Research on the robust emergency facility location

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Key words: emergency location; uncertain demand; robust optimization

Abstract: This paper apply robust optimization (RO) to the emergency granaries location. We consider a model of demand uncertainty within a bounded and symmetric multi-dimensional box, then apply RO approach to find out the optimal solution. Finally, we evaluate the potential benefits of applying the RO approach in numerical experiment, and we find that the model with box uncertainty set opening fewer, larger emergency facility, and the box uncertainty case can provide small.

Introduction

China is one of the countries with largest natural disasters in the world. Various of disasters happen in our vast land, such as earthquakes, typhoons, mudslides, floods, fires and droughts, bringing an enormous effect on people's lives and massive demands of emergency logistics. Today, although the technology and economy develop rapidly, there are large gaps in our emergency system and emergency supplies preparation. Strengthening the research of the emergency logistics system can improve our capability in disasters response and preparedness. And appropriate emergency granary location will help us be well prepared for the possible disasters.

Emergency Logistics refers to the special dynamic logistics activities in order to avoid severe natural disasters, sudden public health incidents, public safety events, military conflicts and other emergencies becoming worse. In emergency logistics context, people can take unconventional actions to ensure the demand of personnel, materials and money [1].

At present, research on emergency food logistics is few, so there is no uniform definition. As emergency food logistics including emergency logistics and food logistics, it can be defined as: emergency food logistics includes two aspects, one is doing emergency plan for possible emergency situations, and the other is doing related logistics activities to ensure effective food supply when the emergency event occurs [2].

To settle the emergency granary location problem, we should consider the actual situation, integrate the existing social resources and select the place that can timely supply the goods to the surrounding areas.

Facility location refers to the argumentation and decision-making process of location before construction. Fixed facilities location is a very important decision problem to the entire logistics network, which determines the model, structure and shape of whole logistics system. Location affects the way, quality, efficiency and cost of service, at the same time, which lead to an impact on the profitability and market competitiveness. It even determines the fate of the enterprise. On the contrary, the design of logistics system defines associated methods and costs during the operation. Facility location problem includes facilities number, location and size [3].

Snyder et al [4] describes two types of problems: stochastic location problems and robust location problems. In general stochastic optimization problems, the solution concept is to optimize the expected value of the objective function. Robust location problems seek to determine locations that are in some sense robust to uncertainty in problem parameters. Often this takes the form of a worst-case performance over a set of possible scenarios or intervals of uncertainty. For these problems, minimax-regret and minimax-cost are applied.

Baron et al [5] analyze the facility location problem using a robust optimization (RO) modeling approach where demand is uncertain and no probability distribution is assumed. On the basis of this, we apply RO approach to solving emergency granary location problems. In order to simplify the model, we only consider the box uncertainty case. At the same time, we make two improvements, firstly, we consider the cost of different roads. secondly, we consider not only the inventory expense but also the loading and unloading charges.

The paper is organized as follows: In section 2, we describe the setting, model the problem, and discuss our assumptions. In section 3, we compare the performance of alternate RO models and that of the nominal model in order to demonstrate the expected potential benefits of applying the RO approach to emergency granary location. Finally, We highlight the insights gained in this study and propose several future research directions.

Emergency Granary Location Problem

Nominal Problem Formulation

We formulate the nominal problem assuming that all data in the problem are deterministic and known.

- ***Sets***

N = Number of nodes in the network

A = Set of arc

- ***Index***

i, j = index for nodes

t = index for periods

- ***Parameters***

d_{ij} = Distance of shipping units between node i and j, d_{ii}=0, d_{ij}=d_{ji}

D_{it} = Demand in period t at node i

K_i = Cost of opening a granary at node i

C_{i0} = Cost per unit of loading and unloading established at the beginning

c_{it} = Cost per unit of loading and unloading at node i in period t

s_{ij} = Cost per unit of shipping between node i and j, s_{ii}=0, s_{ij}=s_{ji}

g_{it} = Cost per unit of inventory at node i in period t

T = Length of the horizon

η = Revenue per unit of fulfilled demand, η>0 l ***Decision Variables***

$$I_i \begin{cases} 1 \; if \;\; a \; granary \; is \; opened \; at \; node \; i \\ \qquad\qquad 0 \; otherwise \end{cases}$$

