The Dynamic Game Analysis of Managers’ Investment Volume of Innovation Project under Financial Constraint Condition

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Abstract. Studying the reasons for the underinvestment of innovation project is a hot topic in corporate finance. Based on the study and analysis of dynamic models between investors and managers, this paper reveals that managers cut down the investment volume of innovation project under financial constraint condition; agency problem can enlarge the degree of underinvestment under financial constraint condition. These game results are also proved by examples.

Introduction

Innovation is the engine of keeping long-term competitive advantages for firms and rapid economic growth for countries [1–2]. Globally, specialists from various areas have been struggling to find out the ways of improving the investment volume of innovation project [3]. However, underinvestment of innovation project is still a common phenomenon all over the world [4–6].

Researchers in corporate finance suggest that financial constraint may be a key reason for the pervasive phenomena of the underinvestment of innovation project [3–5]. However, most of the related research is based on static views [3–5], and little research is established on dynamic views. Especially, scarce research devotes to study the dynamic game of investors and managers on the investment volume of innovation project under financial constraint condition. However, managers are the top directors who are responsible for executing innovation project, while investors are the vital capital providers who provide fund to initiate innovation project. Both investors and managers are the two key roles related to the investment volume of innovation project [6–7]. Therefore, it is necessary and interesting to analyze the dynamic game of investors and managers under financial constraint condition, and find out the reasons why the investment of innovation project is always severely cut down by managers in the end. Hence, this paper builds dynamic game models to analyze the game relationship between investors and managers about innovation project under financial constraint condition. We find that underinvestment of innovation project is the result of the dynamic game between managers and investors under financial constraint condition, and agency problem between investors and managers exacerbates the underinvestment degree of innovation project under financial constraint condition. The results are also proved by the examples in this paper.

Establishment of the Dynamic Game Model

Establishment of Functions and Parameters in the Dynamic Game Model. This paper builds the models based on the classic research [7]. (a) Assumptions about financial constraint: Financial constraint is a common situation that most firms are confronting with [8]. When firms suffer financial constraint, firms don’t process enough resources to invest into all projects, including innovation project and ordinary project, to the optimal investment volumes. Assume that firms only process enough resources to invest $I$ in total. When firms are under financial constraint condition, it is reasonable to assume that $I < I_0^* + I_1^*$ ($I_0^*$, $I_1^*$ are the optimal investment volumes of innovation project and ordinary project in efficient market.). Then, managers have to choose one of innovation project and ordinary project, or both of them, to cut down the investment volume. This paper is
interested in whether managers will finally choose to cut down the investment volume of innovation project under financial constraint condition.

(b) Assumptions about the profit function: $\pi$ is the profit function of the firm. $a$ is the amount of the effort devoted by managers. $I_1$ is the investment volume that managers invest in innovation project. $I_2$ is the investment volume that managers invest in ordinary project, such as constructing factory buildings, purchasing machines and inventory. $p_1$ and $p_2$ are the return ratios of each unit investment invested in innovation project and ordinary project respectively. Generally, innovation project is more profitable than ordinary project, hence $p_1 > p_2$. $\theta_1$ and $\theta_2$ are the random factors that related to each unit investment of innovation project and of ordinary project respectively, and obey normal distribution ($\theta_1 \sim N(0,\sigma_1^2)$; $\theta_2 \sim N(0,\sigma_2^2)$), and are independent with each other. The risk of innovation project is higher than that of ordinary project, hence $\sigma_1 > \sigma_2$. $f_1(\theta_1)$ and $f_2(\theta_2)$ are the density functions of $\theta_1$ and $\theta_2$ respectively. The cost functions of innovation project and ordinary project are $c_1(I_1) = \frac{b_1}{2}I_1^2$ and $c_2(I_2) = \frac{b_2}{2}I_2^2$ ($b_1 < b_2$). Therefore, the expression of the profit function is:

$$\pi = a + p_1I_1(1+\theta_1) + p_2I_2(1+\theta_2) - \frac{b_1}{2}I_1^2 - \frac{b_2}{2}I_2^2$$

