Forecast Model of Housing Prices Based on Grey System Theory

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Abstract. In this paper, the problem of forecasting housing price is studied. Based on the grey system theory, a mathematical model for forecasting housing price is established, and a forecasting example of housing price is given according to the historical data of housing price in Wuhan City to show the feasibility and validity of the presented forecast model. This paper tries to forecast the trend of housing price changes and to provide the reference basis for house-purchase people.

Introduction

Housing price is an important part of a city or region's competitiveness. Housing price plays very important influence on a country's industrial structure of the national economy, industrial policy and coordinated development of industry. Therefore, a comprehensive understanding of housing prices is the key to the whole operation of the housing market.

The housing problem is closely associated with a lot of economic and social problems. Thus, under the condition of rapid economic development, urbanization progresses, and the housing industry is becoming the economic pillar industries, how to deal with housing prices has become a difficult issue for all levels of local government. Too high prices will reduce the purchasing power of residents, and cause overheating and excessive investment in residential real estate speculation. And on the other hand, the economic boom phase usually causes rising housing prices. Therefore, to clarify the relationship between housing prices and the various factors, and to study the price change trend become urgently needs to be solved.

Rapid rise in house prices would reduce the life quality of residents. For the government, to stabilize the residential real estate prices is the primary task of macro-control. The premise is to clarify the relationship between the macro-control housing prices and the influencing factors, and to predict future price trends. Based on the above purpose, this paper attempts to forecast the trend of price changes, and provide the guide for government's macro-control and the reference basis for house-purchase people.

For the study of housing prices, many experts gave their opinions and views. The research shows that in terms of commercial housing price formation mechanism, some relative research are presented, especially in the theoretical modeling. They fully consider the impact of relevant variables and parameters on the final price. However, to use these theories to empirical research is still rare. In this paper, we will present a mathematical model for forecasting housing price, and give a forecasting example of housing price according to the historical data of housing price in Wuhan City to show the feasibility and validity of the presented forecast model.

Forecast model of Housing Price Based on GM(1,1) Model

In this section, we present a forecast model of housing price based on GM(1,1) model.

Suppose that there are n housing prices in past n year, we denote

$$X^0=\left\{X^0(1),X^0(2),\mathbf{L},X^0(n)\right\}$$

By the accumulated generating operation(AGO), we have

$$X^1=\left\{X^1(1),X^1(2),\mathbf{L},X^1(n)\right\}.$$

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The forecast model of housing price can be expressed by

$$\frac{dX^1}{dt}+aX^1=m,$$

where a is the development of grey number, m is the endogenous control grey number. Let $a=\begin{bmatrix} a \\ m \end{bmatrix}$, by using the least square method, we have

$$a=(B^TB)^{-1}B^TY_n,$$

where

$$B=\begin{bmatrix} -\frac{1}{2}\left[X^1(1)+X^1(2)\right] & 1 \\ -\frac{1}{2}\left[X^1(2)+X^1(3)\right] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left[X^1(n-1)+X^1(n)\right] & 1 \end{bmatrix},$$

$$Y_n=\begin{bmatrix} X^0(2) \\ X^0(3) \\ \vdots \\ X^0(n) \end{bmatrix}.$$

Solve this equation, we obtain the prediction formula of AGO sequence as follows.

$$\hat{X}^1(k+1)=\left[X^0(1)-\frac{m}{a}\right]e^{-ak}+\frac{m}{a},(k=1,2,\ldots,n). \tag{1}$$

Thus we get the prediction formula of original sequence as follows.

$$\hat{X}^0(k)=\left[X^0(1)-\frac{m}{a}\right]\times\left[e^{-a(k-1)}-e^{-a(k-2)}\right],(k\geq 2). \tag{2}$$

Now we select the average selling prices (in 10^{-2} Yuan) from 2009 to 2014 in Wuhan City as an application example to apply above forecast model.

