Disturbance Source Location Method of Low Frequency Oscillation With Time-varying Steady Points

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Keywords: low frequency oscillation, disturbance source location, energy decomposition, dissipation energy, steady point identification

Abstract. Low frequency oscillation has been a serious threat to security and stability of power grid, and how to locate the disturbance source accurately is an important issue to suppress low frequency oscillation. The existing location method has poor adaptability to oscillation with time-varying steady points because of the limitations in the derivation of location criterion. This paper presents a disturbance source location method of low frequency oscillation with a better versatility. Firstly, the mechanism and response characteristics of oscillation with time-varying steady points is analyzed, then the energy function of a specific construction form is decomposed into state energy, reciprocation energy and dissipation energy by mathematical derivation. The flow of dissipation energy shows the origin and destination of disturbance energy, so it can be used to identify the specific location of disturbance source. A recognition method for electrical quantities’ steady points is also proposed based on cubic spline interpolation in order to meet the needs of energy calculation. The calculation results of examples show that the derivation and analysis of energy structure in this paper is correct, and it is able to locate the disturbance source accurately according to the dissipation energy.

Introduction

Low frequency oscillation has become an important problem that affects the security and stability of the modern power systems’ operation. During the operation period of the domestic and foreign power grid, there have been many serious low frequency oscillation accidents, which include the negative damping oscillation [1-2] and the forced power oscillation [3-5]. Negative damping oscillation is caused by small disturbance to the negative damping of the generator control system with the system’ sustained increase oscillation for the uncontrolled swing of the disturbed generator rotor[6-8]. The negative damping oscillation , caused by local disturbance, can not been eliminated and attenuates unless the the disturbance source is removed[9].

It is a significant job to locate the source of the oscillation rapidly and accurately after the low frequency oscillation caused by local disturbance. At present, some achievements have been made in the research on the method of the source location of the disturbance. A practical method for calculating the energy flow is proposed[10-12], which reduces the transient energy component in the network and can only calculate the energy consumption or the energy generated by the branch, and locate the disturbance source according to the direction of the energy flow.There also have literatures introduces that different levels of network cut set can be constructed by the WAMS...
dynamic information based on the key line, and the vibration source is located by searching the network cut sets whose disturbance energy flow outwards[13-14]. Also, the abnormal condition of the control system can be troubleshooted by the vibration energy of the excitation torque and speed torque. Besides, paper[15] proposes a method to extract the wave propagation delay time of the perturbed traveling wave in transmission line by using the waveform similarity of multi point sampling data, which can be used to calculate the position of the low frequency oscillation disturbance source in the grid.

The low frequency oscillation of power system is a kind of symmetrical reciprocating motion at the center of steady state operation. When the steady points is fixed, the central axis of the oscillating curve is a horizontal line. However, due to the changes of the network structure, the output of the generator during the oscillation process, and the load of the transmission network( the load increases during the morning and evening peak ), the steady points of the generator and transmission network will change with the time. When the low frequency oscillation occurs with time-varying steady points, the oscillation curve will drift. At present, it is difficult to apply the existing disturbance source location criterion to oscillation with time-varying steady points, and those methods can only guarantee the accuracy of the location in the condition of small variation of the steady state operation point.

In this paper, a new method for disturbance source location of low frequency oscillation with time-varying steady points is proposed, and it also can be applied to the low frequency oscillation with stationary steady points. The energy function of a specific construction form is decomposed into state energy, reciprocation energy and dissipation energy by mathematical derivation. The flow of dissipation energy shows the origin and destination of disturbance energy, so it can be used to identify the specific location of disturbance source. Simulation of actual system of central China power grid shows that his method can be used to identify the disturbance source of drift oscillation and can be applied to the on-line monitoring and off-line analysis.

