Sparse beamforming in peer-to-peer relay networks

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Abstract. To reduce active relays in peer-to-peer relay networks, a sparsity promoting penalty term is introduced into the objective function to obtain beamforming weights through minimization of the total relay transmit power while the signal-to-interference-plus-noise ratio (SINR) at the destinations are guaranteed to be above certain thresholds. To deal with the non-convexity of the problem, we use semidefinite relaxation to turn this problem into a semidefinite programming (SDP) problem. Then we can efficiently solve the SDP problem using interior point method based software tools. Simulation results show that the proposed method can efficiently conduct relay selection with a mild increase in minimum total transmit power of the relays compared with the traditional full cooperative relay network.

I. Introduction

To enhance the utilization of the wireless network resources, energy efficient designs that enable the communication of multiple source-destination pairs over a shared channel are demanded. As a result, cooperative relay has attracted much attention due to its capability of power saving, throughput improvement and coverage extension. In the multi-relay amplify-and-forward (AF) networks, the system performance can be improved by the cooperation of relays to assist the source-destination transmission[1-5]. Generally, using all relays in the cooperative transmission can achieve the best throughput performance for an AF relay network. However, the cost of such full cooperation can be large due to power consumption and signaling overhead. To tackle this problem, another way is partial cooperation by using only a subset of the relays to assist the source-destination transmission[6-9].

In this paper, we consider the communication of multiple soure-destination pairs with the assistance of single-antenna relays. The joint problem of transmit beamforming and relay selection is considered. The goal is to obtain sparse beamforming weights through minimization of the total relay transmit power while satisfying the SINR requirements at the destinations. To deal with the non-convexity of the problem, we use semidefinite relaxation to turn this problem into a semidefinite programming (SDP) problem. Then we can efficiently solve the SDP problem using interior point method based software tools.

II. System model

We consider a wireless network in which communications between K source-destination pairs (S_k, D_k) take place in two stages with the aid of M relay nodes $\{R_m\}_{m=1}^{M}$, as shown in Fig. 1. We assume there is no direct link between any source and destination. We denote the channel from S_k to R_m as $f_{m,k}$ and the channel from R_m to D_k as $g_{k,m}$. A rich-scattering environment is assumed so that all the channels undergo independent Rayleigh flat fading.

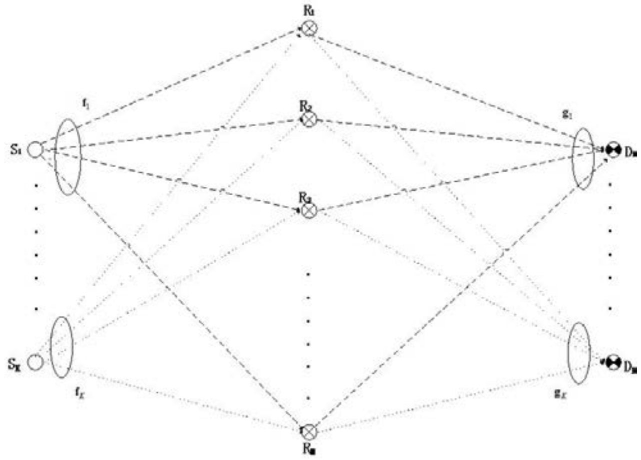

Fig.1 The system model of peer-to-peer relay network

In the first stage, all sources transmit their messages to the relays simultaneously so that the received signal at relay node R_m is

$$r_m = \sum_{k=1}^{K} f_{m,k} s_k + n_m\,, \quad \text{for } m = 1,2,\cdots,M \tag{1}$$

where s_k denotes the data symbol transmitted by source node S_k with $E\left\{\left|s_i\right|^2\right\} = p_i$ and n_m is the zero-mean complex Gaussian noise with variance σ_R^2 at R_m.