(c) Assumptions about the sharing rule: $s(\pi)$ is the sharing rule contained in contract. $\alpha$ is the fixed salary that investors pay to managers. $\beta$ is sharing ratio, which is the ratio of the profit that investors assign to managers, and $0 \leq \beta \leq 1$. Therefore, the expressions of the sharing rule are:

$$s(\pi) = \alpha + \beta \pi; \quad s(\pi) = \alpha + \beta \left[ a + p_1I_1(1+\theta_1) + p_2I_2(1+\theta_2) - \frac{b_1}{2}I_1^2 - \frac{b_2}{2}I_2^2 \right].$$

(d) Assumptions about investors’ income: Investors are risk neutral. Hence, investors’ risk premium is 0. After assigning profit to managers, investors keep the left profit by themselves. Therefore, the expression of investors’ income is:

$$Y_i = -\alpha + (1-\beta) \left( a + p_1I_1 + p_2I_2 - \frac{b_1}{2}I_1^2 - \frac{b_2}{2}I_2^2 \right).$$

(e) Assumptions about managers’ income: Managers are risk aversion, and have a constant absolute risk aversion coefficient $\rho$ [7]. Hence, managers’ risk premium is $\frac{\rho \beta^2(I_1^2\sigma_1^2 + I_2^2\sigma_2^2)}{2}$. Suppose the cost function of managers’ effort is $c_v(a) = \frac{b_0}{2}a^2$. Therefore, the expression of managers’ income is:

$$Y_u = \alpha + \beta \left[ a + p_1I_1 + p_2I_2 - \frac{b_1}{2}I_1^2 - \frac{b_2}{2}I_2^2 \right] - \frac{b_0}{2}a^2 - \frac{\rho \beta^2(I_1^2\sigma_1^2 + I_2^2\sigma_2^2)}{2}$$

Set $\sigma^2 = I_1^2\sigma_1^2 + I_2^2\sigma_2^2$, then $Y_u = \alpha + \beta \left[ a + p_1I_1 + p_2I_2 - \frac{b_1}{2}I_1^2 - \frac{b_2}{2}I_2^2 \right] - \frac{b_0}{2}a^2 - \frac{\rho \beta^2 \sigma^2}{2}$.

The Action Order of Investors and Managers. Dynamic game means that the actions of the subjects follow an order. This paper assumes that there are only investors and managers in the game. All the investors’ benefit are the same, and all the managers’ benefit are also without differences. The design of incentive system includes the following game order and strategies:

Step1: The investors design a contract which contains fixed salary $\alpha$ and sharing ratio $\beta$.

Step2: When the managers get the content of the contract, managers need to decide whether to accept the contract or not. If managers accept the contract, managers will choose the effort level $a$, the investment volume of innovation project $I_1$, and that of the ordinary investment $I_2$.

Step3: The profits realize, and both investors and managers observe the profits.

Step4: According to the observation, investors and managers execute the contract.

Investors design different contracts under different situations, such as the situations that whether financial constraint and agency problem are exist or not. Hence, in Step 2, managers will also choose
different investment volumes of innovation. The profits in Step 3 vary with investors’ decisions in Step 1 and managers’ choices in Step 2.

Equilibrium in Dynamic Game

Because managers’ investment strategy of innovation project is the research object of this paper, we mainly focus on the investment volume of innovation project $I$ in the equilibrium of the dynamic game.

Equilibrium in Dynamic Game Problem with Financial Constraint and without Agency Problem. In this situation, both investors and managers have common knowledge about financial constraint (FC). Managers choose the investment volume of innovation project to maximize investors’ income $I$. Suppose the opportunity cost for managers to take the contract is $\bar{w}_m$. Investors should pay at least $\bar{w}_m$ to managers to get managers involve in, which is the individual rationality constraint (IR). Therefore, the dynamic game between investors and managers can be expressed as the following equations:

$$\max_{\alpha, \beta, \omega, h, I_1, I_2} Y_1 = -\alpha + (1-\beta) \left( a + p_1I_1 + p_2I_2 - \frac{b_1}{2} I_1^2 - \frac{b_2}{2} I_2^2 \right)$$

(\text{IR}) $$\alpha + \beta \left[ a + p_1I_1 + p_2I_2 - \frac{b_1}{2} I_1^2 - \frac{b_2}{2} I_2^2 \right] - \frac{b_1}{2} a^2 - \frac{p\beta^2}{2} = \bar{w}_m$$

(FC) $$I_1 + I_2 = I$$, $I$ is a fixed number, $I < I_1^* + I_2^*$.

