Optimized Analysis on Process Capability of Fuzzy Control Chart Based on Beta Distribution

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Abstract: This paper focused on the question of optimizing process capability under the Beta distribution. First of all, analyze process capability index under Beta distribution, and analyzing the features of process capability index. Then, to decide the effect of process capability, Methodologically, this paper optimized analysis on 3 special situation, including nonconforming rate, loss of product quality, and product value.

Introduction

When production process is under statistical control, Process Capability Index is measurement on measuring how well the capability of production process can satisfy product quality standard. Based on the feature of Beta Distribution, random variable, as rate of rejection, rate of machine maintenance, rate of market share, and rate of shooting hits, ranges between 0 and 1.[1] Analyzing quality feature under Beta distribution can reveal different forms of quality feature distribution in one form. Moreover, analyzing quality feature under Beta Distribution can describe regulations of quality distribution more precisely, compared to analyzing quality feature under Normal Distribution.[2]

According Boyles(1996) and Hubble (2002) , it is restrictive to analyze process capability by single process capability index, more considerations need to get involved. [3]According to the content of research, different values of m and s have significant reflection on process capability. Thus, the method of analyzing process capability from Hubele(1994) and the method of optimized analysis on process capability from Flaig(2002) can be used for reference.[4]

Analysis of Process Capability Index under Beta Distribution

Assume fuzzy set distance as the random variable which follows Beta distribution $\mathrm{Beta}(a,b)$. According to abnormal distribution, analyzing process capability indexes.[5] Set $a = 0.135\%$, Then get x_a and x_{1-a} correspond to 0.135% and 99.865% quantile of fuzzy set distance, respectively. Then Process Capability Index under Beta Distribution C_P' is:

$$C_P' = \frac{T}{\xi_{1-\alpha} - \xi_\alpha} \tag{1}$$

In Equation (1), T is tolerance, the range of production. Based on control chart of fuzzy set distance, Tolerance range is usually $0^{+T_U}_{-0}$ ($T_U \le 1$), and $T = T_U$. T_U is the upper limit of common difference. Only if unilateral process capability index C_{PU}' is below the upper limit of common difference, it has significant practical value in the control chart of fuzzy set distance. C_{PU}' is shown below:

$$C'_{PU} = \frac{T_U - x_{0.5}}{x_{1-a} - x_{0.5}} \quad (2)$$

Similar to the method above, when the tolerance center and the center of quality characteristic value distribution are different, define skew process capability index C'_{PK} as below:

$$C'_{PK} = \frac{T - |x_{0.5} - M|}{x_{1-a} - x_a} \quad (3)$$

In Equation (3), $x_{0.5}$ is the median of Beta Distribution. M is the tolerance center. According to these formulas, when quality characteristic value follows normal distribution, C'_P and C'_{PK} equal to C_P and C_{PK}, respectively.

Analysis on the features of process capability index

In the process of producing, fuzzy set distance is variable. According to this feature, fuzzy set distance index can be fitted Based on Beta distribution. The relations among the mean value of fuzzy set distance,the standard deviation of fuzzy set distance , and distribution index are shown below:

$$a = \frac{m(m - m^2 - s^2)}{s^2}, \quad b = \frac{(1-m)(m - m^2 - s^2)}{s^2} \quad (4)$$

When m and s change, process capability index changes, then analyze the feature of process capability index. Assume common difference $T = 0.5$, in other words, tolerance range is $0_0^{+0.5}$. Then get the curve surface chart of C'_P , when and changes, is shown in Fig. 1:

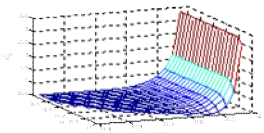

Fig. 1. When m and s get different value, the curve surface of C'_P

When the lower deviation of tolerance range is $d_L = 0$, and the upper deviation is $d_U = 0.5$, m and s should follow some restrictions. In Figure 1, the lower curve surface reveals the effects of these restrictions. When Standard deviation of fuzzy set distance index decreases, C'_P increases. It means that the change of C'_P reflects process capability to an extent.Meanwhile, when decreases to the ideal value of fuzzy set distance, C'_P increases at the first, then decreases a little bit. And lower the value of m is, better the process capability is. According to the analysis above, the change of can reveal the change of process capability in some kind, but not properly. Therefore, C'_P is restrictive when used as the index of revealing process capability.Still set standard as $0_0^{+0.5}$. When and change, get the curve surface chart of C'_{PU} and C'_{PK}, corresponding to Fig. 2 and Fig. 3 respectively.