Z_{i0} = maximum inventory amount of an open granary at node *i*

X_{ijt} = Proportion of demand at node *j* in period t that is satisfied by an open granary at node *i*

Z_{it} = Inventory amount at node *i* in period *t*

t = Profit over the horizon

The problem is formulated as follows:

$$\max_{X,Z,I,Z_0} t$$

Subject to

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}[(h-d_{ij}s_{ij})D_{jt}X_{ijt}]-\sum_{i=1}^{N}\sum_{t=1}^{T}[(c_{it}+g_{it})Z_{it}]-\sum_{i=1}^{N}(C_{i0}Z_{i0}+K_iI_i)\geq t \qquad (1)$$

$$\sum_{j=1}^{N}(D_{jt}X_{ijt})\leq Z_{it} \qquad \text{for all } i,\ t, \qquad (2)$$

$$\sum_{i=1}^{N}X_{ijt}\leq 1 \qquad \text{for all } j,\ t, \qquad (3)$$

$$Z_{i0}\leq MI_i \qquad \text{for all } i, \qquad (4)$$

$$Z_{it}\leq Z_{i0} \qquad \text{for all } i,\ t, \qquad (5)$$

$$X_{ijt}\geq 0; \quad I_i\in\{0,1\} \qquad \text{for all } i,\ j,\ t. \qquad (6)$$

Constraint (1) represents the objective, which is placed in the constraint set to aid the RO formulation below. The first term expresses the revenue less delivery costs; the second term, the operation costs consist of load, unload and inventory expense; the third, the initial investment. Constraint (2) guarantees the demand satisfied by a granary is less than its inventory. Constraint (3) implies at most 100% of the demand at a node is fulfilled. Constraint (4) ensures that only open granaries is able to supply food. Constraint (5) implies the inventory at a granary in each period is less than the established maximum inventory amount.

The Robust Problem

For the case of box uncertainty, we assume that demand in each period are unknown and bounded on symmetric intervals around known nominal values.

Let $\tilde{D}_{jt}$ be the uncertain demand from node *j* in period *t* and let $\bar{D}_{jt}$ be its nominal value, i.e., the center point of the interval. Then we assume $\bar{D}_{jt}(1-e_t)\leq\tilde{D}_{jt}\leq\bar{D}_{jt}(1+e_t)$, for $0\leq e_t\leq 1$, for all *t*, where e_t means the uncertainty size in period *t*.

Therefore, the robust counterpart of problem with uncertainty sets are given by

$$\max_{X,Z,I,Z_0} t$$

Subject to

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}[(h-d_{ij}s_{ij})\bar{D}_{jt}(1-e_t)X_{ijt}]-\sum_{i=1}^{N}\sum_{t=1}^{T}[(c_{it}+g_{it})Z_{it}]-\sum_{i=1}^{N}(C_{i0}Z_{i0}+K_iI_i)\geq t \tag{7}$$

$$\sum_{j=1}^{N}\left[\bar{D}_{jt}(1+e_t)X_{ijt}\right]\leq Z_{it} \tag{8}$$

$$\sum_{i=1}^{N}X_{ijt}\leq 1 \qquad \text{for all } j,\ t, \tag{9}$$

$$Z_{i0}\leq MI_i \qquad \text{for all } i, \tag{10}$$

$$Z_{it}\leq Z_{i0} \qquad \text{for all } i,\ t, \tag{11}$$

$$X_{ijt}\geq 0;\quad I_i\in\{0,1\} \qquad \text{for all } i,\ j,\ t, \tag{12}$$

We let $M=\max_t\left\{\sum_{j=1}^{N}\left[\bar{D}_{jt}(1+e_t)\right]\right\}$ be an appropriately large number in (10).

Numerical Experiment

We randomly generate N nodes on a unit square, representing the demand points and potential granary locations. $\bar{D}_{jt}$ is assumed constant over the T periods. $\bar{D}_{jt}$ is drawn from the uniform distribution, Uniform [17,500, 22,500]. We let N=20 and T=10. the parameters summarized in Table 1.

