Multi-period Dynamic Sequential Optimization Model for Prefabricated Affordable Housing Production Scale

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Abstract. With the rapid development of living standard, the demand for housing is rising. Prefabricated affordable housing (PAH) is an important way to solve the problem. In this paper, the factors of sequential PAH demand and the future development trend in Shenyang is analyzed by regression equation. By mathematical programming theory, multi-period dynamic sequential optimization model for PAH production scale is established. The minimum difference between demand and supply is the objective function. It gives full consideration of the biggest supply capacity of PAH, the total investment in fixed assets restrictions, government-mandated minimum land area which must be used for PAH. For government departments, the model can provide quantitative decision-making optimization method for multi-period dynamic sequential PAH production scale.

Introduction

With the steady development of economy, housing demand standard is improving. Thereby, affordable housing demand standard for low income urban residents will also change. Currently, prefabricated construction (PC) becomes the main build way with higher speed, excellent quality, less climate condition influence, smaller resource burden, more obvious labor-saving features.

On protection of housing construction decisions, Zhangchen Chen provides new insights for demand forecasting of indemnificatory housing [1]. Ma Jian discusses the scale of investment in the construction of affordable housing [2]. Zhu Jiajie discusses the technology to analyze housing demand and the influencing factors [3]. Wang Xianlei makes deep discussion on how to establish a sustainable public housing supply system [4]. On PC aspect, Qi Baoku and Zhang Yang analyzes the existing bottleneck in the development of PC in view of four aspects [5]. Xu Songlin, Tan Yuang describes the development prospects of industrial technology in the application of PAH [6]. Lv Qing builds an economic estimation system model of fabricated buildings of low energy consumption[7].

Total Sequential Demand Estimating for PAH in Shenyang City

Usually, there are 3 people in a Chinese urban family, so in design, the mode of PAH should be dominated by two rooms and one hall. According to architectural design of the lowest standard, 45m^2 to 56m^2 of building area is advisable. In this paper, it is on the basis of 56m^2 for related research.

About the average housing area of urban households, the National Bureau of Statistics conducted a large-scale survey research in 2000. Based on current experience, the proportion of housing construction area and actual area is 1.3. Distribution of housing construction area is shown in table 1.

Table 1 Urban Households Housing Construction Area Distribution in Shenyang City in 2000

Building area (m^2)	< 26	26-52	52-78	78-104	104-130	> 130
Proportion (%)	7.8%	32.7%	35.5%	14.1%	5.4%	4.5%

Suppose the proportion in the area range is uniformly distributed, then in 2000, family proportion of house area is less than 56m^2 = 7.8% + 32.7% + (56-52) / (78-52) × 35.5% = 45.96%. As living standard continues to improve, the proportion of households below 56m^2 will drop. Taking the annual rate of 2% lower, the proportion is 19.96% in 2013. According to data Shenyang Municipal Bureau of

Statistics as shown in table 2, we can use regression equation (1) to predict Shenyang urban households from the year 2014 to 2019, namely, 271.8, 285.5, 300.7, 317.4, 335.6 and 355.2.

$$y = 0.7332x^2 - 1.6462x + 214.91 \qquad R^2 = 0.9262 \tag{1}$$

Table 2 Non-agricultural and Urban Population Between Year 2005 to 2013 in Shenyang City

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013
Non-agricultural population (ten thousand people)	584.5	590.5	599.7	618.8	620.5	624.7	643.2	656.8	662.3
The average urban household population (people)	2.78	2.74	2.7	2.75	2.81	2.8	2.66	2.81	2.55
Urban households Households (million)	210.3	215.5	222.1	225	220.8	223.1	241.8	250.8	259.7

According to the predicted value of urban households, PAH standard and the proportion of demand, accumulated and new PAH demand can be calculated as shown in Table 3.

Table 3 Cumulative and New PAH Demand from Year 2013 to 2019 (Million m^2)

Year	2013	2014	2015	2016	2017	2018	2019
Cumulative demand for PAH	2903.16	3037.71	3191.42	3361.51	3548.00	3750.87	3970.14
PAH in new housing stock	--	134.56	153.70	170.09	186.48	202.88	219.27

Analysis and Forecast to Supply Capacity Constraints of PAH in Shenyang City

Production-related Constraints Analysis of PAH in Shenyang. Production scale of PAH related constraints includes many factors. As shown in Table 4.

Table 4 Statistics Data in Shenyang City from Year 2005 to 2013

Year	Per capita GDP (¥)	Per capita revenue (¥)	Per capita disposable income (¥)	Per capita housing consumer spending (¥)	Per capita living space standard (m^2)	Commercial housing price (¥/m^2)	Urbanization rate (%)	Fixed assets investment (hundred million ¥)	Residential land supply (hectares)	Affordable housing land supply (hectares)
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	Y
2010	62357	7449.24	20541.23	1364.94	30.16	5109	71.60	1004.34	1095	51
2011	72648	9641.24	23326.20	1502.46	28.05	5614	78.50	1260.96	1303	56
2012	80486	10886.69	26430.83	1624.09	30.50	6174	78.80	1331.43	1329	71
2013	86850	12094.21	29074.42	2378.00	31.22	6242	80.21	1574.58	1004	53

PAH is different from economic product. It is necessary to build an optimization model to achieve the optimal state. We can calculate the linear correlation coefficient to screen the factors.

As long as the government investment and policy support, the scale of PC component production can meet the requirement of the construction in Shenyang city. From year 2005 to 2013, the influence degree of impact factors to the Shenyang PAH scale by linear correlation coefficient analysis is calculated, and the linear correlation coefficients and the key factors are shown in Table 5.