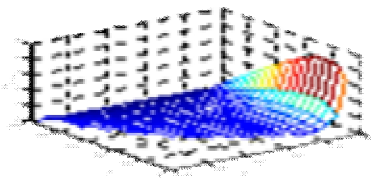

Fig. 2. When m and s get different values, the curve surface of C'_{PU}

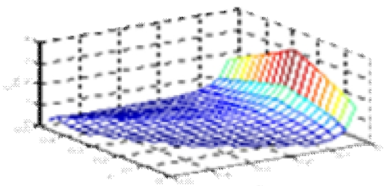

Fig.3. When m and s get different values, the curve surface of C'_{PK}

According to Fig.2 and Fig.3, when s decreases, both C'_{PU} and C'_{PK} increase. It means the change of and kind of reflects the change of process capability. Meanwhile, when decreases to the ideal value of fuzzy set distance, both of C'_{PU} and C'_{PK} increase at the first, then decrease. Moreover, C'_{PU} and C'_{PK} decrease faster than C'_P. Based on the analysis above, C'_{PU} and C'_{PK} have the similar restrictions to C'_P's, they can not reveal process capability properly.

Optimized Analysis on process capability

Optimized Analysis on process capability based on nonconforming rate

Make p represent nonconforming rate. Assume fuzzy set distance X follow $Beta(a,b)$. The mean of $Beta(a,b)$ is m and the standard deviation of $Beta(a,b)$ is s. In order to low down the value of p, affecting m and s should be taken. On one side, the value of m should approach the ideal value of quality technical standard, which is L. At this moment fuzzy set distance is 0. On the other hand, reflecting s, then low down s to low down nonconforming rate p. Their relation is shown in function below:

$$p = P(X > T_U) \tag{5}$$

$$p = \int_{T_U}^{1} f(x;a,b)\,dx \tag{6}$$

Equation (6) is another form of Equation (5), In response to Equation (5), nonconforming rate p can be transformed into function of m and s. Due to the complexity of function calculation, in this paper, take advantage of chart to reveal their relation. Figure1 shows below, As m and s decrease, the value of p decreases.

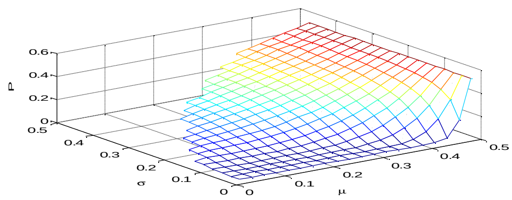

Fig.4. When m and s get different values, the curve surface of p

Optimized Analysis on process capability based on lose of product quality

Under the Asymmetric quality loss function given by Li and Chou(2001),

$$L(x) = \begin{cases} k_1(x-L)^2 & x \le L \\ k_2(x-L)^2 & x \ge L \end{cases} \tag{7}$$

In this Equation, $k_1 = A_1/d_L^2$, $k_2 = A_2/d_U^2$. A_1 is the quality loss at the lower limit of common difference. A_2 is the quality loss at the upper limit of common difference. Obviously, in the common situation of asymmetric common difference, $k_1 \ne k_2$. For the situation that fuzzy set distance follows $Beta(a,b)$ distribution, the average quality loss is:

$$E[L(x)] = \int_0^{T_U} k_2 x^2 f(x;a,b)\,dx \tag{8}$$

Obviously, the lower the average quality loss is, the better the process capability is. In Equation (8), k_2 is constant, the value of $E[L(x)]$ is determined by a and b from $Beta(a,b)$. In other words, it is determined by mean value m and standard deviation s. In Equation (7), the value of m should approach L which is the ideal value of fuzzy set distance. Ideally, m and L should equal. Then, when s decreases, $E[L(x)]$ decreases. The relationship among m, s, and $E[L(x)]$ is shown in Equation 9.

