Automatic measurement of discontinuity spacing based on 3D digital traces model

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Abstract. The common measurement of discontinuity spacing is to use scanline surveys in field, which is inefficient and fallible. This paper proposes an automatic method to acquire discontinuity spacing based on 3D digital traces model. The results obtained are used to generate a 3D fracture network model and the developed method is applied to verify the accuracy of 3D fracture network model. This study shows the efficiency and accuracy of the presented method.

Introduction

A rock mass comprises rock material and discontinuities that include fractures, joints, faults and bedding planes. The spatial distribution of discontinuities has great influence on the deformability, strength and permeability of rock mass. The discontinuity spacing, defined as the vertical distance between two adjacent discontinuities from a particular discontinuity set, is one of important indexes which can express the distribution features of discontinuity and is often used in classification schemes [1]. At present, scanline survey used in field is the most common way to obtain the information of discontinuity spacing, which is inefficient and fallible. Whereas, other advanced methods for acquiring discontinuity information, e.g., digital close rang photogrammetry [2], 3D laser scanning [3], have no mature algorithms to measure spacing currently. Min Yang et al. [4], put forward a non-contact measurement of discontinuity spacing based on image processing and application results shows feasibility of the measurement.

This paper presents an approach to measure discontinuity spacing automatically by developed algorithm. It is based on 3D digital traces model [5], which is established using traces data obtained by GPS-RTK. With this model, an algorithm, programed using C++, has been developed to calculate discontinuity spacing. Results obtained are used to modelling a 3D fracture network. Then algorithm is applied to verify the accuracy of network.

Explanation of method

The method contains several steps: 1.establish 3D digital traces model; 2.input information of scanlines; 3.calculate the intersection point of traces represented by line segments and scanlines; 4.calculate discontinuity spacing. The latter three steps are processed by developed algorithm. Details are described as following.

3D digital traces model. This study has a detailed measurement of discontinuity traces in Beishan area in Gansu province of China. By means of GPS-RTK, dataset of 512 traces has been obtained. Based on traces orientation, they are grouped into 4 sets. Directions of average unit normal vectors for the 4 set are: set 1, 89.101° $\angle$ 62.120°; set 2, 303.369° $\angle$ 71.358°; set 3, 129.325° $\angle$ 65.806°; set 4, 219.589° $\angle$ 78.735°, respectively. A 3D digital traces model is established, shown in Fig.1. The model is generated on X (east direction)-Y (north direction)-Z (elevation) coordinate system. The blue mesh
represents terrain and the line segments in different colors represent discontinuity traces belong to different sets; see more details of this model in [5].

**Calculation of discontinuity spacing.** Noted calculation of discontinuity spacing for each set is independent, we consider one discontinuity set (set 3) as an example. Fig.2 is shown as a reference of calculation.

Start program by inputting coordinate of point O\((x_0, y_0, z_0)\), which decides the position of scanline. Then a plane (Plane P in Fig.2) whose orientation is \(219.325^\circ(129.325^\circ+90^\circ)\) \(\leq 90^\circ\) is calculated to intersect with the digital terrain and a intersecting line (i.e., the scanline) could be got.

The next work is to compute the intersections of traces and scanline and finally discontinuity spacing can be figured out. Indeed, it is difficult to calculate the intersection points of two lines as they do not intersect with each other certainly. This difficulty comes from simplification of traces and the fitting error of digital terrain. Thus, we calculate intersection points of traces and plane P (where the scanline located) instead of intersections of traces and scanline. Actually, it’s appropriate as digital terrain fitted based on endpoints of traces, therefore intersection points of scanline plane and traces are very close to intersections of traces and scanline. The calculation of intersections is shown as following.

Let \(H(x_1, y_1, z_1)\) and \(E(x_2, y_2, z_2)\) denote two endpoints of a particular trace. The parametric equations of trace line can be expressed by Eq. (1).

\[
\begin{align*}
x &= x_i + k(x_2 - x_i) \\
y &= y_i + k(y_2 - y_i) \\
z &= z_i + k(z_2 - z_i)
\end{align*}
\]  
(1)

Where \(k\) is an unknown parameter which need to be figured out. Let \(n_0(N_x, N_y, N_z)\) denotes the unit normal vector of scanline plane, so equation for this plane can be written as Eq. (2).

\[
(x_0 - x_0)N_x + (y_0 - y_0)N_y + (z_0 - z_0)N_z = 0
\]  
(2)

By substituting Eq. (1) to Eq. (2), \(k\) is immediately obtained by Eq. (3).

\[
k = \frac{(x_0 - x_i)N_x + (y_0 - y_i)N_y + (z_0 - z_i)N_z}{(x_2 - x_i)N_x + (y_2 - y_i)N_y + (z_2 - z_i)N_z}
\]  
(3)

The intersection point of one trace and scanline plane can be figured out when substitute \(k\) acquired by Eq. (3) to Eq. (1). In addition, lots of intersection points should be deleted by considering limitation of trace length. After acquiring all intersection points of traces belong to one set and a scanline plane, sort points and calculate distance of two adjacent points (represented by \(A(x_a, y_a, z_a), B(x_b, y_b, z_b)\)), then revise distance by Eq.(4).

\[
d_i = \frac{|A\hat{B} \cdot n_0|}{|n_0|} = |(x_b - x_a)n_x + (y_b - y_a)n_y + (z_b - z_a)n_z|
\]  
(4)

Where \(d_i\) is discontinuity spacing defined in section 1, and \(n_0(n_x, n_y, n_z)\) is the average unit normal vector of discontinuities belongs to one set.
Besides, the algorithm is programed to receive coordinates of any number of points which decided the position of scanline planes in one time. The position and number of scanline plane are determined by the area of study region. In this case (set 3), ten scanline planes are assigned parallel to each other with a distance (3.00m) slightly larger than mean trace length (2.40m). Therefore, most traces would not be intersected by scanline planes more than once and scanlines cover most area of study region. By the method described above, the dataset of discontinuities spacing of study area has been acquired.