### Analysis of Low Frequency Oscillation with Time-varying Steady Points

**Small Perturbation Response.** Taking a single machine infinite bus system as an example, the response analysis of the low frequency oscillation with time-varying steady points is carried out. The generators are based on the classical two order model, the rotor motion equation is:

\[
\begin{align*}
M \frac{d\omega}{dt} &= P_m - P_e - D(\omega - 1) \\
\frac{d\delta}{dt} &= \omega - 1
\end{align*}
\]  

(1)

Linearize the Eq.1 at the working point, the equation is:

\[
M \Delta \dot{\delta} + D \Delta \delta + K \Delta \delta = \Delta P_m .
\]

(2)

In the Eq.2, \( K = (E'U \cos \delta_0) / X_e \).

In the actual power grid, the low frequency oscillation with time-varying steady points is often caused by the change of the mechanical power of the generator. It can be considered approximatively that the B is a constant and the generator output increases linearly with time. The Eq.2 is a two order constant coefficient nonhomogeneous linear differential equation, the general solution of the corresponding homogeneous equation is:
\[
\Delta \delta_1(t) = A e^{\left(\frac{(D/2M)^2}{2}\right) t} \cos(\omega t + \varphi). 
\]

In the Eq.3, \(\omega = \sqrt{4MK - D^2 / 2M}\); \(A\) and \(\varphi\) are two constants determined by the initial conditions.

The special solution of formula is:
\[
\Delta \delta_2(t) = ct. 
\]

In the Eq.4, \(c\) is constant. Thus, in the case of time-varying steady points, the small disturbance response of the generator rotor angle can be decomposed into the free component represented by the Eq.3 and the forcing component represented by the Eq.4. The forcing component is a straight line with a constant slope, which is the main reason for the drift of the oscillating curve. If that the damping coefficient \(D\) of the generator contains the damping of the control system, the negative damping of the excitation system will increase with the increase of the generator output. So the damping of the free component will diminish gradually.

**Oscillation Characteristics.** Fig.1 is for the active power waveform of the disturbance source unit for the two simulations.

Fig.1 shows that the oscillation 1 belongs to the low frequency oscillation with stationary steady points. The active power curve of the oscillation is around the horizontal axis, and the average value of the active power of each oscillation cycle is maintained by 500MW. Oscillation 2 belongs to the low frequency oscillation with time-varying steady points. The oscillation curve rise gradually with time and the average active power of each oscillation cycle increases gradually, by first 500MW increasing to 600MW. The low frequency oscillation with stationary steady points can be seen as a special case of the low frequency oscillation with time-varying steady points. Therefore, the method of locating the source of the oscillation, proposed in this paper, is equally applicable to the type of oscillation with the stationary steady points. Prony algorithm is a kind of algorithm for the analysis of steady signal and can get the accurate identification results by extracting data only from the steady state operation points[16]. Therefore, the accuracy of the results can not be ensured by using Prony for the curve of low frequency oscillation with time-varying steady points.

**Energy decomposition and disturbance source location**

**Energy Fuction.** The literature[17-18] presents a method for constructing the energy function of generator, line and load, and derives the energy conservation equation. The energy flowing from the bus \(i\) to the bus \(j\) after the oscillation is:
\[ E_{ij} = \int \text{Im}(\mathbf{f}_{ij}^\dagger d\mathbf{U}) = \int \text{Im}\left[ \frac{P_{ij} + jQ_{ij}}{U_i} (dU_i e^{j\theta_i}) \right] = \int \text{Im}\left[ \frac{P_{ij} + jQ_{ij}}{U_i} e^{j\theta_i} dU_i + jU_i e^{j\theta_i} d\theta_i \right] = \int P_{ij} d\theta_i + \frac{Q_{ij}}{U_i} dU_i \, . \quad (5) \]

In the Eq.5, \( \mathbf{f}_{ij}^\dagger \) is the conjugate of the current phase of the branch \( L_{ij} \); \( \theta_i \) is the voltage phase of the bus \( i \); \( P_{ij} \) is the active power transmitted from bus \( i \) to bus \( j \), and \( Q_{ij} \) is the reactive power transmitted from bus \( i \) to bus \( j \).