$$E[L(x)] = \int_0^{T_U} k_2 x^2 f(x;a,b)\,dx \tag{9}$$

When $T_U = 1$, $E[L(x)] = k_2(m^2 + s^2)$. Then as m and s decrease, $E[L(x)]$ also decreases. When $T_U < 1$, get the Equation below.

$$E[L(x)] = \frac{a(a+1)}{(a+b)(a+b+1)} C(a+2, b; T_U) k_2 \quad (10)$$

In Equation (10), $C(a, b; T_U) = \int_0^{T_U} f(x; a, b)dx$ .Under the consideration of Equation (9), $E[L(x)]$ can be transformed into the function of m and s . Due to the complexity of function calculation, the relationship is shown in Fig. 5.

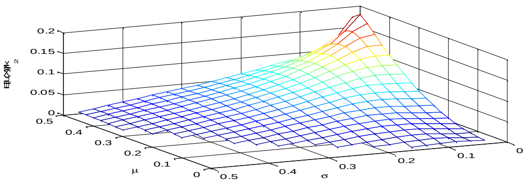

Fig.5. The curve surface of $E[L(x)]$ based on function (9)

In Fig.5, as m and s decreases, $E[L(x)]$ decreases. In addition, the marginal impact of average quality loss decline is similar.

Optimized analysis on process capability based on product value.

Based on value function, product value can be seen as the level of process capability. In other words, the higher the product value is, the higher the level of process capability is. The function of product value is shown below.

$$E[W(X)] = \int_0^{T_U} \left(W - \frac{W}{T_U^2} x^2\right) f(x; a, b)dx \quad (11)$$

In Equation (9), W_L and T_U^2 is constant. Thus, a and b of $Beta(a,b)$ determine the value of $E[W(X)]$. In other words, m and s determine the value of $E[W(X)]$. So, in order to get higher product value, some effects should be taken on m and s . The get the following Equation.

$$E[W(X)] = WC(a, b; T_U) - \frac{W}{T_U^2} \frac{a(a+1)}{(a+b)(a+b+1)} C(a+2, b; T_U) \quad (12)$$

In Equation (12), then $C(a, b; T_U) = \int_0^{T_U} f(x; a, b)dx$. Under the consideration of (12), $E[W(X)]$ can be transformed into the function of m and s . This function is shown in Fig.6.

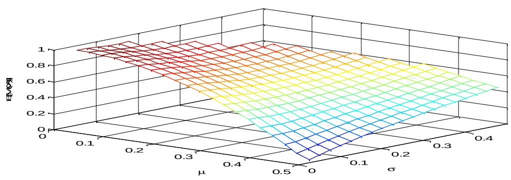

Fig.6. curved surface chart of Formula (12)

In Fig.6, as s decreases, $E[W(X)]$ decreases. When m decreases, $E[W(X)]$ increases. The margin impact of product value by decline of m is larger than the margin impact of product value by decline of s . Although , their order of magnitude are different. Under the effect of m and s , in asymmetric deviation the level of quality improvement under the change of m is higher than the level of quality improvement under the change of s .

Conclusions

This paper analyzes Optimization of process capability by the features of random variable in fuzzy set distance. Based on correlation analysis, different values of m and s have significant reflection on process capability. This paper do Optimized Analysis 3 special situation. First of all, basing on rate of rejection, as m and s decrease, the value of nonconforming rate decreases. Then, loss of product quality, as m and s decreases, average quality loss decreases, in addition, the marginal impact of average quality loss decline is similar. At last, basing on product value, bring quality improvement under the change of m is higher than the level of quality improvement under the change of s .

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