

The Study of Intervals Sorting Method Based on close degree

Er-dun Bai¹, Yu-e Bao^{2, a}

¹ College of Computer Science and Technology, Inner Mongolia University for Nationalities, Tongliao 028043, China

² College of Mathematics, Inner Mongolia University for Nationalities, Tongliao 028043, China

^abyebed@163.com

Abstract. Against the selection of best option, the sorting of programs and other issues, we discuss the sorting issue of interval numbers based on close degree. Then we propose a sorting method of interval numbers based on close degree. Though an example, we verify the rationality and effectiveness of the sorting methods.

Keywords: interval number; close degree; sorting method.

1. Introduction

In the multi-attribute decision making, due to the complexity of objective things, the finiteness of human knowledge and ability, a lot of decision-making information in the form of interval numbers to represent is more reasonable. Interval multi-attribute decision making usually is converted to interval scheduling problem [1]. However, because the difference of dimension and type of each attribute index values, we cannot directly use the initial index interval values comparing and sorting programs, you need to remove the incommensurability and contradiction between each index value. So, the decision-making index values are normalized to interval number on $[0,1]$ [2-5]. Therefore, it has great significance that sort reasonably the interval number on $[0,1]$. In paper [6], we give the general representation of close degree between interval numbers, and illustrates that we can construct a variety of nearness according to the general representation. On this basis, this paper discusses the sorting indicators of interval number $[0,1]$, and gives a sorting method of interval numbers based close degree.

2. Preliminary

Let R denote all the real numbers. For arbitrarily $a^L, a^U \in R, a^L \leq a^U$, we call $[a^L, a^U]$ is a interval number, i.e. $a = [a^L, a^U]$.

If $0 \leq a^L \leq a^U$, then $a = [a^L, a^U]$ is a non-negative interval number. The whole interval numbers on R denote by $[R]$, the whole interval numbers on $[0,1]$ denote by $[I]$, then $[I] \subset [R]$.

Theorem 1.1^[7] Set $a, b \in [R]$,

$$d(a, b) = \sqrt{\left(\frac{a^L + a^U}{2} - \frac{b^L + b^U}{2}\right)^2 + \frac{1}{12}[(a^U - a^L) - (b^U - b^L)]^2}, \quad (1)$$

Then

$$N(a, b) = \begin{cases} \frac{1}{\sqrt{1 + d(a, b)}} & , \quad d(a, b) \in [0, 1] \\ 0 & , \quad d(a, b) \geq 1 \end{cases}, \quad (2)$$

is the close degree of a and b .

3. The sorting method of interval numbers based on close degree

Set $a_i = [a_i^L, a_i^U]$ ($i = 1, 2, \dots, n$) are non-negative interval numbers, through some appropriate normalized such as specific gravity variation method in paper [5], we get the interval numbers on $[0, 1]$, $r_i \in [I]$ ($i = 1, 2, \dots, n$). The steps of sorting them are as:

Step 1: According to the actual situation, we select positive and negative ideal interval numbers using appropriate methods. The usual conventional method is following. The positive ideal interval number of $r_i = [r_i^L, r_i^U]$, $i = 1, 2, \dots, n$. is

$$r^+ = [\max\{r_1^L, r_2^L \dots r_n^L\}, \max\{r_1^U, r_2^U \dots r_n^U\}].$$

The negative ideal interval number is

$$r^- = [\min\{r_1^L, r_2^L \dots r_n^L\}, \min\{r_1^U, r_2^U \dots r_n^U\}].$$

Step 2: Calculating the close degree $N(r_i, r^+)$, $N(r_i, r^-)$ according to close degree formula. On this basis, according to the idea that the program is better if the distance with positive ideal solution is smaller and with negative ideal solution is greater, we give sorting index formula

$$T(r_i) = \frac{N(r_i, r^+) + (1 - N(r_i, r^-))}{2}, \quad (3)$$

And find the corresponding values of $T(r_i)$.

Step 3: Sorting them according to the size of $T(r_i)$. The value of $T(r_i)$ is greater, the corresponding a_i is more front.

4. Application examples

Example 3.1 Paper [4] gives the comprehensive evaluation interval values of each program. The data are as follows:

$$r_1 = [0.1890, 0.1976]; \quad r_2 = [0.2022, 0.2154];$$

$$r_3 = [0.2021, 0.2112]; \quad r_4 = [0.1865, 0.1964];$$

$$r_5 = [0.1888, 0.1983].$$

We sort the comprehensive evaluation interval values.