**Energy decomposition.** The energy transmitted in the line is transformed relative to steady points as follows:

\[ E_{ij} = \int P_{ij} d\theta_i + \frac{Q_{ij}}{U_i} dU_i = \int (P_{ij,s} + \Delta P_{ij}) d(\theta_{i,s} + \Delta \theta_i) + \int (Q_{ij,s} + \Delta Q_{ij}) d(ln U_{i,s} + ln U_i - ln U_{i,s}) \, . \quad (6) \]

In the Eq.6: \( P_{ij,s}, Q_{ij,s} \) respectively, the steady active power and the steady reactive power of the branch \( L_{ij} \) at each time point. \( \Delta P_{ij}, \Delta Q_{ij} \), respectively, for the variation of the active power and the variation of the reactive power to the steady points at each moment. \( lnU_i, lnU_{i,s} \), respectively, for the natural logarithm of the fluctuant voltage and the steady voltage of bus \( i \) at each time point.

Transform the Eq.6:

\[ E_{ij} = \int P_{ij,s} d\Delta \theta_i + \int Q_{ij,s} d(ln U_i - ln U_{i,s}) + \int \Delta P_{ij} d\theta_i + \int \Delta Q_{ij} d(ln U_{i,s}) + \int P_{ij,s} d\theta_{i,s} + \int Q_{ij,s} d(ln U_{i,s}) + \int \Delta P_{ij} d\Delta \theta_i + \int \Delta Q_{ij} d(ln U_i - ln U_{i,s}) \, . \quad (7) \]

By Eq.7, the energy components in the transmission energy of the branch can be classified, and the calculating formula of the discretization can be achieved.

**State energy.** The state energy of the branch \( L_{ij} \) can be defined as follows:

\[ E_{sta} = \int P_{ij,s} d\theta_{i,s} + \int Q_{ij,s} d(ln U_{i,s}) = \sum_{k=1}^{n} \left[ P_{i,k} (\theta_{i,k} - \theta_{i,k-1}) + Q_{i,k} (ln U_{i,k} - ln U_{i,k-1}) \right] \, . \quad (8) \]

In the Eq.8, \( n \) is the number of sampling points.

The state energy is generated by the change of the steady points of the branch, which is absorbed by the branch to make it in a new steady state. If the steady points do not change, this part of energy will kept unchanged.
Dissipation energy. The dissipation energy of the branch Lij can be defined as follows:

\[
E_{\text{dis}} = \int \Delta P \, d\Delta \theta + \int \Delta Q \, d(\ln U_1 - \ln U_{i-1})
\]

\[
= \sum_{j=2}^{n} \left[(P_{ij} - P_{j,i})\theta_{k-1} - \theta_{k,i} - \theta_{k,j} + \theta_{k,j-1} + (Q_{ij} - Q_{j,i})(\ln U_j - \ln U_{i,j} - \ln U_{k-1} + \ln U_{i,k-1})\right].
\]  

The dissipation energy has a clear flow, which is generated by the disturbance source and transmitted to the system's positive damping element. The dissipation energy can reflect the position of the oscillating source and the damping property of the network element.

Reciprocation energy. The adjustment of the branch steady points and the propagation of the dissipation energy are carried out with the reciprocating oscillation. The reciprocation energy of the branch Lij can be defined as follows:

\[
E_{\text{rec}} = \sum_{k=2}^{n} \left[P_{k,j}(\theta_{k,i} - \theta_{k,j} - \theta_{k,j-1} + \theta_{k,j-1}) + (P_{i,j} - P_{j,i})(\ln U_i - \ln U_{i,j} - \ln U_{k-1} + \ln U_{i,k-1}) + (Q_{i,j} - Q_{j,i})(\ln U_{i,j} - \ln U_{j,i})\right].
\]  

Reciprocating oscillation energy can be understood as the carrier of energy transmission. In general, it does not propagate in one direction, but in periodic reciprocating flow. Its fluctuation curves are symmetric and have a horizontal symmetric axis, and the average energy is close to 0.

**Disturbance source location criteria.** In the low frequency oscillation, the dissipation energy, generated by the negative damping element, flows to the network with the oscillating energy and is consumed in the positive damping element. If the total energy consumption of the system is greater than the energy increase, the oscillation amplitude of the system will be reduced gradually; If the total energy consumption of the system is less than the energy increase, the oscillation amplitude of the system will becomes greater. If the energy consumption and the energy increase are equal, the oscillation of the system will be equal amplitude oscillation[10].

