Evaluation on frequency sensitivity of Young's modulus of a steel transmission tower

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Abstract. Sensitivity analysis as an alternative approach can take structural parameters as variable and achieve the relationship only with one time analysis. The study on frequency sensitivity of a transmission tower is actively carried out in this study. The three dimensional analytical model of a transmission tower is established by using the finite element method. The sensitivity coefficients to natural frequencies are deduced based on the equation of motion of the tower. A real transmission tower constructed in China is taken as an example to examine the feasibility and reliability of the proposed sensitivity computation approach. The made observations demonstrate that the Young's modulus of the vertical major members plays an important role in the first two vibration modes in comparison with all the other vibration modes.

Introduction

For a transmission tower with determined parameters, it is troublesome to attain the relationship be-tween dynamic characteristics and changed parameters by numerous recalculation which will be almost impossible for large scale structures [1-3]. While sensitivity analysis, as an alternative approach, can take structural parameters as variable and achieve the relationship only with one time analysis. The study on frequency sensitivity of a transmission tower is actively carried out in this study. The three dimensional analytical model of a transmission tower is established by using the finite element (FE) method [4-6]. The sensitivity coefficients to natural frequencies are deduced based on the equation of motion of the tower [7-8]. A real transmission tower constructed in China is taken as an example to ex-amine the feasibility and reliability of the proposed sensitivity computation approach. The made obser-vations demonstrate that the Young's modulus of the vertical major members plays an important role in the first two vibration modes in comparison with all the other vibration modes.

Frequency sensitivity analysis

As a typical type of spatial structures, the transmission tower can be modeled by using three dimensional beam elements. The element stiffness matrix $\mathbf{K}^{(i)}$ and mass matrix $\mathbf{M}^{(i)}$ of the *i*th element in the global coordinate system can be determined by transforming the element stiffness matrix $\mathbf{K}_e^{(i)}$ and mass matrix $\mathbf{M}_e^{(i)}$ in the local coordinate system with the aids of coordinate transformation matrix. The global stiffness matrix $\mathbf{K}_t$ and mass matrix $\mathbf{M}_t$ of a transmission tower in the global coordinate system can be constructed by using the position matrix of element freedom following the finite element connection information of each element

$$\mathbf{K}_t = \sum_{i=1}^{ne} \mathbf{T}^{(i)T}\mathbf{K}^{(i)}\mathbf{T}^{(i)} \tag{1}$$

$$\mathbf{M}_t = \sum_{i=1}^{ne} \mathbf{T}^{(i)T}\mathbf{M}^{(i)}\mathbf{T}^{(i)} \tag{2}$$

Where ne is the total element number of the finite element model of a transmission tower; $\mathbf{T}^{(i)}$ is the freedom transform matrix from element coordinate system to the GCS, which is the product of coordinate transformation matrix $\mathbf{T}_a^{(i)}$ and position matrix $\mathbf{T}_c^{(i)}$ of the ith element.

Sensitivity coefficients are defined as the rate of change of a particular response quantity R with respect to a change in a structural parameter x. A differential sensitivity coefficient is the slope of the response R_j with respect to parameter x_i, computed at a given state of the parameter. When this differential is computed for all selected responses with respect to the selected parameters, the element of the sensitivity matrix **S** is obtained:

$$S_{ji} = \frac{\partial R_j}{\partial x_i} \quad (j = 1, 2, ..., n;\ i = 1, 2, ..., m) \tag{3}$$

In which n is the number of the responses; m is the number of the structural parameter. The absolute sensitivity coefficient S_{ji} is the (j, i)th element in the sensitivity matrix **S**.