For convenience, (1) is rewritten in vector form as

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{f}_k s_k + \mathbf{n}_r \tag{2}$$

where $\mathbf{r} = [r_1, r_2, \cdots, r_M]^{\mathrm{T}}$, $\mathbf{f}_k = [f_{1,k}, f_{2,k}, \cdots, f_{M,k}]^{\mathrm{T}}$ and $\mathbf{n}_r = [n_1, n_2, \cdots, n_M]^{\mathrm{T}}$.

In the second stage, R_m forwards its received noisy signal weighted by a complex coefficient w_m. The transmit signal vector from all the relays can be written as

$$\mathbf{t} = \mathbf{W}\mathbf{r} \tag{3}$$

where $\mathbf{W} = diag(w_1, w_2, \cdots, w_M)$ is a diagonal matrix. Therefore, the total transmit power of the relays is

$$P_T = E\left\{\left\|\mathbf{t}\right\|^2\right\} = \sum_{m=1}^{M} \left|w_m\right|^2 E\left\{\left|r_m\right|^2\right\} = \mathbf{w}^{\mathrm{H}}\mathbf{D}\mathbf{w} \tag{4}$$

where $\mathbf{w} = [w_1, w_2, \cdots, w_M]^{\mathrm{T}}$, $\mathbf{D}$ is a diagonal matrix with diagonal elements

$$[\mathbf{D}]_{m,m} = \sum_{k=1}^{K} \left| f_{m,k} \right|^2 p_k + \sigma_R^2 \tag{5}$$

Let's define $\mathbf{g}_k = [g_{k,1}, g_{k,2}, \cdots, g_{k,M}]^{\mathrm{T}}$, then the signal received at D_k is

$$d_k = \mathbf{g}_k^{\mathrm{H}}\mathbf{t} = \mathbf{g}_k^{\mathrm{H}}\mathbf{W}\mathbf{f}_k s_k + \mathbf{g}_k^{\mathrm{H}}\mathbf{W}\sum_{\substack{i=1\\ i\neq k}}^{K} \mathbf{f}_i s_i + \mathbf{g}_k^{\mathrm{H}}\mathbf{W}\mathbf{n}_r + v_k \tag{6}$$

where v_k is the zero-mean complex Gaussian noise with variance σ_D^2 at D_k. Therefore the SINR at D_k can be written as

$$SINR_k = \frac{\mathbf{w}^{\mathrm{H}}\mathbf{R}_k\mathbf{w}}{\mathbf{w}^{\mathrm{H}}\mathbf{Q}_k\mathbf{w} + \mathbf{w}^{\mathrm{H}}\mathbf{S}_k\mathbf{w} + \sigma_D^2} \tag{7}$$

where

$$\mathbf{R}_k = p_k(\mathbf{g}_k \circ \mathbf{f}_k)(\mathbf{g}_k \circ \mathbf{f}_k)^{\mathrm{H}} \tag{8}$$

$$\mathbf{Q}_k = p_k \sum_{\substack{i=1 \\ i \neq k}}^{K} (\mathbf{g}_k \circ \mathbf{f}_i)(\mathbf{g}_k \circ \mathbf{f}_i)^{\mathrm{H}} \tag{9}$$

$$\mathbf{S}_k = \sigma_R^2 diag([\mathbf{g}_k \mathbf{g}_k^{\mathrm{H}}]_{11}, [\mathbf{g}_k \mathbf{g}_k^{\mathrm{H}}]_{22}, \cdots, [\mathbf{g}_k \mathbf{g}_k^{\mathrm{H}}]_{MM}) \tag{10}$$

where the notation $\circ$ denotes the element-wise Hadamard product and $[\cdot]_{kl}$ gives the (k,l) th element of a matrix.