According to these equations, we can get that the investment volume of innovation project at the equilibrium point $I_{eq}$ is equal to $[(p_1 - p_2) + \beta I + p\beta^2(\sigma_1^2 + \sigma_2^2)]/\left[\left(\frac{h_1}{2} + \frac{h_2}{2} + p\beta^2(\sigma_1^2 + \sigma_2^2)\right)\right]$. In the expression of $I_{eq}$, $\beta_{eq}$ is the equilibrium result of $\beta$ and $\beta_{eq} = 0$.

Hence, $I_{eq} = \frac{(p_1 - p_2) + \beta I}{h_1 + h_2}$.

Equilibrium in Dynamic Game Problem with Financial Constraint and Agency Problem. In this situation, both investors and managers have common knowledge about financial constraint (FC) and agency problem. Agency problem rises from that the benefits of investors and managers are not exactly converged with each other, and that information asymmetry exists between investors and managers (specifically, managers process more information about investment than investors) [9, 10].

Therefore, when agency problem exists, managers will choose the investment volume of innovation project which can maximize managers’ own income $Y_m$ but not investors’ income $Y_I$. This paper names it the first incentive compatibility constraint (IC$_1$).

What’s more, when agency problem exists, managers may not only choose the investment volume of innovation project which can maximize managers’ own income $Y_m$, but also further expropriate firms’ benefit from investment $Y_I$, such as building a larger scale firm to enlarge their power and social influence. This paper names it the second incentive compatibility constraint (IC$_2$).

(1) Equilibrium with the first incentive compatibility constraint (IC$_1$). The dynamic game between investors and managers can be expressed as the following equations:

$$\max_{\alpha, \beta, \omega, h, I_1, I_2} Y_1 = -\alpha + (1-\beta) \left( a + p_1I_1 + p_2I_2 - \frac{b_1}{2} I_1^2 - \frac{b_2}{2} I_2^2 \right)$$

(\text{IR}) $$\alpha + \beta \left[ a + p_1I_1 + p_2I_2 - \frac{b_1}{2} I_1^2 - \frac{b_2}{2} I_2^2 \right] - \frac{b_1}{2} a^2 - \frac{p\beta^2}{2} = \bar{w}_m$$

(\text{IC$_1$}) $$\max_{\alpha, \beta, \omega, h, I_1, I_2} Y_m = \alpha + \beta \left[ a + p_1I_1 + p_2I_2 - \frac{b_1}{2} I_1^2 - \frac{b_2}{2} I_2^2 \right] - \frac{b_1}{2} a^2 - \frac{p\beta^2}{2}$$

(FC) $$I_1 + I_2 = I$$, $I$ is a fixed number, $I < I_1^* + I_2^*$.

According to these equations, we can get that the investment volume of innovation project at the equilibrium point:
\[ I_{\text{IFA}}^* = \frac{(p_1 - p_2) + b_1 I + \rho b_2 \beta_{FA}^* \sigma_j^2 I}{b_1 + b_2 + \rho b_2 \beta_{FA}^* (\sigma_j^2 + \sigma_2^2)}. \]

In the expression of \( I_{\text{IFA}}^* \), \( \beta_{FA}^* \) is the equilibrium result of \( \beta \). It is very difficult to obtain the analytical solution of \( \beta_{FA}^* \). However, we can get that \( \beta_{FA}^* \neq 0 \) (specifically, \( 0 < \beta_{FA}^* \leq 1 \)). Because if \( \beta_{FA}^* = 0 \), the equations in 3.2 (1) cannot coexist. What’s more, as long as the values of \( p_1 \), \( p_2 \), \( b_1 \), \( b_2 \), \( b_3 \), \( \rho \), \( I \), \( \sigma_j \) and \( \sigma_2 \) are given, the value of \( \beta_{FA}^* \) can be uniquely determined.