The eigenvalue equation of a MDOF transmission tower can be expressed as

$$\left(\mathbf{K} - \omega^2 \mathbf{M}\right)\boldsymbol{\varphi} = 0 \tag{4}$$

Where $\mathbf{M}$, $\mathbf{K}$ and $\boldsymbol{\varphi}$ are the mass matrix, stiffness matrix and modal vector of the transmission tower respectively. The eigenvalue equation of the rth mode vibration is

$$\left(\mathbf{K} - \omega_r^2 \mathbf{M}\right)\boldsymbol{\varphi}_r = 0 \tag{5}$$

The mass matrix, stiffness matrix, and modal vector are the function of physical parameters. Thus, the first derivative of Equation (5) to the parametric x_i of the ith structural member for the rth mode vibration results in

$$\left(\frac{\partial \mathbf{K}}{\partial x_i} - \frac{\partial \omega_r^2}{\partial x_i}\mathbf{M} - \omega_r^2 \frac{\partial \mathbf{M}}{\partial x_i}\right)\boldsymbol{\varphi}_r + (\mathbf{K} - \omega_r^2 \mathbf{M})\frac{\partial \boldsymbol{\varphi}_r}{\partial x_i} = 0 \tag{6}$$

In which: ω_r and $\boldsymbol{\varphi}_r$ are the rth circular frequency and modal vector of the system. Since the mass and stiffness matrices are a symmetric matrix. The sensitivity of the frequency to the ith structural parameter x_i is

$$\frac{\partial f_r}{\partial x_i} = \frac{1}{8\pi^2 f_r} \cdot \frac{\boldsymbol{\varphi}_r^T \left(\sum_{i=1}^{ne} \frac{\partial \mathbf{K}_i^e}{\partial x_i} - 4\pi^2 f_r^2 \sum_{i=1}^{ne} \frac{\partial \mathbf{M}_i^e}{\partial x_i}\right)\boldsymbol{\varphi}_r}{\boldsymbol{\varphi}_r^T \mathbf{M} \boldsymbol{\varphi}_r} \tag{7}$$

Equation (7) provides a way to calculate the sensitivity of the rth natural frequency to the change in the ith structural parameter x_i. Assuming that the transmission tower is linear and the change in natural frequency due to the change of structural parameter is small, the change in the rth natural frequency Δf_r, due to the variations of the structural parameter Δx_i, can be expressed as

$$\Delta f_r = \frac{\partial f_r}{\partial x_i}\Delta x_i \tag{8}$$

The sensitivity of the rth natural frequency to the Young's modulus of the ith structural element E_i can be rewritten by using the element stiffness matrix $\mathbf{K}_i^e$ and mass matrix $\mathbf{M}_i^e$ of the ith element in the global coordinate system. The sensitivity of the rth natural frequency to the Young's modulus of the ith structural element E_i is given by

$$\frac{\partial f_r}{\partial E_i} = \frac{1}{8\pi^2 f_r} \cdot \frac{\boldsymbol{\varphi}_r^T \left(\sum_{i=1}^{ne} \frac{\partial \mathbf{K}_i^e}{\partial E_i}\right)\boldsymbol{\varphi}_r}{\boldsymbol{\varphi}_r^T \mathbf{M} \boldsymbol{\varphi}_r} \tag{9}$$

Structural description and dynamic properties

Displayed in Figure 1 is the model of a transmission tower constructed in the southern China. The height of the tower is 38.4m. The structural members used in the transmission tower are made of Q235 steel with a yielding stress of 235 MPa. The Young's modulus of the steel is 2.01×10^{11} N/m^2 and the density is 7800 kg/m^3. The vertical major members, the skew members, the cross arms and the platform of the tower are formed as a spatial truss tower. Three platforms are connected to the vertical major members to form the tower body and the skew members are incorporated to increase the vertical and lateral stiffness of the entire tower. A three dimensional FE model is constructed based on the FE method with the aids of the commercial package. The in-plane and out-of-plane directions are denoted as X and Y directions, respectively.

