The tunneling spectrum of Einsein Born-Infeld Black Hole

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ABSTRACT: The tunneling emission spectrum of massless particles via tunneling from the Einsein-Born-Infeld black hole has been researched in this paper. It is shown that that, when the particle's self-gravitation interaction is included, the emission spectrum of the EBI black hole is not precisely thermal, but satisfies the unitary theory.

INTRODUCTION

In 1974, Hawking first proved that the black hole can radiate particle, and the spectrum is precisely thermal, which had an important significance in the development of black hole physics[1,2]. Since then, several derivations of Hawking radiation appear in the literature, but most of them are based on quantum field theory on a fixed background spacetime without considering the fluctuation of the spacetime geometry. When considering energy conservation, it seems that the background geometry of a radiating black hole should be altered with the loss of energy, but this dynamical effect is often neglected in formal treatments. Recently, a new method to describe Hawking radiation as a tunneling process, where a particle moves in dynamical geometry, was initiated by Kraus and Wilczek and developed by Parikh and Wilczek[3-5]. There are two significant points in the tunneling picture. Firstly, they pointed out there is no preexisting barrier, and the barrier is created by the outgoing particle itself. Secondly, energy conservation , which was often neglected in the former treatments of Hawking radiation, was included. In the tunneling framework, they investigated Hawking radiation of static spherically symmetric Schwarzschild black hole and Reissner- Nordström black hole. The result shows the exact radiation spectrum deviates from the pure thermal, but is consistent with an underlying unitary theory. Following this method, many papers appeared to support the Parikh- Wilczek's opinion and presented a correct amendment to the exact emission spectrum of black hole[6-15]. In this paper, we develop the Parikh-Wilczek's method to study the Hawking radiation of massless particle via tunneling from the Einsein-Born-Infeld (EBI) black hole.

EBI BLACK HOLE IN DRAGGING COORDINATE SYSTEM

The space-time metric for the EBI black hole can be written as

$$ds^2 = -\frac{\Delta^2}{r^2}\left(dt_{EBI} - a\sin^2 q\, d j\right)^2$$

$$+\frac{r^2}{\Delta^2}dr^2 + r^2 dq^2$$

$$+\frac{\sin^2 q}{r^2}\left[a dt_S - \left(r^2 + a^2\right)d j\right]^2 \qquad (1)$$

where t_{EBI} is the coordinate time for EBI spacetime, and

$$\Delta^2 = r^2 - \left(2M + B\right)r + Q^2 + a^2 + C$$

$$r^2 = r^2 + a^2\cos^2 q\ . \qquad (2)$$

where,

$$a = \frac{J}{M},\quad C = c^2(0.8576\frac{c^2Q^2}{r_0^4}),$$

in which, c is a parameter. When $r_0 \to \infty$, the EBI black hole reduces to Kerr-Newman black hole. The ADM mass is related to the mass parameter of the black hole with

$$M_{ADM} = (1+\frac{B}{2M})M.$$

From the null super-surface equation, we can obtain the event horizon equation, i.e.

$$r^2 - (2M+B)r + Q^2 + a_{KN}^2 + C = 0 \qquad (3)$$

Solving Eq. (3) , we have

$$r_h = M + \frac{B}{2} \pm \sqrt{\left(M+\frac{B}{2}\right)^2 - Q^2 - a^2 - C} \qquad (4)$$

where “+”and “–” are internal and external horizon of the black hole respectively. The event horizon area of EBI black hole is given by

$$A_h = \int dA' = \int \sqrt{g}\, dq\, dj = 4p\left(r_h^2 + a^2\right) \qquad (5)$$

where

$$g = \sin^2 q\left(r_h^2 + a^2\right)^2.$$

The infinite red-shift surface is given by the equation

$$g_{00} = 0,$$

i.e.

$$\Delta^2 - a^2\sin^2 q = 0.$$

Obviously, the infinite red-shift surface is not consistent with the event horizon, so the geometrical optical approximation cannot apply here. In order to make coincidence between the infinite red-shift surface and the event horizon, we perform dragging coordinate transformation as

$$\dot{j} = \frac{dj}{dt_{EBI}} = -\frac{g_{03}}{g_{33}} = \Omega, \qquad (6)$$

for the line element (1) yields

$$ds^2 = \hat{g}_{00}dt_{EBI}^2 + \frac{r^2}{\Delta^2}dr^2 + r^2 dq^2 \qquad (7)$$

where

$$\hat{g}_{00} = g_{00} - \frac{g_{03}^2}{g_{33}} = -\frac{\Delta^2 r^2}{\left(r^2+a^2\right)^2 - \Delta^2 a^2 \sin^2 q}. \qquad (8)$$

Obviously, in dragging coordinate system, the infinite red-shift surface coincides with the event horizon.

GENERAL PAINLEVE COORDINATE AND QUANTUM TUNNELING

In (7), there is also a coordinate singularity at the event horizon, so it is necessary to eliminate the singularity. Here, we perform general Painleve coordinate transformation

$$dt_{EBI} = dt + F(r,q)dr + G(r,q)dq \qquad (9)$$

where $F(r,q)$ and $G(r,q)$ are two undeterminated function of r and q, and satisfy condition of integrability

$$\frac{\partial F(r,q)}{\partial q} = \frac{\partial G(r,q)}{\partial r}. \qquad (10)$$

Substituting (9) into line element (7), and Considering flat Euclidean space in radial, the line element in general Painleve-EBI spacetime can be written as

$$ds^2 = \hat{g}_{00}dt^2 + dr^2 \pm 2\sqrt{\hat{g}_{00}(1-g_{11})}dtdr$$

$$+\left[\hat{g}_{00}G^2(r,q)+g_{22}\right]dq^2 + \qquad +2\sqrt{\hat{g}_{00}(1-g_{11})}G(r,q)\,drdq\,,$$
$$+2\hat{g}_{00}G(r,q)\,dtdq\,, \qquad (11)$$

where "+" and "−" denote the line element of outgoing and ingoing particles at the event horizon of the black hole respectively.