(2) Equilibrium with the second incentive compatibility constraint (IC2). According to Motta (2003), managers expropriate \( ml^2 \) through the investment of ordinary project [11], \( m \) is the expropriation degree, and \( m > 0 \). Compared to ordinary project, innovation project contains more invisible property \( (s_1 \neq 0) \). Therefore, we can get that \( I_{\text{FAE}}^* \neq 0 \) and \( I_{\text{FAE}}^* = 0 \). However, we can get that \( FAE^* \) is not only determined by the return ratio \( \rho \) and the cost function’s coefficient of innovation project \( \sigma_j \), but also affected by the return ratio of ordinary project \( p_2 \), and the coefficient of innovation project \( \sigma_j \).

Results Analysis

In this session, we examine whether underinvestment of innovation project exist in the equilibrium of the dynamic game, and calculate the underinvestment degree if it exists. First, we calculate the perfect investment volume of innovation project \( I_i^* \). Second, we calculate the differences between the optimal investment volume of innovation project \( I_i^* \) and the equilibrium investment volumes of innovation project in the dynamic game under different situations \( I_{IF}^*, I_{IFA}^*, I_{FAE}^* \).

The perfect investment volume of innovation project satisfies the rule that the marginal return of the investment of innovation project is equal to the marginal cost of it [7]. That is \( p_i = c_i'(I_i) \). Therefore, we can get \( I_i^* = p_i / b_i \). (In the same way, we can also get the optimal investment volume of ordinary project \( I_1^* = p_1 / b_1 \). According to the expression of \( I_i^* \), only the return ratio \( p_i \) and the cost function’s coefficient of innovation project \( b_i \) can affect the optimal investment volume of innovation project.

Results Analysis of the Equilibrium in Dynamic Game Problem with Financial Constraint and without Agency Problem. According to the equilibrium in the dynamic game, the investment volume of the innovation project \( I_i^* \) is not only determined by the return ratio \( p_i \) and the cost function’s coefficient of innovation project \( b_i \), but also affected by the return ratio of ordinary project \( p_2 \), and the
cost functions’ coefficient of each project $b_1, b_2$, and the total investment volume $I$ which is determined by the available resources.

In order to examine whether underinvestment exists in innovation project, we compute the difference $\Delta R_i$ between $I_i^*$ and $I_{i_{FA}}^*$. And if $\Delta R_i > 0$, underinvestment exists. The larger $\Delta R_i$ is, the larger underinvestment degree of innovation project is.

The difference $\Delta R_i$ is: $\Delta R_i = I_i^* - I_{i_{FA}}^* = \frac{p_1}{b_1} \left( (p_1 - p_2) + b_1 I + p_1 b_1 I^* \right) - \frac{p_2 b_1 + p_1 b_1 - b_2 I}{b_1 + b_2} \left( b_1 I^* + b_2 I_{i_{FA}}^* \right)$.

Suppose $\Delta R_i = \frac{p_2 b_1 + p_1 b_1 - b_2 I}{b_1 (b_1 + b_2)}$, $\Delta R_i = \frac{b_1 b_2 + b_1 b_2^* (I_i^* + I_{i_{FA}}^*)}{b_1 (b_1 + b_2)} = 0$.

For $I < I_i^* + I_{i_{FA}}^*$, $\frac{p_2 b_1 + p_1 b_1 - b_2 I}{b_1 (b_1 + b_2)} > \frac{p_2 b_1 + p_1 b_1 - b_2 I}{b_1 (b_1 + b_2)}$ and $\Delta R_i > \Delta R_{i_{FA}}$.

Therefore, $\Delta R_i > 0$ and $I_i^* > I_{i_{FA}}^*$.

In conclusion, when firms suffer financial constraint, managers will choose to cut down $\Delta R_i$ of innovation project, generating underinvestment in innovation project.