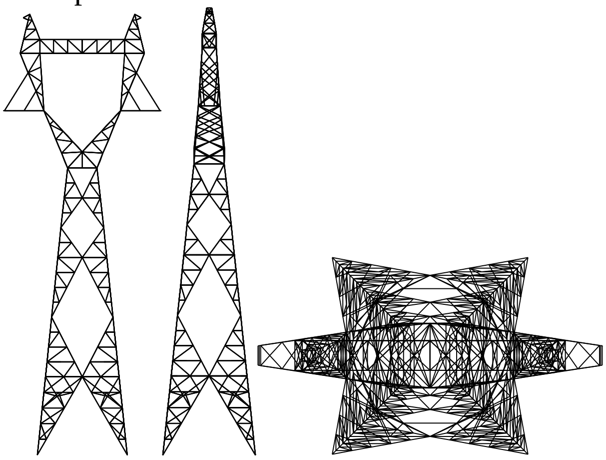

Figure 1. Model of the transmission tower-line system

A three dimensional finite element model is established for the steel transmission tower using a commercial computer package. The model has a total of 1223 three dimensional beam elements and 454 nodes with 6 degrees of freedom at each node. All the joints in the finite element model are assumed to be rigid. The movement of all the supports in the three orthogonal directions is restricted. For the sake of convenience in the subsequent discussion, the beam elements in each component of the transmission tower are numbered differently. The beam elements in the vertical major members are numbered from 1 to 73 (denote zc), the elements in the platforms are numbered from 74 to 185 (denote pt), the elements in the skew members are counted from 186 to 819 (denote xg), the elements in the lower cross arm are counted from 820 to 1223 (denote hd). The number of vertical major members is only 5.96% of the total number of elements used in the structure while the number of skew members is 51.8% of the total number of elements used in the structure. The dynamic characteristics analysis is conducted based on the established finite element model of the steel transmission tower.

Table 1 Dynamic properties of the transmission tower

No.	Frequency (Hz)	Properties of the mode shape
f_1	2.0769	1st global vibration mode in the y direction
f_2	2.6174	1st global vibration mode in the x direction
f_3	3.6403	1st global torsional vibration mode
f_4	7.7039	2nd global vibration mode in the y direction
f_5	7.7627	Local vibration mode of the tower body
f_6	9.2659	2nd global vibration mode in the x direction
f_7	9.2678	Local vibration mode of the tower body
f_8	10.334	Local vibration mode of the tower body
f_9	10.426	3rd global vibration mode in the x direction
f_{10}	10.931	Local vibration mode of the tower body

The first ten natural frequencies and vibration modes of the tower are depicted in Table 1 and Fig-ure 2, respectively. The first natural frequencies of the transmission tower for the out-of-plane and in-plane vibration are 2.0769 Hz and 2.6174 Hz, re-spectively. It is seen that the first and second vibra-tion modes are the global vibration mode in the y di-rection and x direction, respectively. The third vibration mode is a global torsional vibration mode in the x-y plane due to the tower rotation. The fourth to six vibration modes are the high order transla-tional and torsional vibration modes of the tower for both global and local vibration. The first ten natural frequencies computed indicate that the natural fre-quencies of the structure are not closely spaced. The dynamic responses of the first three mode shapes are the major parts of the entire dynamic responses of the transmission tower.

