Analysis of plastic zone boundary of circular tunnel under the asymmetric load

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Keywords: Circular tunnel; plastic zone; lateral pressure coefficient; Mohr Coulomb criterion

Abstract. In order to study the stress distribution features of the surrounding rocks a long the circle tunnel after the roadway excavated and the radius size of there leased circle, first analysis of the stress distribution characteristics of the surrounding rock, on this basis, circular tunnel for the elastic and plastic zone is derived based on the mohr coulomb strength criterion. The plastic zone in the surrounding rock of the tunnel under the conditions of different side pressure, different stress, different cohesion and different friction angle was obtained with the theoretical calculation. The evolution of the plastic zone is summed up from plastic zone equation. The lateral pressure coefficient has a great influence on plastic radius; with the increase of stress, the plastic zone increases; with the cohesion increases the plastic zone reduced; with the cohesion increases the plastic zone reduced.

Introduction

The size of the plastic zone of surrounding rock and the stress distribution law are an important basis for tunnel design, so selecting reasonable calculation method is very important. So far, Many people have researched on elastoplastic calculation of underground caverns in many aspects, have gotten some valuable achievements. these studies are mostly limited to axisymmetric load, While λ is not often is equal to 1 in practical engineering. It is so far little research under λ is equal to 1.we can Only use of practical engineering model test results or use formula when λ is is equal to 1. So it is an important problem that Calculates the plastic zone of the surrounding rock under the asymmetric load. In this paper gets tunnel plastic zone based on the Mohr Coulomb strength criterion under the asymmetric load. Research on different side pressure, depth, rock properties and other parameters influence of the plastic zone of the roadway based on the mohr coulomb strength theory.

Tunel plastic radius

The basic assumption. Rock is a very complex material structures, it is difficult to use a unified constitutive equations to describe the mechanical behavior, we make the following assumptions for using Mohr Coulomb strength theory.

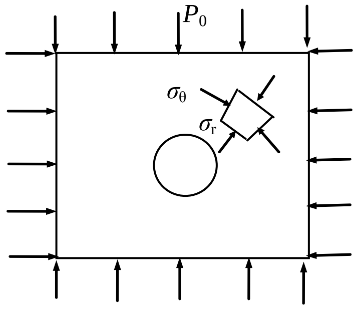

Fig. 1 Theoretical calculation model of the plastic zone

Firstly assumes that rock is continuous, homogeneous, isotropic elastic-plastic material and tunnel axial unchanged,

Secondly assumes that In order to facilitate the calculation, we assumptions that is plane stress problem., a circular Chamber is excavated in the infinite plate as a computational model.

Thirdly assumes that calculation unit is not weigh, The weight stress of the rock mass is as the initial stress state.

Through the above assumption, Stress distribution of surrounding rock after the circular tunnel is excavated is considered as a biaxial compression infinite plate hole. The radius of tunnel is ra, The vertical stress is p_0 , The lateral pressure is λp_0, λ is lateral pressure coefficient, the calculation model is shown in figure 1.

Plastic zone theoretical calculations. The asymmetric load of circular tunnel can be divided into uniform stress field and uniaxial compressive stress field. The vertical and the horizontal stresses are equal under uniform stress field of circular tunnel, The vertical and the horizontal stresses are both is equal to （$1+\lambda$） $p_0/2$, the vertical and the horizontal stresses are equal under uniaxial compressive stress field of circular tunnel, The vertical and the horizontal stresses are both is equal to $p_0/2$,but the vertical stresses is compressive stress ,the horizontal stresses is tensile stress. If add uniform stress field and uniaxial compressive stress field together, and you will get stress field under the asymmetric load that the vertical stresses are is equal to p_0 and the horizontal stresses are is equal to λp_0. The stress at any point will be getten after adding two kinds stress together, The radial stress is σ_r , r is the distance from the center, The tangential stress is is equal to $\tau_{r\theta}$, The stress formula is as follows.

$$\sigma_r = \frac{p_0}{2}\left[(1+\lambda)\left(1-\frac{r_a^2}{r}\right)-(1-\lambda)\left(1-4\frac{r_a^2}{r^2}+3\frac{r_a^4}{r^4}\right)\cos 2\theta\right] \tag{1}$$

$$\sigma_\theta = \frac{p_0}{2}\left[(1+\lambda)\left(1+\frac{r_a^2}{r}\right)+(1-\lambda)\left(1+3\frac{r_a^4}{r^4}\right)\cos 2\theta\right] \tag{2}$$

$$\tau_{r\theta} = -\frac{p_0}{2}\left[(1-\lambda)\left(1+2\frac{r_a^2}{r^2}-3\frac{r_a^4}{r^4}\right)\sin 2\theta\right] \tag{3}$$