III. Problem formulation and proposed method

In this section, the joint problem of transmit beamforming and relay selection is considered. We suppose only the best L relays are selected out of the M relays($L \le M$). The goal is to find the corresponding beamforming vector $\mathbf{w} = [w_1, w_2, \cdots, w_M]^{\mathrm{T}}$ so that the total relay transmission power is minimized, subject to receive-SINR constraints per subscriber.

$$\begin{aligned} &\min_{\mathbf{w}} \ \mathbf{w}^{\mathrm{H}} \mathbf{D} \mathbf{w} \\ &\text{s.t. } \frac{\mathbf{w}^{\mathrm{H}} \mathbf{R}_k \mathbf{w}}{\mathbf{w}^{\mathrm{H}} \mathbf{Q}_k \mathbf{w} + \mathbf{w}^{\mathrm{H}} \mathbf{S}_k \mathbf{w} + \sigma_D^2} \ge \gamma_k \ , \ \ k = 1, 2, \cdots, K \\ &\|\mathbf{w}\|_0 \le L \end{aligned} \tag{11}$$

where γ_k denotes the SINR target at D_k, the l_0-(quasi)norm is the number of nonzero entries of $\mathbf{w}$. Instead of the hard sparsity constraint, an l_0 penalty can be employed to promote sparsity, leading to

$$\begin{aligned} &\min_{\mathbf{w}} \ \ \mathbf{w}^{\mathrm{H}} \mathbf{D} \mathbf{w} + \lambda \|\mathbf{w}\|_0 \\ &\text{s.t. } \frac{\mathbf{w}^{\mathrm{H}} \mathbf{R}_k \mathbf{w}}{\mathbf{w}^{\mathrm{H}} \mathbf{Q}_k \mathbf{w} + \mathbf{w}^{\mathrm{H}} \mathbf{S}_k \mathbf{w} + \sigma_D^2} \ge \gamma_k \ , \ \ k = 1, 2, \cdots, K \end{aligned} \tag{12}$$

where λ is a positive real tuning parameter that controls the sparsity of the solution, and thus the number of selected relays. Problem (12) strikes a balance between minimizing the transmission power and minimizing the number of selected relays, where a larger λ implies a sparser solution. Note that for any λ, there is a corresponding L for which problems (11) and (12) yield the same sparse solution.

Because of the ℓ0-(quasi)norm, solving (12) requires an exhaustive combinatorial search over all $\binom{M}{L}$ possible sparsity patterns of $\mathbf{w}$, where the NP-hard problem (12) has to be solved for each of these patterns. This motivates the pursuit of computationally efficient solutions. To this objective, we will use the convex l_2-norm squared as a sparsity inducing regularization to replace the nonconvex ℓ0-norm in (11). Also notice $\mathbf{w}^{\mathrm{H}} \mathbf{Q}_k \mathbf{w} + \mathbf{w}^{\mathrm{H}} \mathbf{S}_k \mathbf{w} + \sigma_D^2 \ge 0$, problem (12) is then reformulated as

$$\begin{aligned} &\min_{\mathbf{w}} \ \ \mathbf{w}^{\mathrm{H}} \mathbf{D} \mathbf{w} + \lambda \|\mathbf{w}\|_2^2 \\ &\text{s.t. } \mathbf{w}^{\mathrm{H}} (\mathbf{R}_k - \gamma_k (\mathbf{Q}_k + \mathbf{S}_k)) \mathbf{w} \ge \gamma_k \sigma_D^2 \ , \ \ k = 1, 2, \cdots, K \end{aligned} \tag{13}$$

Unfortunately problem (13) is not a convex optimization problem in general. We thus resort to a semidefinite relaxation approach to solve a relaxed version of (13). Let us define $\mathbf{X} = \mathbf{w}\mathbf{w}^{\mathrm{H}}$, the optimization problem in (13) is rewritten as

$$\begin{aligned} &\min_{\mathbf{X}} \ \ trace((\mathbf{D} + \lambda \mathbf{I})\mathbf{X}) \\ &\text{s.t. } tr(\mathbf{R}_k - \gamma_k (\mathbf{Q}_k + \mathbf{S}_k)\mathbf{X}) \ge \gamma_k \sigma_D^2 \ , \ \ k = 1, 2, \cdots, K \\ &rank(\mathbf{X}) = 1 \text{ and } \mathbf{X} \ge 0 \end{aligned} \tag{14}$$