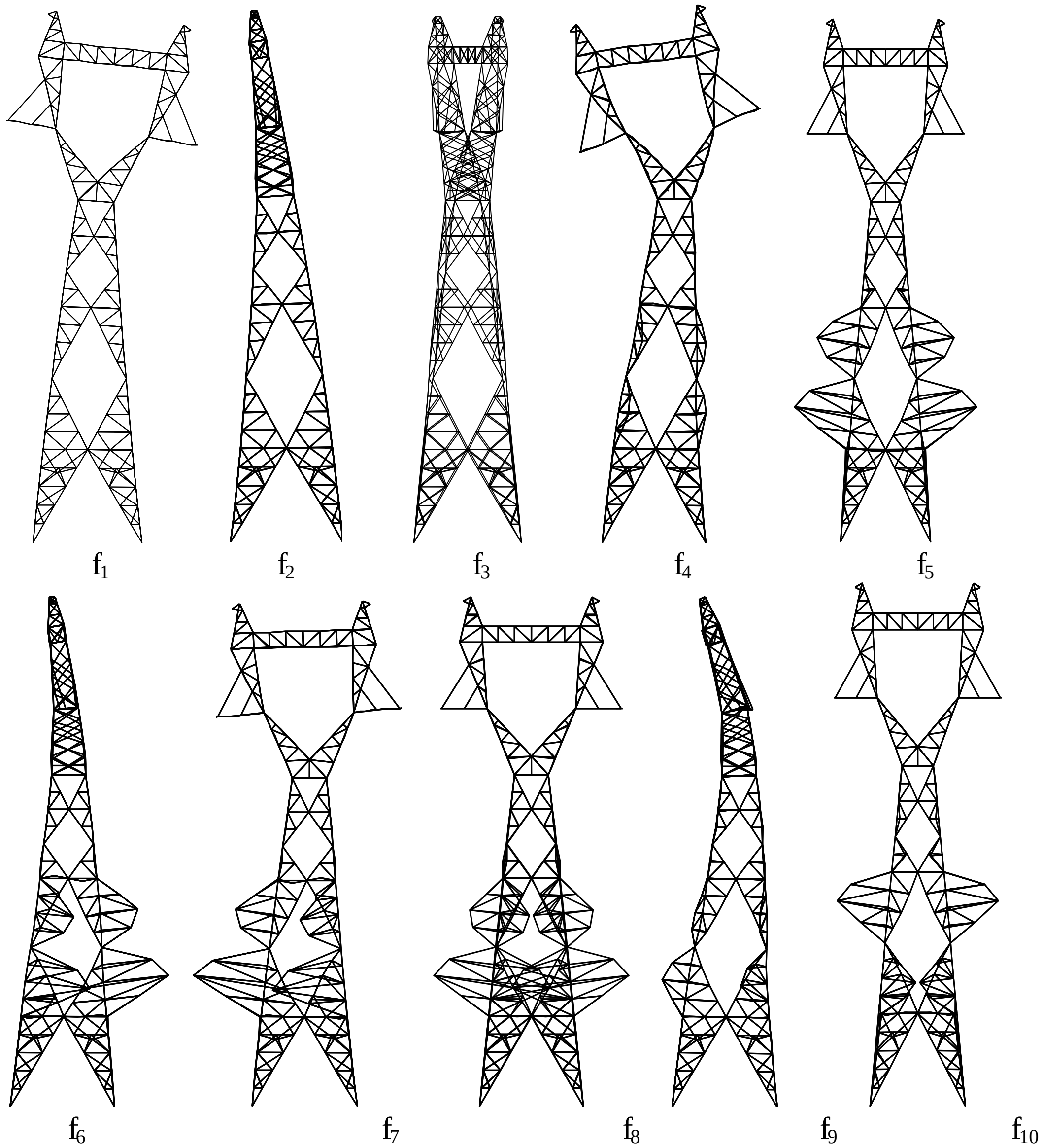

Figure 2 Mode shapes of the transmission tower

Frequency sensitivity coefficients

Figures 3 and 4 display the sensitivities of the first ten natural frequencies to the Young's modulus of each member. It is seen from the first two figures that the first two natural frequencies are more sensitive to the Young's modulus change of members in the vertical major members (zc) than other members. The curves in the third figure indicate that the sensitivity coefficients of the first torsional frequency of the skew members are much larger than those of the other members. All the higher natural frequencies of the global vibration are more sensitive to the Young's modulus change of both vertical major members and the skew members. This is consistent with the structural configuration and the modes of vibration: the first two translational modes of vibration are mainly due to the deformation of the vertical major members and the first torsional mode is due to the deformation of the skew members. The made observations demonstrate that the Young's modulus of the vertical major members plays an important role in the first two vibration modes in comparison with all the other vibration modes. The sensitivity coefficients of the skew members are slightly smaller than those of the vertical major members.