**Results Analysis of the Equilibrium in Dynamic Game Problem with Financial Constraint and Agency Problem.** (1) Results analysis of the equilibrium with the first incentive compatibility constraint (IC$_1$). According to the expression of $I_i^*_{FA}$, managers’ decision about the investment volume of innovation project is not only determined by the return ratio $p_1$ and the cost $b_1$ of innovation project, but also affected by the return ratio of ordinary project $p_2$, the cost of ordinary project $b_2$, the risk of innovation project $\sigma_i^2$ and of ordinary project $\sigma_2^2$.

In order to examine whether underinvestment exists in innovation project, we compute the difference between $I_i^*$ and $I_{i_{FA}}^*$: The difference $\Delta R_i$ is:

$$\Delta R_i = I_i^* - I_{i_{FA}}^* = \frac{p_1}{b_1} \left( (p_1 - p_2) + b_1 I + p_1 b_1 I^* \right) - \frac{p_2 b_1 + p_1 b_1 - b_2 I}{b_1 + b_2} \left( b_1 I^* + b_2 I_{i_{FA}}^* \right).$$

The cost of innovation project is less than that of ordinary project ($b_1 < b_2$). The risk of innovation project is less than that of ordinary project ($\sigma_i^2 > \sigma_2^2$). Therefore, $b_1 \sigma_i^2 - b_2 \sigma_2^2 > 0$.

The return ratio of innovation project is higher than that of ordinary project ($p_1 - p_2 > 0$).

Therefore, $\frac{p_2 b_1 + p_1 b_1 - b_2 I}{b_1 (b_1 + b_2)} > 0$ and $\Delta R_i > 0$, hence $I_i^* > I_{i_{FA}}^*$.

In conclusion, when firms suffer financial constraint and agency problem, with the first incentive compatibility constraint (IC$_1$), managers will choose to cut down $\Delta R_i$ of innovation project in the game, generating underinvestment in innovation project.

What’s more, compared to the equilibrium of the game with financial constraint and without agency problem, the investment volume of innovation project will change $\Delta R_i^*$:

$$\Delta R_i^* = I_i^* - I_{i_{FA}}^* = \frac{(p_1 - p_2) + b_1 I}{b_1 + b_2} \left( (p_1 - p_2) + b_1 I + p_2 b_1 I^* \right) - \frac{(p_1 - p_2) + b_1 I + p_2 b_1 I^*}{b_1 + b_2} \left( b_1 I^* + b_2 I_{i_{FA}}^* \right).$$

Therefore, $\Delta R_i^* > 0$, hence $I_i^* > I_{i_{FA}}^*$. For $I_i^* > I_{i_{FA}}^*$, we can get $I_i^* > I_{i_{FA}}^*$. 

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(2) Results analysis of the equilibrium with the second incentive compatibility constraint (IC₂).

According to the expression of $I_{IFAE}^*$, the investment of innovation project is further affected by the expropriation degree $m$. In order to examine whether underinvestment exists in innovation project, we compute the difference between $I^*_i$ and $I_{IFAE}^*$. The difference $\Delta R_i$ is:

$$\Delta R_i = I^*_i - I_{IFAE}^* = \frac{p_i}{h_i} \left( \frac{(p_i - p_f) + b_f I - 2mI + \rho_{EFAE}^* \sigma_1^2 I}{b_i + b_f - 2m + \rho_{EFAE}^* \sigma_1^2 + \sigma_2^2} \right)$$

$$= \left\{ \frac{p_i b_f + p_i b_f - b_f I + (p_i - p_f)(\sigma_1^2 + \sigma_2^2) \rho_{EFAE}^* I + (b_i \sigma_1^2 - b_f \sigma_2^2)}{(b_i + b_f) \left[ b_i + b_f - 2m + \rho_{EFAE}^* \sigma_1^2 + \sigma_2^2 \right]} \right\} \frac{p_i}{h_i} \left( p_i - p_f \right) \sigma_1^2 + \sigma_2^2 \rho_{EFAE}^* I^2 > 0 \text{ and } b_i + b_f > 0.$$