**Application**

The method proposed cannot only be used in 3D digital traces model, but also 3D fracture network model to measure discontinuity spacing. And automation of the method makes optimization for model convenient. This paper establishes a 3D fracture network model based on dataset of discontinuity spacing and makes comparison between spacing data obtained from virtual model and original measurement. In fact, an error (relates with discontinuity spacing) function could be constructed to assess and increase the goodness of simulation for fracture network model. Apparently, optimization needs more studies, but it is not concerns of this paper.

**Construction of 3D fracture network model.** Representing discontinuities by circular discs and generating diameter and orientation according to their probability distribution function (PDF) by Monte-Carlo method, a fracture network model has been generated in 3D space with size 50m×50m×30m (by considering size of 3D digital traces model), shown in Fig.3. Additional, the mean 3D density of discontinuity is derived based on mean 1D density by means of Eq.(5)[6].

\[
(\lambda_i) = \frac{4(\lambda_i)}{\pi E(D^2)}
\]

Where \((\lambda_i)\) is the mean 3D density of \(i\)th set, and \((\lambda_i)\) is the mean 1D density of \(i\)th set, which is the inverse of the mean spacing that obtained. And D is the diameter of discontinuities while \(E(D^2)\) is the mean square of D, whose value relies on trace length PDF[7].

![Fig.3 3D fracture network model](image)

**Verification of simulated model.** Reliability of the model can be assessed by error of spacing obtained from simulated model and original measurement. There, a fitting plane of digital terrain has been figured out to intersect the generated discontinuity discs. Considering the intersection lines as traces and using method described in Section 2 with the same scanline planes, dataset of discontinuity spacing can be obtained. Noted that it is not necessary to establish a 3D traces model again as the position of scanline planes are known. The error of mean spacing is defined as Eq. (6).

\[
\text{error}_r = \frac{l_r - l_s}{l_r} \times 100%
\]

Where \(l_r\) and \(l_s\) denote the mean discontinuity spacing obtained from original measurement and simulated model, respectively.
The comparison of discontinuity spacing obtained from original measurement in 3D digital traces model (Original) and 3D fracture network model (Model) and error are shown in Fig. 4 and Table. 1. Fig. 4 shows distribution of discontinuity spacing and Table. 1 shows spacing information of original measurement and simulated model.

![Fig. 4 Distribution of discontinuity spacing obtained from original measurement and simulated model](image)

**Table. 1 Comparison of spacing information between original measurement and simulated model**

<table>
<thead>
<tr>
<th>Set number</th>
<th>Mean spacing (m)</th>
<th>Standard deviation (m)</th>
<th>Error of spacing (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>Model</td>
<td>Original</td>
</tr>
<tr>
<td>1</td>
<td>2.35</td>
<td>2.32</td>
<td>3.34</td>
</tr>
<tr>
<td>2</td>
<td>3.17</td>
<td>2.72</td>
<td>3.81</td>
</tr>
<tr>
<td>3</td>
<td>2.89</td>
<td>3.03</td>
<td>4.17</td>
</tr>
<tr>
<td>4</td>
<td>4.38</td>
<td>4.82</td>
<td>4.28</td>
</tr>
</tbody>
</table>

Table. 1 shows that the absolute values of error are within 15%. And from Fig. 4, distribution of spacing data of each set acquired from simulated model is similar to that from original measurement. Obviously, it is necessary to modify 3D fracture network model which is now underway. The error of simulated model which is acceptable in this paper may come from the determination of diameter of discontinuity disc, $\lambda$, and simplification of model whose 3D density is considered the same everywhere.
In future, an error function considering mean value and standard deviation of discontinuity spacing could be constructed to modify fracture network by minimizing the function value. The methods for minimization could be adjusting parameters of distribution of diameters of discs and 3D density.

Conclusions
This paper has proposed an automatic measurement of discontinuity spacing based on 3D digital traces model. Algorithm developed to do the calculation is programed by C++. Compared to traditional method (i.e., scanline surveys in field), the arrangement of digital scanline is more flexible and acquisition of discontinuity spacing is much more efficient. Thus, this method would play a critical role in dealing with huge amount of information of discontinuity.

In addition, this paper has generated a 3D fracture network model using spacing data obtained from digital traces model and verified the network by method described in this paper. The comparison of discontinuity spacing obtained from digital traces model and simulated model shows the necessary of optimization, which can take advantage of error function considering mean value and standard deviation of discontinuity spacing. The optimization is now underway.

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References