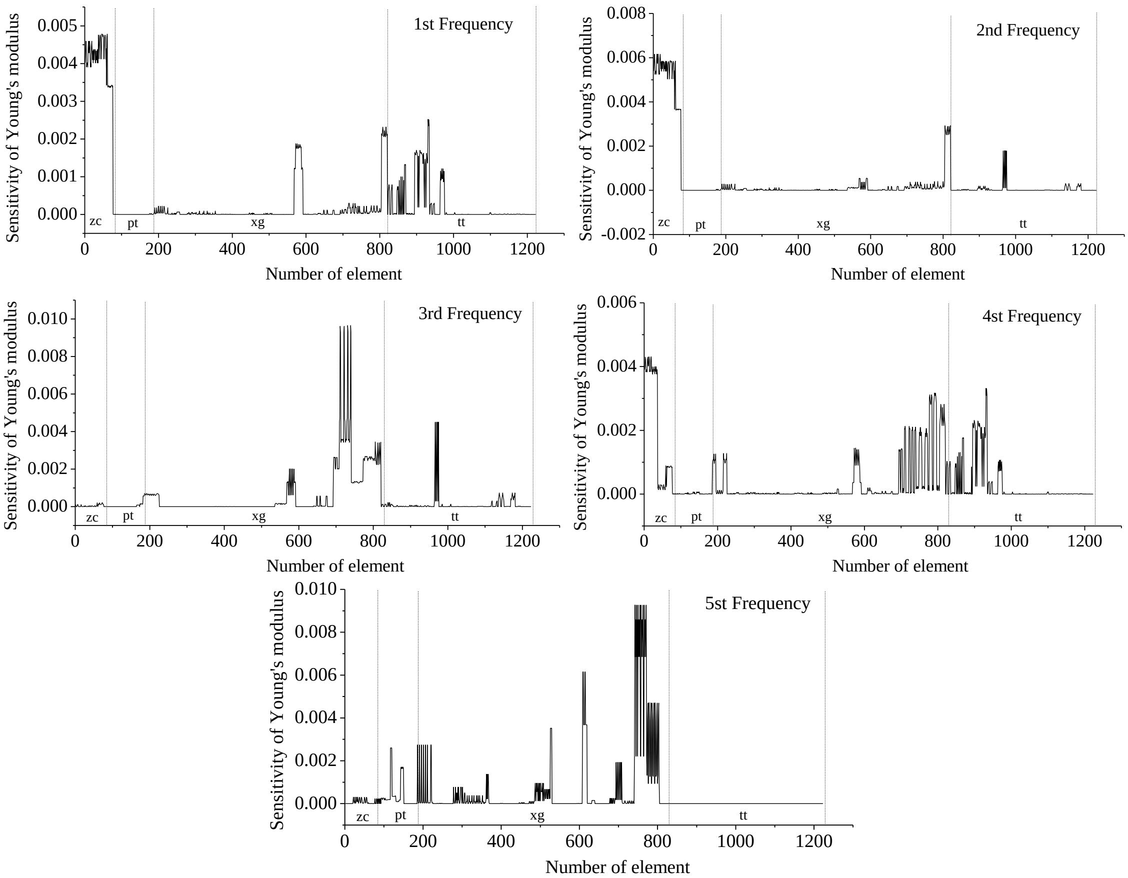

Figure 3 Sensitivity coefficient of the Young’s modulus of the first five natural frequencies