Table 1 Parameter Values

Parameters	Meaning	Value
N	Quantity of nodes	20
T	Length of the horizon	10
$\bar{D}_{jt}$	Nominal demand at each demand point in period *t*	~ U[17,500, 22,500]
h	Revenue	2
C_{i0}	Cost per unit of loading and unloading established at the beginning	0.5
K_i	Cost of opening a granary at node *i*	30,000
g	Initial (first period) uncertainty	0.15
e_t	Uncertainty size in period *t*	$e_0=0$ $e_t = g + (1-g)e_{t-1}$
c_{it}	Cost per unit of loading and unloading at node *i* in period *t*	0.06
s_{ij}	Cost per unit of shipping between node *i* and *j*	~U[1,2]
g_{it}	Cost per unit of inventory at node *i* in period *t*	~U[0,0.1]

For the box uncertainty set, we assume $U_{jt}^{BD} = \bar{D}_{jt}[1 \pm e_t]$ and compare the solutions of the problem to those of the box uncertainty problem. We do so by considering the topology of the solutions, such as the number of granaries established, the number of average demand nodes and so on.

We use LINGO to solve the problem and get the topology of 5th period in Fig.1 and Fig. 2.We present typical solutions of the nominal model and Robust model. In the figure, a dot represents a demand node, and a circle denotes an open granary (co-located with a demand node). A line indicates that in at least one period the granary is used to satisfy demand at the node. The size of the circles is proportional to the inventory amount of the granary. The Solution of the Two Models see Table 1.

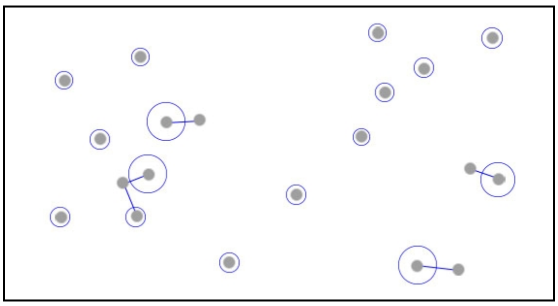

Fig.1 The Network Representation of Nominal Model

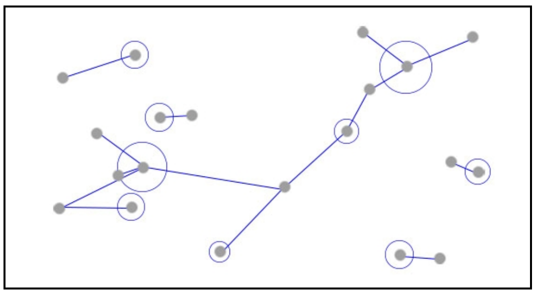

Fig.2 The Network Representation of Box Uncertainty Case

Table 2 Comparing the Solution of the Two Models

Model	Quantity of Opened Granaries	Average Quantity of Supply Nodes	Average Maximum Inventory Amount	Average Inventory Amount
Nominal	16	1.42	26421.07	24991.79
Robust	9	2.48	74625.48	67517.94

Base on Table 1 and Fig.1, 2, the robust model is better than the nominal model when faced with uncertain demand. Firstly, the robust model opens fewer granaries with larger supply capacities. Secondly, the robust model establishes more edges per open granary than the nominal model. Importantly, it allows more than one granary to serve the same demand node, expressing flexibility in the service to increase robustness to demand uncertainty. This indicates the robust models are performing as expected in terms of their ability to address uncertainty.

Conclusion

The objective of this paper is to investigate potential means by which RO techniques could be used to solve emergency granary location problems.

Firstly, we consider a multi-period fixed-charge emergency facility location problem. Secondly, by using the RO approach, we formulate the problem to include alternate levels of uncertainty over the periods. Thirdly, we consider the box uncertainty model. Finally, in numerical study, we evaluate the potential benefits of applying the RO approach. We find that the model with box uncertainty set opening fewer, larger emergency granaries, and it can provide small but significant improvements over the solution to the problem when demand is deterministic and set at it's nominal value.

Such a problem with 20 nodes and 10 periods may be considered to be of only semi-realistic size, but the results of the work reemphasize the need to evaluate the uncertainty in the emergency granary location problem. Future work may consider additional uncertainty sets such as a polyhedral and apply the uncertainty model to realistic emergency granary location problem by using the realistic data.

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