A New Method Based on Wavelet Packet Transformation in 3D Face Feature Extraction

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Abstract. Transition from 2D to 3D in face pattern recognition in order to seek richer human face information but feature extraction as an important step among them can be seen as information compression. Obviously the two are contrary. For the fast effective locate and extract features in the large-scale 3D face data set, the highly effective digital compression computation becomes very important. This paper study a feature extraction method based on the wavelet packet transformation compression, proposed the specific method and the princip. When the actual data which distributed in 2D or higher dimensional projection space disobeyed assuming requirement or sample parameters inaccurate estimated, it cannot be correctly classified drawbacks or be correctly classified. Our experiment is indicated that our method can not only conquer this disadvantage but also meet the actual demand at the processing time. The experiment is indicated that our method towards 3D face feature extraction data sets compress processing is effective and feasible.

Introduction

The wavelet theory is rapidly expanded to be a new mathematical theory and method in the recent several years, but the wavelet analysis is rapidly expanded emerging disciplines in the nearly more than ten year. It simultaneously has the double meaning that the theory is profound and the application is very widespread. The wavelet packet theory is developed from the foundation of wavelet theory, which has overcome flaw and the insufficiency that the wavelet transformation in high frequency signal or the signal's high-frequency component frequency resolution low [1]. In the 3D face extract, large scale feature data sets must be compression which has the high request to the compression efficiency and the compression quality. As a result of digital image including massive information content, moreover increases along with the resolution increases suddenly, this gave the image without doubt the memory and the transmission has brought the difficulty. For the information memory and the transmission, the highly effective digital compression computation becomes very important.

But in the practical application, we are always interesting in the specific region, the special time or the frequency range section's signal; therefore we hoped to enhance resolution in these place that is interested. The wavelet analysis has been meeting this kind of requirement, and it carries on the frequency band to divide the multi-level[2]. The resolution analysis did not decomposed high-frequency unit decomposed in further, and auto-adapted to select the corresponding frequency band according to the characteristic which is analyzed, caused this to match with the signal frequency spectrum, thus raises the time-frequency resolution. Therefore, the wavelet packet has widespread application value.

The developing process wavelet packet analysis theory

Wavelet packet theory is developed from the foundation of wavelet theory, which has overcome flaw and the insufficiency that the wavelet transformation in high frequency signal or the signal's high-frequency component frequency resolution low [3]. For the information memory and the transmission, the highly effective digital compression computation becomes very important. But in the practical application, we are always interesting in the specific region, the special time or the
frequency range section's signal, therefore we hoped to enhance resolution in these place that is interested. The wavelet analysis has been meeting this kind of requirement. Many research show the wavelet packet function is the promotion of wavelet function [4]. Wavelet packet decomposition and restructuring algorithm will now be given let \( n \) be the negative binary integer,

\[
\sum_{i=-\infty}^{\infty} e_i 2^{-i} = \text{or} 1
\]

then the wavelet packet function \( \psi_n(t) \)'s Fourier transformation nation is given by the equation below:

\[
\hat{\psi}_n(\omega) = \frac{1}{2} \sum_{k=-\infty}^{\infty} h_k e^{-i\omega \cdot k} \cdot m_n(\omega) = \frac{1}{2} \sum_{k=-\infty}^{\infty} g_k e^{-i\omega \cdot k}
\]

It suppose \( \{u_n(t)\}_{n \in \mathbb{Z}} \) is orthogonal escale function \( \phi(t) \) under has the orthogonal wavelet packet, then \( <u_n(-k),u_n(-l)> = \delta_{k,l} \), namely \( \{u_n(t)\}_{n \in \mathbb{Z}} \) constitution the standard orthogonal basis of \( L^2(\mathbb{R}) \).