The Eq.9 can be used for the calculation of the dissipation energy of the generator, line and load. When the dissipation energy of the line Lij, which flows from bus i to bus j, is positive, the disturbance source is near the end of the bus i. Drawing on the concept of cut set proposed by [13], the whole power grid is divided into A and B two subsystems. When the total dissipated energy of the whole cut set flows from A system to B system, the disturbance source can be located in the A system. Then, the A system can be further divided into smaller subsystems and the suspicious region of the disturbance source can be further reduced. When the cut set contains only the generator outlet, if the dissipation energy is positive, the generator is a disturbance source; if the dissipation energy is negative, the generator is not a disturbance source.

**Steady points identification**

The calculation of each energy component requires the identification of the electric quantities’ steady points at each time point. These electric quantities include the active power, reactive power, the voltage and angle of the bus. It is considered that the central axis of the oscillation curve is its steady points, and the central axis can be obtained by fitting the upper and lower envelope of the
oscillation curve. The steady points identification can be divided into 3 steps: “turning point” identification, envelope fitting, and the central axis obtaining.

**Identification of the turning points.** For the low frequency oscillation with time-varying steady points, some oscillation curves have obvious maxima and minima, and some always keep the positive slope. When fitting the upper and lower envelope of the oscillation curve, the turning points, having the fastest change in the slope in an oscillating period, must be contained. The tangent slope \( f'(t_i) \) and the velocity variation of the tangent slope \( f''(t_i) \) of the sampling point \( i \) are approximately represented by first difference and second order difference:

\[
\begin{align*}
  f'(t_i) &= \frac{f(t_{i+1}) - f(t_i)}{t_{i+1} - t_i} \\
  f''(t_i) &= \frac{f'(t_{i+1}) - f'(t_i)}{t_{i+1} - t_i}, \quad i = 1, 2, \ldots, n \tag{11}
\end{align*}
\]

In the Eq. 11, \( t_i \) is the time for the electric quantity of the \( i \) sampling point; \( f(t_i) \) is the electric quantity value of the \( i \) sampling point.

Fig. 2 contains the oscillating curve of the voltage angle of a generator at 25~30s in a low frequency oscillation with time-varying steady points and the curve of the change speed of its slope. Among them, the voltage angle is the per-unit value, divided by the average of the segment data and subtract 1, and the slope change rate magnified 5 times to observe more conveniently.

![Fig. 2. Per unit curve of oscillation and the change speed of its slope](image)

Fig. 2 shows that, the change speed of the tangent slope of the “turning point” in the upper envelope curve is minimum point, and the change speed of the tangent slope of the “turning point” in the lower envelope is the maximum point, respectively, as shown in the first row and second row of the Eq. 12. According to this criterion, the “turning point” of the oscillation curve can be found:

\[
\begin{align*}
  f'(t_{i-1}) &> f'(t_i) < f'(t_{i+1}) \\
  f'(t_{i-1}) &< f'(t_i) > f'(t_{i+1}). \tag{12}
\end{align*}
\]

**Envelope fitting and steady points obtaining.** Assuming that there are five “turning point” in the upper envelope of the oscillation curve, they can be expressed as \( [t_k, f(t_k)](k = 1, 2, \ldots, m) \). The
three spline interpolation function is constructed to obtain the upper envelope of the oscillation curve, and the expression is:

\[
S_{up}(t) = a_k t^3 + b_k t^2 + c_k t + d_k, \\
t \in [t_k, t_{k+1}], \quad k = 1, 2L \quad m-1.
\] (13)

According to the interpolation conditions, continuity conditions and boundary conditions to obtain the undetermined coefficient [19]. The method to obtain the lower envelope of the oscillation curve is the same as the upper envelope.