In the formula, λ is lateral pressure coefficient, p_0 is crustal stress.

$$\sigma_1 = \frac{\sigma_r+\sigma_\theta}{2}+\sqrt{\left(\frac{\sigma_r-\sigma_\theta}{2}\right)^2+\tau_{r\theta}^2} \tag{4}$$

$$\sigma_3 = \frac{\sigma_r+\sigma_\theta}{2}-\sqrt{\left(\frac{\sigma_r-\sigma_\theta}{2}\right)^2+\tau_{r\theta}^2} \tag{5}$$

We can obtain the principal stress of $\sigma 1$ and $\sigma 3$ according to equation (4) and equation (5)

$$\sigma_1 = \frac{(1+\lambda)p_0}{2}-(\lambda-1)\frac{p_0 r_a^2}{r^2}\cos 2\theta+\frac{p_0}{2}\beta \tag{6}$$

$$\sigma_3 = \frac{(1+\lambda)p_0}{2}-(\lambda-1)\frac{p_0 r_a^2}{r^2}\cos 2\theta-\frac{p_0}{2}\beta \tag{7}$$

In the formula, β can be obtained according to the following formula

$$\beta = \sqrt{I} \tag{8}$$

$$I = \left[(1+\lambda)\frac{p_0 r_a^{\ 2}}{r^2} - (\lambda - 1)\left(1 - \frac{2r_a^{\ 2}}{r^2} + 3\frac{r_a^{\ 4}}{r^4}\right)\cos 2\theta \right]^2 + \left[(\lambda - 1)\left(1 + 2\frac{r_a^2}{r^2} - 3\frac{r_a^4}{r^4}\right)\sin 2\theta \right]^2 \quad (9)$$

Using Mohr-Coulomb strength criterion, σ1 can be obtained form following formula.

$$\sigma_1 = \xi\sigma_3 + \sigma_c \quad (10)$$

In the formula,σc is uniaxial compressive strength, numerically equal 2ccosφ/(1-sinφ),ξ is slope, numerically equal (1+sinφ)/(1-sinφ), φ is inner friction angel of rock.

Using the formula (6) and formula (7), the formula（10）, the formula（11）can be rewritten According to Mohr-Coulomb strength criterion.

$$\left[\frac{(1+\lambda)p_0}{2} - (\lambda - 1)\frac{p_0 r_a^{\ 2}}{r^2}\cos 2\theta \right](1-\xi) + \frac{p_0}{2}\beta(1+\xi) = \sigma_c \quad (11)$$

p_0 can be calculated according to the engineering geological data, λ numerical is according to the experimental results, if an angle value is given, then we can obtained a plastic radius , When calculating the point enough, you can find plastic zone boundary map, this process can be programmed to complete.

The effect of different parameters on the plastic zone of rock. There are five independent factors affect the roadway plastic zone radius, they are the lateral pressure coefficient λ, internal friction angle φ, cohesion c, original rock stress p_0 , tunnel radius r_a。 This article cited a simple example calculation of different parameters on the circular roadway of the plastic zone.In order to facilitate analysis assuming that the radius of the circular tunnel is equal to 1 meter, cohesion is equal to 10 Mpa and internal friction angle is equal to 30 degree. Analysis of the influence of the four main parameters are as follows.

The effect of different depth on the plastic zone of circular tunnel. Assumed lateral pressure coefficient λ is equal to 1.0, c is equal to 10Mpa. We can draw plastic zone boundary line according to the formula (11), Plastic zone shown in Figure 2 when lateral pressure is equal to 10MPa,15 MPa,20 MPa,30 MPa .

If the average weight of the overburden calculated by γ is equal to 25 kN/m^3, 10MPa rock pressure is the equivalent of 400m depth, 15 MPa,20 MPa,30MPa corresponds approximately 400m, 600m, 800m, 1200m depth.

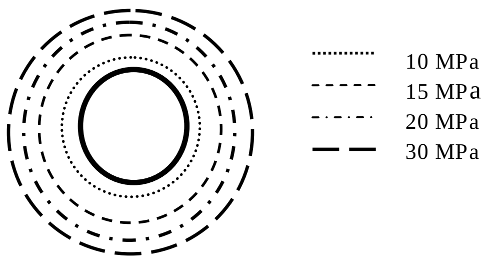

Fig.2 plastic zone of different depth

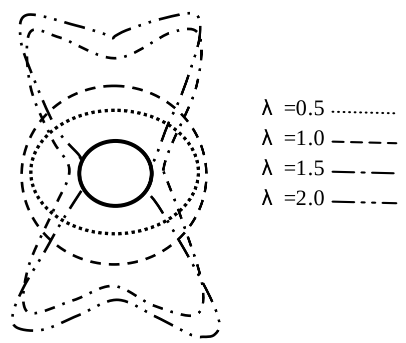

Fig.3 plastic zone of different lateral pressure

Figure 2 shows the range of plastic zone of surrounding rock increases with depth, this is one of the main factors surrounding rock in deep difficulty shoring.