Drop the rank constraint(which is not convex), we aim to solve the following optimization problem

$$\min_{\mathbf{X}} \quad trace((\mathbf{D}+\lambda\mathbf{I})\mathbf{X})$$
$$\text{s.t. } tr(\mathbf{R}_k - \gamma_k(\mathbf{Q}_k + \mathbf{S}_k)\mathbf{X}) \geq \gamma_k \sigma_D^2 , \quad k = 1, 2, \cdots, K$$
$$\mathbf{X} \geq 0 \tag{15}$$

We see that the objective function and the first K constraints are all affine, and therefore, convex in $\mathbf{X}$. Also, the positive semidefinite constraint on $\mathbf{X}$ is convex. As a result, the optimization problem (14) is convex and it can be efficiently solved using software tools such as SeDuMi. In general, the matrix $\mathbf{X}_{opt}$ obtained from (15) is not necessarily of rank one. When the matrix rank is one, then its principal eigenvector is the optimal solution to the original problem. Otherwise, we have to use randomization technique to obtain a suboptimal rank-one solution from $\mathbf{X}_{opt}$ [10].

IV. Simulations

In this section, simulation results are presented to evaluate the effectiveness of the proposed approach. In all examples, the parameters are set as $K = 2$ and $M = 20$. For convenience, the target SINRs are set to be the same, i.e., $\gamma_k = \gamma$, ($k = 1, 2, \cdots, K$). We also assume that $p_k = 1$ and both channels are in Rayleigh flat fading with $E\left\{\left| f_{m,k}\right|^2\right\} = E\left\{\left|g_{k,m}\right|^2\right\} = 10$.

We plot the average total transmit power of the relays nodes against required target SINR in Fig.2 for three different penalty parameter $\lambda = 0$, $\lambda = 10$ and $\lambda = 50$. The case with $\lambda = 0$ means omitting the penalty part in (15), this is the full cooperation case in reference [3] and all relays are active. The performance comparisons in Fig.2 show that the average total transmit power of the relays nodes of the proposed method is only slightly higher than that of the full cooperation method in reference [3] when the penalty parameter λ is set to 10 and 50, which verifies the effectiveness of the proposed method.

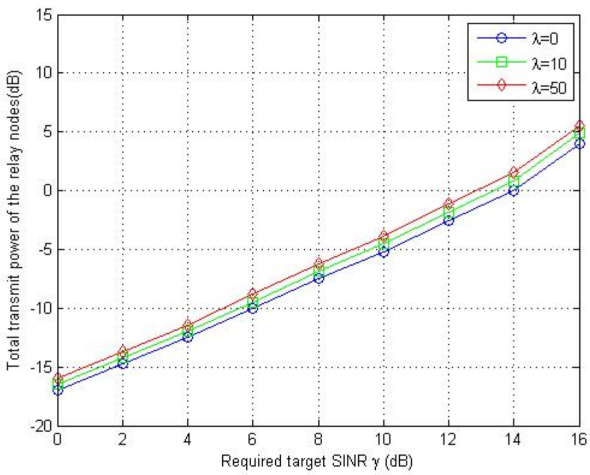

Fig.2 Average total transmit power of the relays nodes versus γ

Fig. 3 shows that more relays are involved in the cooperative transmission if the required target SINR γ increases, and larger λ refers to less active relays. From Fig.1 and Fig.2, we can see that compared with the full cooperation case, there is only mild increase of the average total transmit power of the relays nodes(less than 1.5 dB) using the proposed method. At the same time, the proposed method involves much less relay nodes. At $\gamma = 6\ dB$ and $\lambda = 10$, the proposed method yields similar average total transmit power of the relays nodes while using only about 13 relays out of the total 20 relays. At $\gamma = 6\ dB$ and $\lambda = 50$, the proposed method yields similar average total transmit power of the relays nodes while using only about 11 relays out of the total 20 relays.