Step 1 The positive ideal interval number is

$$r^+ = [\max\{r_1^L, r_2^L \dots r_5^L\}, \max\{r_1^U, r_2^U \dots r_5^U\}]$$

$$= [0.2022, 0.2154]$$

and the negative ideal interval number is

$$r^- = [\min\{r_1^L, r_2^L \dots r_5^L\}, \min\{r_1^U, r_2^U \dots r_5^U\}]$$

$$= [0.1865, 0.1964].$$

Step 2 Using formula (1), we get

$$\begin{aligned} d(r_1, r^+) &= \sqrt{\left[\frac{(0.1890+0.1976)}{2} - \frac{(0.2154+0.2022)}{2}\right]^2 + \frac{1}{3}\left[\frac{(0.1976-0.1890)}{2} - \frac{(0.2154-0.2022)}{2}\right]^2} \\ &= \frac{1}{2}\sqrt{[(0.1890+0.1976) - (0.2154+0.2022)]^2 + \frac{1}{3}[(0.1976-0.1890) - (0.2154-0.2022)]^2} \\ &= \frac{1}{2}\sqrt{(0.3866-0.4176)^2 + \frac{1}{3}(0.0086-0.0132)^2} \\ &= \frac{1}{2}\sqrt{0.031^2 + \frac{1}{3} \times 0.0046^2} \\ &= 0.0156 \end{aligned}$$

In the same way we get

$$d(r_1, r^-) = 0.0014 .$$

Therefore using formula (2), we get

$$N(r_1, r^+) = \frac{1}{\sqrt{1+d(r_1, r^+)}}$$

$$= \frac{1}{\sqrt{1+0.0156}}$$

$$= 0.9922,$$

$$N(r_1, r^-) = \frac{1}{\sqrt{1+d(r_1, r^-)}}$$

$$= \frac{1}{\sqrt{1+0.0014}},$$

$$= 0.9993.$$

Using formula (3), we obtain the sorting index values

$$T(r_1) = \frac{N(r_1, r^+) + (1 - N(r_1, r^-))}{2}$$

$$= \frac{0.9922 + (1 - 0.9993)}{2}$$

$$= 0.49325.$$

The same can be other sort index values:

$$T(r_2) = 0.50855 ; T(r_3) = 0.506 ;$$

$$T(r_4) = 0.49145 ; T(r_5) = 0.49345.$$

Step 3 Sorting them according to the size of $T(r_i)$. The sorting results is

$$r_2 \succ r_3 \succ r_5 \succ r_1 \succ r_4,$$

That is

Program 2 \succ program 3 \succ program 5 \succ program 1 \succ program 4.

So, scheme 2 is the best option.

The result is consistent with literature [4]. However, the method need a secondary comparison, and method 2 also need to give risk coefficient. The calculation process of method we present is relatively simple, easy and practical.

Example 3.2 We use the Example in [8]. For getting the "Service Star", a bus company investigate the company's five buses L_1, L_2, L_3, L_4, L_5 through questionnaires and other forms, and get the comprehensive evaluation interval values of passenger satisfaction

$$r_1 = [0.8024, 0.8110]; r_2 = [0.7846, 0.7978];$$

$$r_3 = [0.7888, 0.7971]; r_4 = [0.8036, 0.8135];$$

$$r_5 = [0.8017, 0.8112].$$

The final results in [8] is

$$r_1 \succ r_4 \succ r_5 \succ r_3 \succ r_2,$$

That is

bus $L_1 \succ$ bus $L_4 \succ$ bus $L_5 \succ$ bus $L_3 \succ$ bus L_2 .

So L_1 obtain the "Service Star". However, from the point of interval data, the comprehensive evaluation value of r_4 is better than r_1 .

So, the results are not realistic.

Using our method, we get

$$T(r_1) = 0.5069 ; T(r_2) = 0.4917 ;$$

$$T(r_3) = 0.4938; T(r_4) = 0.5087;$$

$$T(r_5) = 0.5067.$$

Thus, the sorting results is

$$r_4 \succ r_1 \succ r_5 \succ r_3 \succ r_2,$$

That is

$$\text{bus } L_4 \succ \text{bus } L_3 \succ \text{bus } L_5 \succ \text{bus } L_1 \succ \text{bus } L_2.$$

So, L_4 obtain the "Service Star". This result is more in line with the objective reality.

5. Summary

Close degree is an important concept of uncertainty mathematical theory, and has been widely applied in solving practical problems. Against the selection of best option, the sorting of programs and other issues, we give the interval number sorting method based on close degree. Firstly, according to comprehensive evaluation interval values of various programs, we determine the positive and negative ideal interval values. Secondly, we calculate the close degree between comprehensive evaluation interval numbers of per program and the positive and negative ideal solution interval numbers. Then, according to the idea that the program is better if the distance with positive ideal solution is smaller and with negative ideal solution is greater, we give sorting index formula of each program. Finally, we calculate the sorting index values of each program, and sort them according to the size of sorting index values. Examples show that the sorting method of interval numbers based on close degree is a reasonable and practical sorting method.

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