the electricity grid without management, meet the requirements of power quality and achieve remarkable economic benefits.

It's necessary to develop measurement work on the distortion rate of AC arc furnace harmonic current frequency product to meet the needs of the construction before the assessment.

REFERENCES

[1] Huazhi Bai, Rie Xu, Riliang Xu. Approach of forecasting evaluation methodology to harmonic wave emitting level of current electric arc furnace. Inner Mongolia Electric Power, 2007, 25(1):26-29.

[2] Wei Hu, Xiaoming Zha, Jianjun Sun. Harmonic sources modeling and harmonic superposition simulation of power system with multi-harmonic sources. Electric Power, 2006, 39(3):61-65.

[3] Standard Voltages, current ratings and frequencies. GB/T 14549-1993 Quality of electric supply. Harmonics in public supply network. Chinese Standard SN for Books. Beijing, China: China Standard Press, 1993.

[4] Standard EMC technical committee. GB/T 1024.1-2000. Electromagnetic compatibility-Limits-Assessment of emission limits for distorting loads in MV and HV power systems. Chinese Standard SN for Books. Beijing, China: China Standard Press, 2000.

[5] Wei Zhang, Honggeng Yang. A method for assessing harmonic emission level based on binary linear regression. Proceedings of the Csee,2004,24(6):50-53.

[6] Jin Hui, Honggeng Yang, Shunfu Lin, et al. Assessment method of harmonic emission level based on covariance characteristic of random vectors. Automation of Electric Power Systems, 2009, 33(4):27-31.

[7] Sherman W.G. Summation of harmonics with random phase angles IEE Proc.1972, 119 (11):1643-1648.

[8] Jing Zhang. Studies of harmonics superposition method in multi—harmonic sources system.Power System Technology, 1995, 19(3):23~27.

[9] Honggeng Yang, Dong Qin, Zhengshu Zhang, et al. A study on summation of harmonics with random phases angles based on laguerre polynomials. Power System Technology, 2005, 29 (14): 26- 29.

[10] Cavallini A, Montanari G.C, Cacciari M. Stochastic evaluation of harmonics at network buses. IEEE Trans. on Power Delivery, 1995, 10(3): l606—1613.
[11] Cavallini A, Montanari G.C.A deterministic/stochastic framework for power system harmonics modeling. IEEE trans. on Power Systems, 1997, 12(1): 407—415.
[12] Limin Weng, Yunping Chen, Liping Shu. Influence of electric arc steel furnace on the power system and its suppression. Power System Technology, 2004, 28(2):64-67.
[13] Yufu Xu, Jinding Cai, Tianyou Li. Prediction and suppression for steel mill arc furnace influence to distribution networks power quality. Journal of Electric Power Science and Technology, 2008, 23(4):83-88.
[14] Weihu Cheng, Xiangrong Lai. Covariance and Correlation Coefficient of Complex Random Variables. Journal of the CUN (Natural Sciences Edition), 2001, 10 (1):19-21.
[15] Chun Luo, Xiaodi Wang, Yingshan Zhang. Orthogonality and independence—new thinking of dealing with complex systems series three. Journal of Shanghai Institute of Technology (Natural Science), 2010, 10(4):271-277.
[16] Huangguang Zhao. Introduction to functional analysis .Sichuan University Press, 2005.6, 95-96.
[17] Qixiang Cheng, Dianzhou Zhang, Guoqiang Wei. Real Variable Function Theoty and Functional Analysis foundation (2nd).Higher Education Press ,2004.2:242-243.