Since the tunneling processes take place near the event horizon, we may consider a particle tunneling from the event horizon as an ellipsoid shell and think the particle should be still an ellipsoid shell during the tunneling process. So, the line element can be written as

$$ds^2 = \hat{g}_{00}dt^2 + dr^2 + 2\sqrt{\hat{g}_{00}(1-g_{11})}dtdr$$

$$+\left[\hat{g}_{00}G^2(r,q)+g_{22}\right]dq^2 + \qquad +2\sqrt{\hat{g}_{00}(1-g_{11})}G(r,q)\,drdq\,,$$
$$+2\hat{g}_{00}G(r,q)\,dtdq\,. \qquad (12)$$

From Eq.(12), we can get the radial null geodesics in these coordinates

$$\dot{r} = \sqrt{-g_{11}} - \sqrt{\hat{g}_{00}(1-g_{11})}\,, \qquad (13)$$

According to quantum field theory, when a pair of virtual particles spontaneously creates inside just inside the horizon, the positive energy virtual particle can tunnel out the horizon and materializes as a real particle; alternatively, for a pair created just outside the horizon, the negative energy particles, which are forbidden outside the horizon, can tunnel inwards. In either case, the negative particles are absorbed by black hole, resulting in a decrease of the mass of the black hole. Considering energy conservation and angular momentum conservation, when a massive particle is tunneled out as an ellipsoid massive shell of energy

$$\left(1+\frac{B}{2M}\right)w\,,$$

(including the static energy) and angular momentum,

$$\left(1+\frac{B}{2M}\right)\omega a$$

the mass parameters will be change to

$$M-\left(1+\frac{B}{2M}\right)w.$$

Thus, after the particle tunnels out, the event horizon of the black hole becomes

$$r_h' = \left(1+\frac{B}{2M}\right)(M-w) \pm \sqrt{\left(1+\frac{B}{2M}\right)^2(M-w)^2 - Q^2 - a^2 - C} \qquad (14)$$

and the dragging angular velocity of the black hole is given by

$$\Omega_h' = \frac{a}{r_h'^2 + a^2}\,. \qquad (15)$$

Since the event horizon coincides with the infinite red-shift surface, geometrical optics limit can be applied here. According to the WKB approximation, the tunneling rate is

$$\Gamma \sim e^{-2\,\mathrm{Im}S}\,, \qquad (16)$$

where the action

$$Im\,S = Im\left[\int_{r_i}^{r_f}\int_0^{P_r} dP_r' dr - \int_{j_i}^{j_f}\int_0^{P_r} dP_j' dj\right] \qquad (17)$$

where r_i and r_f correspond to the event horizons before and after the particle tunneling out. Here, it is treated as two turning points of potential barrier. The distance between them is determined by energy of the tunneling particle. According to Hamilton equation, we have

$$\dot{r} = \frac{dH}{dP_r}\bigg|_{(r;j,P_j)} ,$$

$$\dot{j} = \frac{dH}{dP_j}\bigg|_{(j;r,P_r)} , \qquad (18)$$

Substituting (18) into (17), the imaginary part of the action is given by

$$Im\,S = Im\int_{H_i}^{H_f}\int_{r_i}^{r_f}\left(\frac{dH'}{\dot{r}} - \frac{\Omega_h' dJ'}{\dot{r}}\right)dr$$

$$= Im\int_0^{w}\int_{r_i}^{r_f}\left(1+\frac{B}{2M}\right)\left(1-a\Omega_h'\right)\frac{d\left(M-w'\right)}{\dot{r}}dr$$

$$= -\frac{p}{2}\left(r_f^2 - r_i^2\right) . \qquad (19)$$

Then, the tunneling rate at the event horizon is

$$\Gamma \sim e^{-2\,Im\,S}$$

$$= e^{p\left(r_f^2 - r_i^2\right)} = e^{\Delta S_{BH}} . \qquad (20)$$

Obviously, when energy conservation and angular momentum conservation are considered, the exact spectrum of massless particle via tunneling from EBI black hole is not precisely thermal, but satisfies the unitary theory.

CONCLUSIONS

When self-gravitation interaction is included, the Hawking radiation of massless particle via tunneling from EBI black hole is not precisely thermal, but satisfies the unitary theory. Next, we will show, in some special case, Eq. (20) can recover the well-known results.

(i) When $r_0 \to \infty$, we have

$$\Gamma = e^{-2p\left[M^2-(M-w)^2+M\sqrt{M^2-a^2-Q^2}-(M-w)\sqrt{(M-w)^2-a^2-Q^2}\right]} ,$$

which is the tunneling rate of the Kerr-Newman black hole.

(ii) When $r_0 \to \infty$, Q=0, we have

$$\Gamma = e^{-2p\left[M^2-(M-w)^2+M\sqrt{M^2-a^2}-(M-w)\sqrt{(M-w)^2-a^2}\right]} ,$$

which corresponds to the tunneling rate of the Kerr black hole.

(iii) When $r_0 \to \infty$,Q=0,a=0, we have

$$\Gamma = e^{-4p\left[M^2-(M-w)^2\right]}$$

which is the tunneling rate of the Schwarzschild black hole[3].

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