In the multi-resolution analysis, uses the above definition and property, and supposes:

\[
W_j = U_{j-1}^2 \oplus U_{j-1}^3, U_{j-1}^2 = U_{j-2}^4 \oplus U_{j-2}^5, U_{j-1}^3 = U_{j-2}^6 \oplus U_{j-2}^7
\]

Then we can obtain the wavelet subspace \( W_j \) which can be decomposing as follows:

\[
W_j = U_{j-1}^2 \oplus U_{j-1}^3 (4)
\]

\[
W_j = U_{j-2}^4 \oplus U_{j-2}^5 \oplus U_{j-2}^6 \oplus U_{j-2}^7 (5)
\]

\[
W_j = U_{j-2}^4 \oplus U_{j-2}^5 \oplus \cdots \oplus U_{j-2}^{j-1} (6)
\]

\[
W_j = U_{0}^2 \oplus U_{0}^3 \oplus \cdots \oplus U_{0}^{2^{j-1}-1} (7)
\]

\( W_j \) Space decomposes the subspace sequence may write \( U_{j-1}^{2^m} m=0,1,2 \cdots \), standard orthogonal basis of the subspace sequence \( U_{j-1}^{2^m} \) can be writen as \( \{2^{-j/2} u_{2^m}^{2^j-1} (2^j t - k) ; k \in Z \} \). It's easy to see that an \( l = 0, m = 0 \), the subspace sequence \( U_{j-1}^{2^m} \) are simplified as \( U_{j-1}^1 = W_j \), the corresponding orthogonal basis are simplified as \( 2^{j/2} \psi_{j,k,n} (2^j t - k) \), which exactly are standard orthogonal basis bunch \( \{ \psi_{j,k,n} (x) \} \).

If \( n \) is an octave thin division parameter, namely let \( n = 2^l + m \), and defines the wavelet packet the briefly written as \( \psi_{j,k,n}(t) = 2^{j/2} \psi_{j,k,n}(2^j t - k) \). and \( \psi_{n}(t) = 2^{j/2} u_{2^{m}}^{2^j-1} (2^j t) \). \( \psi_{j,k,n}(t) \) is called scales index \( j \), position index \( k \) and the frequency index \( n \) of wavelet packet. Defines the function \( \psi_{n}(t) \) which produces by the function bunch \( \psi_{j,k,n}(t) \) (and \( n \in Z_+ ; j,k \in Z_+ \)) is called sheaf of functions which constructs by the scale function \( \phi(t) \). Then, for each \( \psi_{j,k,n} \mid j = \cdots , -1,0; n = 2,3, \cdots \), \( k \in Z \) is an orthogonal basis of \( L^2(R) \).

After fig.1 is carries on the third-level young Pood criterion decomposes the result, \( LL_3 \) is the low frequency component; \( HL_j \) is the vertical edge detail; \( LH_j \) is the horizontal edge detail; In \( HH_j \) correspondence 45° ,135° direction detail \( \{i=1,2,3\} \).

The wavelet packet's decomposition algorithm is as simply showed as follows:
a. The first layer of wavelet decomposition

b. The third layer of wavelet decomposition

![Fig1. Level tower system wavelet decomposition](image)

By 

\[ d^{j+1,n}_i = \sum_k h_{k-2} d^{j+1,n}_k \]  

(8)

\[ d^{j+2,n}_i = \sum_k g_{k-2} d^{j+1,n}_k \]  

(9)

wavelet packet restructuring algorithm:

by \( \{d^{j,2n}_i\}\) and \( \{d^{j+2,2n+1}_i\}\) obtain \( d^{j+1,n}_i \) 

(11)

\[ d^{j+1,2n}_i = \sum_k (h_{k-2} d^{j+2,2n+1}_k + g_{k-2} d^{j+1,2n+1}_k) \]  

(12)

3D face Feature extraction Based on wavelet packet

When the wavelet packet carries on the 3D face feature extract, it must choose a quite good wavelet packet base, uses for to express the signal the characteristic. In order to select a quite good wavelet packet base, first assigns a sequence the cost function, then seeks in all wavelet packet base causes the cost function smallest base, assigned the vector regarding one, the minimum cost was the most effective expression, this base was called optimal base. Cost function and optimal base’s selection method as follows [5]:

**Cost Function**

The cost function may define as the sequence real function \( M \), but most practical is that which can survey the concentration degree the additivity cost function \( M \).