From the equations in 3.1, we can get $I_{2F}^* = \left\{ \frac{(p_i - p_f) + b_f I}{b_i + b_f} \right\}$. For $I_{2F}^* > 0$ and $b_i + b_f > 0$, $(p_i - p_f) + b_f I > 0$. From 3.2 (2), we can get $I_{2FAE}^* = \left\{ \frac{(p_i - p_f) + b_f I + \rho_{EFAE}^* \sigma_1^2 I}{b_i + b_f - 2m + \rho_{EFAE}^* \sigma_1^2 + \sigma_2^2} \right\}$. For $I_{2FAE}^* > 0$ and $(p_i - p_f) + b_f I + \rho_{EFAE}^* \sigma_1^2 I > 0$, $b_i + b_f - 2m + \rho_{EFAE}^* \sigma_1^2 + \sigma_2^2 > 0$.

Therefore, $\frac{(p_i - p_f)(\sigma_1^2 + \sigma_2^2) \rho_{EFAE}^* I + (b_i \sigma_1^2 - b_f \sigma_2^2) \rho_{EFAE}^* I^2 + 2m[(p_i - p_f) + b_f I]}{(b_i + b_f) \left[ b_i + b_f - 2m + \rho_{EFAE}^* \sigma_1^2 + \sigma_2^2 \right]} > 0$.

Hence, we can get $\Delta R_i > 0$.

In conclusion, when firms suffer financial constraint and agency problem, with the second incentive compatibility constraint (IC₂), managers will choose to cut down $\Delta R_i$ of innovation project in the game, generating underinvestment of innovation project.

What’s more, compared to the equilibrium of the game with financial constraint and without agency problem, the amount of resources invest in innovation project will change $\Delta R_i$.

$$\Delta R_i = I_{IF}^* - I_{IFAE}^* = \left\{ \frac{(p_i - p_f) + b_f I}{b_i + b_f} \right\} \left( \frac{(p_i - p_f) + b_f I - 2mI + \rho_{EFAE}^* \sigma_1^2 I}{b_i + b_f - 2m + \rho_{EFAE}^* \sigma_1^2 + \sigma_2^2} \right)$$

$$\Delta R_i = \left\{ \frac{(p_i - p_f)(\sigma_1^2 + \sigma_2^2) \rho_{EFAE}^* I + (b_i \sigma_1^2 - b_f \sigma_2^2) \rho_{EFAE}^* I^2 + 2m[(p_i - p_f) + b_f I]}{(b_i + b_f) \left[ b_i + b_f - 2m + \rho_{EFAE}^* \sigma_1^2 + \sigma_2^2 \right]} \right\} > 0,$$ hence $I_{IF}^* > I_{IFAE}^*$.

For $I_i^* > I_{IF}^*$, we can get $I_i^* > I_{IF}^* > I_{IFAE}^*$.

Above all, when firms are with financial constraint and agency problem, managers will choose to cut down the investment volume of innovation project, resulting underinvestment in innovation project. What’s more, compared to the situation that firms suffer financial constraint but not agency problem, when firms suffer financial constraint and agency problem, managers will choose to further cut down the investment volume of innovation project, generating larger underinvestment degree of innovation project. Because the coexistence of financial constraint and agency problem is a more common situation than the situation that with financial constraint but not agency problem in the real world, the equilibrium results in 3.2 can better explain the severe underinvestment degree of innovation project in reality.

Summary

Underinvestment in innovation project is a common phenomenon all over the world. Using dynamic game model between investors and managers under financial constraint, this paper finds that: (1) Managers will choose to underinvest in innovation project under financial constraint condition. (2) Agency problem enhances the negative impact of financial constraint on the investment volume of innovation project. Specifically, when managers choose to invest to maximize managers’ own income (IC₁), the underinvestment degree of innovation project is enlarged. What’s more, when managers not only choose to invest to maximize managers’ own income but also expropriate firms’ benefit from investment (IC₂), the underinvestment degree of innovation project is enlarged further more.

There are several interesting related topics for further research. For example, we can incorporate competitive or collaborative relationship among managers into our model to extend the research.
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