At the same time, the median value of the three spline interpolation function of the upper and lower envelope is used as the steady point at this moment. For example, the active power steady point at \(t_i\) is:

\[
P_{s,i} = \frac{S_{up}(t_i) + S_{down}(t_i)}{2}.
\] (14)

**Calculation example verification**

Taking the central China power grid as an example, the steady points identification, energy decomposition and disturbance source location method are verified. Set up the power plant of Jinxi, Sichuan Province as the disturbance source by modifying the control system parameters and increasing the power plant outlet reactance to weaken the damping of the plant unit. Set up a trouble-free break on one line among the three lines of the power plant to cause a negative damping oscillation. After the oscillation, the mechanical power of the 5 units is increased by the quick closing valve. The climbing speed is divided into 3 stages: 0~7s is 0.92%/s; 7s~12s is 0; 12s~18s is 1.67%/s.

**Steady points identification.** Take the identification of the electromagnetic power steady points of the 1# unit in Jinxi power plant as an example. Based on the principle of acceleration and deceleration of generator rotor, it can be considered that the electromagnetic power of generator is fluctuating around the mechanical power, and the mechanical power is the real steady points of electromagnetic power. The curve of mechanical power and the curve of the identification of the electromagnetic power steady points are shown in Fig. 3.

![Fig. 3. Comparison chart of actual steady point and identification steady point](image)

By Fig.3, the difference between the steady points identified by the electromagnetic power curve and the actual steady points is very small and the identification accuracy is satisfactory. The deviation of the initial stage of the oscillation is caused by the disturbance, and the data from the first oscillation period should be removed when the actual calculation be carried on.
Energy decomposition of the 1# unit in Jinxi power plant. According to the Eq.8~Eq.10, the state energy, the reciprocation energy and the dissipation energy of the 1# unit in Jinxi power plant are calculated. The energy waveforms are shown in Fig.4.

![Fig. 4. Exploded view of energy](image)

Fig.4 shows that, the curves of electromagnetic power, reactive power and voltage phase angle drift greatly due to the great changes of the mechanical power of the 1# unit in Jinxi power plant during the oscillation period. So, the state energy value is very large and increases obviously with time. But the reciprocation energy fluctuates symmetrically around the horizontal 0 axis and the overall does not flow along a certain direction, which is in accordance with the results of this paper. The dissipation energy is the energy that really reflects the damping characteristic of the unit, and the dissipation energy of the 1# unit in Jinxi power plant is obviously higher than 0, which indicates that the unit is the disturbance source, which is the same as the default.

Energy decomposition of the 1# unit in Guandi power plant. The waveforms of the state energy, the reciprocation energy and the dissipation energy of the 1# unit in Guandi power plant are shown in Fig.5.

![Fig. 5. Exploded view of energy](image)

Fig.5 shows that, in the oscillation, the electromagnetic power curve and reactive power curve of the 1# unit in Guandi power plant have a certain degree of drift, but the amplitude is very small. The main reason causing the high state energy is that the voltage phase angle drift greatly with the oscillation of the system. The reciprocation energy oscillates around the horizontal axis that is slightly higher than 0 axis, which shows that the average value of the reciprocation energy is 0 in most of the time, and the positive energy that only exist in the initial stage of the oscillation elevates the overall level of the curve. The dissipation energy of the 1# unit in Guandi power plant is less than 0, and there is a very obvious downward trend when it is amplified, which shows that the 1# unit in Guandi power plant does not absorb the dissipation energy and is not the disturbance source, which is in accordance with the expected results of this paper.
Conclusion

This paper proposes a disturbance source location method of low frequency oscillation with time-varying steady points, decomposes the transmission energy of the line into the state energy, the reciprocation energy and the dissipation energy by mathematical derivation, and presents the criteria for the localization of the disturbance source with the dissipation energy. The simulation results show that the method is effective and the characteristics of the energy components are consistent with the theoretical analysis, and the disturbance source of the low frequency with time-varying steady points can be accurately identified by the flow of the dissipation energy. This method can overcome the limitation of the existing method of the disturbance source localization, and can be applied to the on-line vibration monitoring and off-line oscillation analysis. In addition, this paper mainly analyzes and tests the negative damping oscillation caused by local disturbance sources. In fact, the proposed method is also applicable to forced power oscillation.

Reference