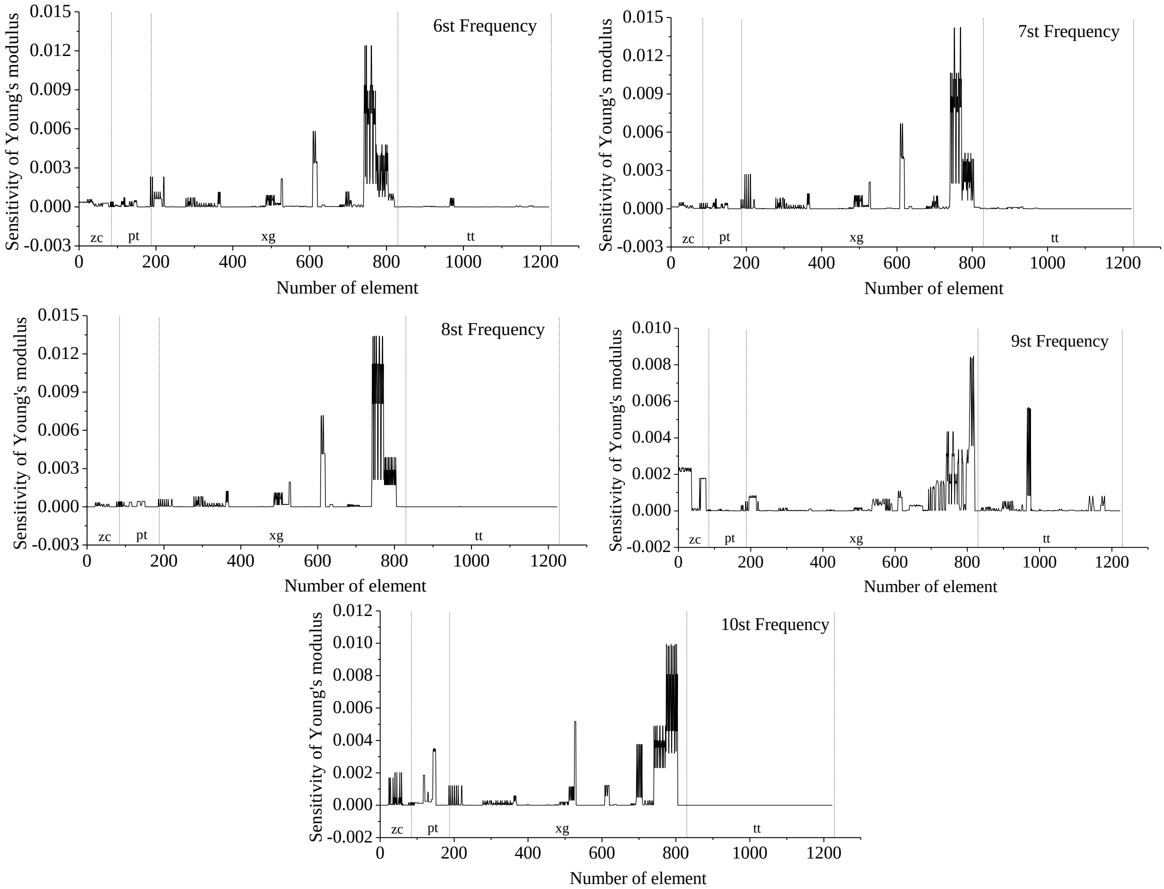

Figure 4 Sensitivity coefficient of the Young’s modulus of the sixth to the tenth natural frequencies

Conclusions

The feasibility of evaluating the parametric effects of a transmission tower based on the frequency sensi-tivity analysis is actively carried out in this study. The 3D analytical model of a transmission tower is first constructed by using the FE method. The differ-ential sensitivity analysis approach is presented based on the differential sensitivity coefficient, the absolute sensitivity coefficient, and the relative sen-sitivity coefficient respectively. A real transmission tower-line system is taken as the example to investi-gate the effects of the structural parameters on the natural frequency through the detailed parametric study. The made observations demonstrate that the Young's modulus of the vertical major members plays an important role in the first two vibration modes in comparison with all the other vibration modes.

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