Let the original sequence be

$$X^0(i)=\{51.99,55.50,66.76,68.95,72.38,78.06\}.$$

and the accumulated generating operation(AGO) is

$$X^1(i)=\{51.99,107.49,174.25,243.20,315.58,393.64\}.$$

so we get

$$\begin{aligned} a&=\left(B^TB\right)^{-1}B^TY_n \\ &=\begin{bmatrix} 0.0000210791 & 0.004482787 \\ 0.004482787 & 1.153333644 \end{bmatrix}\begin{bmatrix} -76124.21878 \\ 341.65 \end{bmatrix} \\ &=\begin{bmatrix} -0.07308 \\ 52.78778 \end{bmatrix}. \end{aligned}$$

Thus the model is

$$\frac{dX^1}{dt}-0.07308X^1=52.78778,$$

$$X^0(1)=51.99,$$

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$$\frac{m}{a}=\frac{52.78778}{-0.07308}=-722.29935,$$

$$X^0(1)-\frac{m}{a}=51.99-(-722.29935)=774.28475.$$

and the prediction formula of AGO sequence is

$$\hat{X}^1(k+1)=774.28475e^{0.07308k}-722.29935.$$

The detailed prediction values are listed in Table 1.

Table 1. The prediction values

$\hat{X}^1(1)$	$\hat{X}^1(2)$	$\hat{X}^1(3)$	$\hat{X}^1(4)$	$\hat{X}^1(5)$	$\hat{X}^1(6)$	$\hat{X}^1(7)$
51.99	110.69	173.85	241.79	314.89	393.53	478.13

Based on the data in Table 1, the prediction values of original sequence are as follows.

$$\hat{X}^0(1)=\hat{X}^1(1)=51.99,$$

$$\hat{X}^0(2)=\hat{X}^1(2)-\hat{X}^1(1)=110.96-51.99=58.70,$$

$$\hat{X}^0(3)=\hat{X}^1(3)-\hat{X}^1(2)=173.85-110.96=63.16,$$

$$\hat{X}^0(4)=\hat{X}^1(4)-\hat{X}^1(3)=241.79-173.85=67.94,$$

$$\hat{X}^0(5)=\hat{X}^1(5)-\hat{X}^1(4)=314.89-241.79=73.10,$$

$$\hat{X}^0(6)=\hat{X}^1(6)-\hat{X}^1(5)=393.53-314.89=78.64.$$

Next we do the modeling results testing as follows.

(1) Residual error test

The original sequence is

$$X^0(i)=\{51.99,55.50,66.76,68.95,72.38,78.06\}.$$

and the sequence of prediction values is

$$\hat{X}^0(i)=\{51.99,58.70,63.16,67.94,73.10,78.64\}.$$

Thus we can get the absolute error sequence is

$$\Delta^0(i)=\{0\ \ 3.20\ \ 3.60\ \ 1.01\ \ 0.72\ \ 0.58\}.$$

and the relative error sequence is

$$\Phi(i)=\left|\frac{\Delta^0(i)}{X^0(i)}\right|=\{0,5.77\%,5.39\%,1.46\%,0.99\%,0.74\%\}.$$

From above results, we can see that the absolute error and relative error are all small. So the residual error test passes.

(2) Grey relevance degree test

From the absolute error sequence, we have

$$\min\{\Delta^0(i)\}=\min\{0\ \ 3.20\ \ 3.60\ \ 1.01\ \ 0.72\ \ 0.58\}=0,$$

$$\max\{\Delta^0(i)\}=\max\{0\ \ 3.20\ \ 3.60\ \ 1.01\ \ 0.72\ \ 0.58\}=3.60.$$

Thus

$$e(i)=\frac{\min\{\Delta^0(i)\}+P\max\{\Delta^0(i)\}}{\Delta^0(i)+P\max\{\Delta^0(i)\}}(i=1,2,\ldots n;P=0.5),$$

Then the grey relevance degree is

$$r = \frac{1}{6}\{e(1)+e(2)+e(3)+e(4)+e(5)+e(6)\}$$
$$= \frac{1.000+0.4500+0.3333+0.6405+0.7143+0.7563}{6}$$
$$= 0.6491.$$

$r = 0.6491$ satisfies the condition $r > 0.6$ when $P = 0.5$, so the grey relevance degree test passes.

According to above test results, we can conclude that the GM(1,1) model can be used to forecast future housing prices. Thus, by using the (1) and (2), we can obtain the forecast housing prices of year 2015, 2016, 2017, 2018 and 2019 are 8459.72 (Yuan/Square meter), 9101.11 (Yuan/Square meter), 9791.13 (Yuan/Square meter), 10533.46 (Yuan/Square meter) and 11332.07 (Yuan/Square meter) respectively.

Conclusions

This paper studied the problem of forecasting housing price, and presented a forecasting model of forecasting the housing price based on the GM(1,1) model. Moreover, an application example of forecasting housing price of Wuhan City is given to show the feasibility and validity of the presented forecast model, and a satisfied forecast results are obtained.

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