The effect of different lateral pressure on the plastic zone of circular tunnel. Assumed vertical stress is is equal to 10 Mpa, cohesion is equal to 10Mpa. We can draw plastic zone boundary line according to the f formula (11), Plastic zone shown in Figure 3 when lateral pressure coefficient λ is equal to 0.5,1.0,15,2.0 .

Fig 3 indicates the two sides of the tunnel plastic area is bigger, The top and bottom of the plastic zone is relatively small when lateral pressure coefficient λ less than 1. The destruction of chamber show that the plastic zone on both sides of chambers is the form of a sliding wedge, the theory is completely the same with the actual situation. plastic zones of tunnel is approximately circular when lateral pressure coefficient λ is equal to 1. With the lateral pressure coefficient λ value is increasing, two sides roadway plastic zone decreases, the rapid increase in the plastic roof and floor plastic zone, when λ = 2 top and bottom larger plastic zone increases, the middle part of both sides of tunnel is no plastic zone, the rapidly development of the plastic zone in the direction of the tunnel corner, and increases significantly.

The cohesion effects on the plastic zone of circular tunnel. Assumed lateral pressure coefficient λ is equal to 1.0, c is equal to 10Mpa. We can draw figure of cohesion and plastic radius according to the formula (11), Figure 4 show plastic radius change when cohesion change.

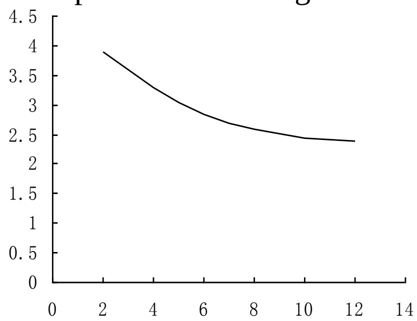

Fig.4 plastic radius curve with cohesion

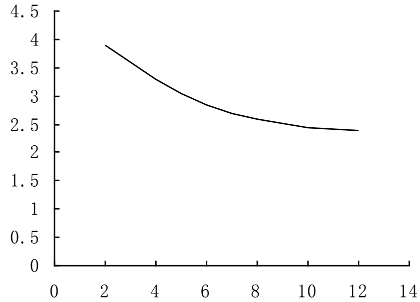

Fig.5 plastic radius curve with friction cohesion angle

Figure 4 indicates the two sides of the tunnel plastic radius significantly decreases with cohesion increase when cohesion is smaller. With the increase of cohesion, plasticity radius decreases, and finally stabilized. So when the surrounding rock strength is smaller, effect of tunnel grouting is significant, conversely, when a higher strength of surrounding rock, the effect of tunnel grouting is not obvious.

The friction angle effects on the plastic zone of circular tunnel. Assumed lateral pressure coefficient λ is equal to 1.0, p is equal to 10Mpa. We can draw figure of friction angle and plastic radius according to the formula (11), Figure 5 show plastic radius change when friction angle.

Figure 5 show the curve of tunnel plastic radius with friction cohesion angle. Figure 5 indicates that plasticity radius decreases with the increase of cohesion angle, but the value decreased less, therefore, internal friction angle limit effect on plastic radius of tunnel.

Summary

On this basis, The formula of circular tunnel plastic radius under the asymmetric load is derived based on the Mohr Coulomb strength criterion.

The lateral pressure coefficient has a great influence on plastic radius of circular tunnel, The results show that when $\lambda = 1$, the plastic zone is circular; when $\lambda < 1$, the plastic zone range is large at roadway side, the plastic zone range is small at roadway top; hen $\lambda > 1$, It is clean contrary, The rapid development of the plastic zone in the corner;

Numerically stress effect on plastic radius of circular tunnel, Area of the plastic zone increases with the increase of ground stress.

The cohesion of tunnel rock effects on the plastic zone of circular tunnel, the tunnel plastic radius significantly decreases with cohesion increase when cohesion is smaller. With the increase of cohesion, the size of the plastic radius stabilized, So when the surrounding rock strength is poor, grouting effect is significant, on the contrary effect is not obvious.

The friction angle effects on the plastic zone of circular tunnel, plasticity radius decreases with the increase of cohesion angle, but the value decreased less, therefore, internal friction angle limit effect on plastic radius of tunnel.

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