The additivity is refers to, if \( M(0) = 0, M(x_i) = \sum_i M(x_i) \) \( M \) is called a additivity information cost function. Commonly used cost function including several kinds as followed:

1. Bigger than some threshold the integer, namely initializes a random threshold \( \varepsilon > 0 \) and calculates the element integer \( \varepsilon \) which in the sequence the absolute value \( x \) is bigger.

2. Basis’s \( l^p, p < 2 \) norm’s denseness, namely chooses one to count \( p < 2 \) and to make the \( M(x) = \|x\|_p \).

3. Information entropy willfully, namely the definition sequence's \( x = \{x_i\} \) Shannon-Weaker entropy is \( M(x) = -\sum_j p_j \log p_j \), where \( p_j = \frac{x_j}{\|x\|} \) and \( p = 0 \). (attention: the information entropy only satisfies half additivity).

4. The logarithm entropy, namely ommand \( M(x) = \sum_j \log |x_j|^2, \log 0 = 0 \).

**Optimal base’s selection Method**

Supposes \( x = \{x_j\} \) for the space vector \( V \), records \( B \) for an orthogonal basis which selects from the storehouse. \( x \) is \( x \) under \( B \) base's coefficient, for \( x \in V \), if \( M(Bx) \) smallest, \( B \) is the optimal base. If the orthogonal basis storehouse satisfies the following condition:
1. The base vector composition's subset equates in the nonnegative integer collection $N$ has the following form sector:

$$I_{nk} = [2^n, 2^n(n + 1)]; k \in Z, n \in N$$  \hspace{1cm} (13)

2. In storehouse's each base corresponds to one $N$ by $I_{nk}$ is composed cover which does not intersect.

3. If $V_{nk}$ equates $I_{nk}$, so $V_{2nk} \oplus V_{2nk+1}$. Then we said that this orthogonal basis storehouse is a dual tree structure. If the storehouse is a tree, then may through to the induction found the optimal base. Records $B_{nk}$ is corresponds to $I_{nk}$ vector base, $A_{nk}$ is limited to $x$ the span $B_{nk}$ optimal base. For $k = 0$, existence single base, immediately most superior base $I_{nk,0}$, this time $A_{nk,0} = B_{nk,0}$ to all $n \geq 0$ establishment. Presently to $n \geq 0$ constructs $A_{nk+1}$ as follows:

$$A_{nk+1} = \begin{cases} B_{nk+1} + M(B_{nk+1}(x)) < M(A_{nk}(x)) + M(A_{nk+2}(x)) \\ A_{nk} \oplus A_{nk+1}, else \end{cases}$$  \hspace{1cm} (14)

So long as we satisfies about the cost function $M$ orthogonal basis $A(x)$ which above equation the algorithm produces $x$ namely for the optimal base.

**Experimental result**

We use the different valve value to carry on the compression to the face image experimental result as shown in Table 1, After different valve value processing image:

<table>
<thead>
<tr>
<th>Decomposition level</th>
<th>Threshold value (kb)</th>
<th>Before (kb)</th>
<th>After (kb)</th>
<th>Compression ratio(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>40</td>
<td>680</td>
<td>353</td>
<td>53.3%</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>680</td>
<td>366</td>
<td>55.2%</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>680</td>
<td>353</td>
<td>44.9%</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>680</td>
<td>370</td>
<td>50.0%</td>
</tr>
</tbody>
</table>

We may know through the table 1, under the different decomposition level, valve value's enlargement, the image compression ratio also increases, human eye obvious feeling image distortion. The method has good performance in improving the speed of extracting feature information, has good adaptive ability, enhance the overall performance as shown in Figure 2.

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**References**