The reason behind is that because different communication links have different fading characteristics, not all relay nodes contribute to the improvement of system performance. By appropriately selecting relay nodes in beamforming we can produce similar system performance.

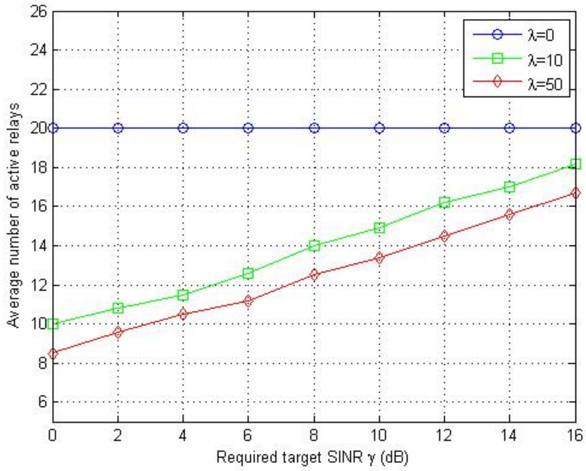

Fig.3 Average number of active relays versus γ

Therefore, we can conclude that the proposed method with reasonable λ can achieve similar minimum average total transmit power of the relays compared with the full cooperation case, but just use a subset of relays.

V. Conclusion

This paper addressed the joint problem of transmit beamforming and relay selection in multiple peer-to-peer networks. The simulation results showed that the proposed method can effectively balance the size of active relays and the total relay transmit power. The proposed approach can effectively perform relay selection while satisfying the target SINR constraints and has only a mild increase in the relay transmit power as compared to the full cooperation case.

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References

[1] Y. Zhao, R. Adve, and T. Lim, "Beamforming with limited feedback in amplify-and-forward cooperative networks," IEEE Trans. Wireless Commun., vol. 7, no. 12, pp. 5145–5149, Dec. 2008.

[2] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," IEEE Trans. Inf. Theory, vol. 55, no. 6, pp. 2499–2517, Jun. 2009.

[3] Siavash Fazeli-Dehkordy, Shahram Shahbazpanahi and Saeed Gazor, "Multiple Peer-to-Peer Communications with a network of relays," IEEE Transactions on Signal Processing, vol. 57, no. 8, pp. 3053-3062, August 2009.

[4] G. Zheng, K.-K. Wong, A. Paulraj, and B. Ottersten, "Robust collaborative-relay beamforming," IEEE Trans. Signal Process., vol. 57, no. 8, pp. 3130–3143, Aug. 2009.

[5] Y.-W. Liang and R. Schober, "Cooperative amplify-and-forward beamforming with multiple multi-antenna relays," IEEE Trans. Commun., vol. 59, no. 9, pp. 2605–2615, Sep. 2011.

[6] T. Tao and A. Czylwik, "Beamforming design and relay selection for multiple MIMO AF relay systems with limited feedback," in Proc. IEEE VTC Spring, 2013, pp. 1–5.

[7] Mingxiong Zhao, Xiangfeng Wang, Yuan Liu, et al.. Sparse Beamforming in Multiple Multi-Antenna Amplify-and-Forward Relay Networks. IEEE Communications letters, vol. 18, no. 6, pp.1023-1026, JUNE 2014.

[8] M. Y. Hong, R. Y. Sun, H. Baligh, and Z.-Q. Luo, “Joint base station clustering and beamformer design for partial coordinated transmission in heterogeneous networks,” IEEE J. Sel. Areas Commun., vol. 31, no. 2, pp. 226–240, Feb. 2013.

[9] J. Zhao, T. Quek, and Z. Lei, “Coordinated multipoint transmission with limited backhaul data transfer,” IEEE Trans. Wireless Commun., vol. 12, no. 6, pp. 2762–2775, Jun. 2013.

[10] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, “Transmit beamforming for physical-layer multicasting,” IEEE Trans. Signal Process., vol. 54, no. 6, pp. 2239–2251, Jun. 2006.