Table 5 The Linear Correlation Coefficients of each Indicator

Indicator	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
Linear correlation coefficient	0.37	0.36	0.32	0.13	0.09	0.54	0.40	0.18	0.73
Ranking	4	5	6	8	9	2	3	7	1

The Production and Supply Capacity Forecast of PAH in Shenyang. Select four most sensitivity indicators, x_9, x_6, x_7, x_1, eatablish multiple linear regression model to fit the relationship between PAH and land area. The expression is as follows. Forecast data in Shenyang city (I) is shown in table 6.

$$y = -6.13 + 9.29\times10^{-3}x_9 - 2.94\times10^{-2}x_6 - 8.06\times10^{-6}x_7 + 1.42\times10^{-3}x_1 \quad (2)$$

$$x_6 = 5103.5n^{0.1536} \qquad (R^2 = 0.9684) \quad (3)$$

$$x_7 = 6.0739\ln(n) + 72.452 \qquad (R^2 = 0.8959) \quad (4)$$

$$x_1 = 163.52n^2 + 5931.1n + 21469 \qquad (R^2 = 0.9975) \quad (5)$$

Table 6 Forecast Data in Shenyang City (I)

Year	2014	2015	2016	2017	2018	2019
x_9 (¥)	1079.03	1147.26	1131.52	1117.19	1120.50	1123.51
x_6 (¥/m^2)	6534.76	6720.35	6881.37	7023.97	7152.20	7268.89
x_7 (%)	82.23	83.33	84.27	85.08	85.80	86.44
x_1 (¥)	97132.00	106497.02	116189.08	126208.18	136554.32	147227.50

Based on the 1.2 multiplied, the largest supply capacity of PAH is shown in Table 7.

Table7 Predictive Value of Construction Scale/Supply Capacity of PAH (Million m^2)

Year	2014	2015	2016	2017	2018	2019
The supply capacity forecast of PAH	94.18	98.64	101.70	104.80	108.22	111.68
The largest supply capacity of PAH	113.02	118.37	122.04	125.76	129.86	134.02

Multi-period Dynamic Sequential Optimization Model for PAH Production Scale

Variables Setting. Variables are set as follows.

x_j' - demand scale of PAH in Shenyang city in year *j*.

y_j - maximum supply capacity of PAH in Shenyang city in year *j*.

x_j - supply scale of PAH in Shenyang city in year *j*.

f_j - unit construction cost of PAH in Shenyang city in year *j*.

F_j - fixed assets investment which and be used for PAH in Shenyang city in year *j*.

r - average floor area ratio (FAR) of PAH in Shenyang city. Here, the average value of FAR is 4.

G_j - land area which can be used for residential in Shenyang city in year *j*.

g - ratio of PAH construction land to total residential land.

q_j - unit market price of PAH in Shenyang city in year *j*.

c_j - unit sale price of PAH in Shenyang city in year *j*.

Q_j - subsidied capital which can be used on PAH construction in Shenyang city in year *j*.

Model Establishing. On the basis of above variable setting, multi-period dynamic sequential optimization model for PAH production scale is established as follows:

$$\text{MinZ} = \sum_{j=1}^{n} \left| x_j' - x_j \right| \qquad j = 1,2,\ldots n \quad (6)$$

$$\text{s.t.} \quad x_j \le y_j \qquad j = 1,2,\ldots n \quad (7)$$

$$f_j x_j \le F_j \qquad j = 1,2,\ldots n \quad (8)$$

$$x_j \ge g G_j / r_j \qquad j = 1,2,\ldots n \quad (9)$$

$$q_j x_j - c_j x_j \le Q_j \qquad j = 1,2,\ldots n \quad (10)$$

$$x_j \ge 0 \qquad j = 1,2,\ldots n \quad (11)$$

The objective function (6) is the demand and supply of construction minimize the difference. Constraint (7) shows the PAH assembly-scale supply of less then maximum supply capacity. Constraint (8) represents the total cost of PAH less than assets investment for the construction. Constraint (9) shows the construction of PAH land area greater than or equal government regulations must be used for construction of affordable housing land area. Constraint (10) represents the

difference between the market price and the actual selling price of PAH is less than government funding subsidies. Constraint (11) indicates construction scale of PAH supply is greater than zero. Forecast data in Shenyang city (II) is shown in table 8.

Table 8 Forecast Data in Shenyang City (II) (million ¥)

Year	2014	2015	2016	2017	2018	2019
Investment in Fixed Assets	1528.30	1643.81	1756.88	1867.75	1976.62	2083.67
Subsidies for the construction of PAH	504.25	443.92	396.86	348.73	298.39	248.25
Units Price of PAH (¥ / m^2)	3000					

By solving above programming model, the optimal solution is as follows: MinZ=96.48, $(x_1, x_2, x_3, x_4, x_5, x_6)^T = (134.56, 153.70, 162.72, 167.68, 173.15, 178.69)^T$

Sequential production scale predict data of PAH in Shenyang are as shown in Table 9.

Table 9 The Sequential Production Scale of PAH in Shenyang (Million m^2)

Year	2014	2015	2016	2017	2018	2019
The time series production scale of PAH in Shenyang	134.56	153.70	162.72	167.68	173.15	178.69

Conclusions

According affordable housing needs and constraints, by establishing production scale multi-stage dynamic sequential optimization model, you can get optimal PAH production scale per year among multi-stage. It benefits for surporting sustainable, healthy and stable development of city PAH. The feasible suggestion obtained by above optimizational model will make PAH and urban to develope in a more scientific way. Thus, it will improve city people's affordable housing utilities.

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