An Optimization Model for Inventory System based on Supply-demand Balance

Qingsong Jiang
College of Information Engineering
Tarim University
Alar 843300, China

Wei Xing*
College of Information Engineering, Tarim University
& College of Computer Science and Technology,
Zhejiang University, Hangzhou 310027, China
*Corresponding author

***Abstract*—In order to investigate the inventory optimization of circulation enterprises, demand analysis was carried out firstly considering supply-demand balance. Then it was assumed that the demand process complied with mutually independent compound Poisson process. Based on this assumption, an optimization model for inventory control of circulation enterprises was established with the goal of minimizing the average total costs in unit time of inventory system. The research results have certain practical significance to the optimal management of the inventory system.**

Keywords-supply-demand balance; inventory system; optimization model

I. INTRODUCTION

Inventory control of warehousing has been widely focused by circulation enterprises and relevant scholars all the time. Owing to the significance of inventory optimization and control in circulation enterprises, it has been increasingly concerned by many relevant scholars. Among these researches, economic order quantity (EOQ) model based on stock-dependent demand was established (as in [1]). Besides, the production-inventory model for perishable items with definite productivity and with demand linearly depending on inventory level was considered (as in [2]). Some scholars explored the inventory issue with allowable shortages under inventory-level-dependent demand, at the same time, they also took monetary value as well as the expansion rate caused by external and internal costs into consideration (as in [3]). In addition, EOQ model for perishable items was established, where the perishable items were under the following conditions: the demand rate was related to inventory level and some stock-outs could be supplemented later (as in [4]). Moreover, the optimal replenishment strategy for perishable items was investigated aiming at maximizing profits (as in [5]), while the inventory optimization for perishable items under stock-dependent demand was studied as well (as in [6]). There are also a lot of research work in this field, not listed here.

Most of the above researches were conducted on the basis of continuous normal population, which made the researches convenient and operable to some extend. However, on the premise of uncertain supply and demand, there were a lot of uncertain factors for inventory optimization. In fact, most of the demand and supply in reality cannot distribute continually but present in the form of discrete random variables usually. As a result, on the assumption that the demand process of each sub-warehouse submitted to the mutually independent compound Poisson process, the authors carried out the researches on some aspects, including the optimization and control of inventory system based on supply-demand balance as well as the optimal inventory costs. In addition, the related researches have a certain value on theoretical research.

II. INVENTORY SYSTEM MODEL

From a practical point of view of research object, a necessary simplification for the research object was conducted during the research combining the actual conditions of regional circulation enterprises. For the underdeveloped regions, the two-echelon inventory system is more common. The sketch map of inventory system model is illustrated in Figure 1.

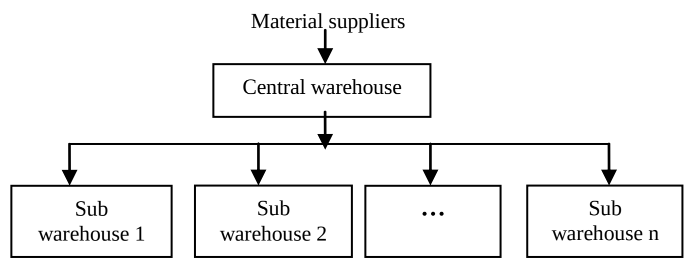

FIGURE I. SKETCH MAP OF INVENTORY SYSTEM MODEL

III. MODEL ASSUMPTION AND SYMBOL DESCRIPTION

A. Model Assumption

- The central warehouse of the two-echelon inventory system mentioned purchases products from material suppliers, while the sub-warehouses order goods from the central warehouse.
- Both the central warehouse and sub-warehouses of the system carry out the (R,Q) ordering (as in [7]) strategy of continuous review inventory.
- The material suppliers can supply unlimitedly and the delivery time for central warehouse is a constant, while the transportation time from central warehouse to sub-warehouses is a random variable. Then the lead time of sub-warehouses consists of random delay and random transportation time.
- The product demand process of sub-warehouses is a mutually independent compound Poisson process, that is to say, Poisson arrival of consumers. In addition, the demand of each consumer is a random integer.
- $R_i \geq -Q_i (i = 0,1,2,\cdots,N)$ are available for all order points.
- All stock-outs in the two-echelon inventory system are waiting. Besides, the delayed order-to-delivery follows the principle of "first come first serve".

B. Symbol Description

The meaning of the symbols in the research is as follows:

- N is the number of retailers.
- L_0 is the fixed delivery time from manufacturers to distribution centers, namely the lead time of distribution centers
- T_i is the random transportation time of goods from distribution centers to retailers i.
- τ is the random delay of retailer's orders in distribution centers.
- L_i is the lead time of retailers i, $L_i = T_i + \tau$.
- Q_0 is the order quantity of distribution centers.
- Q_i is the order quantity of retailers i.
- R_0 is the order point of distribution centers.
- R_i is the order point of retailers i.
- h_0 is the storage costs of unit goods in unit time of distribution center.
- h_i is the storage costs of unit goods in unit time of retailers i.
- P_0 is the stock-out losses of unit goods in unit time of distribution center.
- P_i is the stock-out losses of unit goods in unit time of retailers i.
- $C_0(R_0, Q_0)$ is the average holding costs and shortage costs of distribution center in unit time.
- $C_i(R_i, Q_i)$ is the holding costs and shortage costs of retailers i in unit time.
- TC is the expected gross costs of the inventory system.

IV. Modeling

On the assumption that the lead time of retailers is a random variable, and the demand process is a compound Poisson process, the (R,Q) storage model was established with the aim of minimizing the mean total cost of two-echelon inventory system in unit time (as in[4]).

A. The inventory model of central warehouse

It was assumed that the demand process of sub-warehouse i followed the compound Poisson process with parameter of λ_i, namely λ_i is the mean unit time of purchasers arriving at sub-warehouse i. If j is the demand of purchasers; $f_{i,j}$ is the probability of demand $j(j>0)$ in retailer i; μ_i is the average demand of retailer i in unit time; σ_i^2 is the demand variance of retailer i in unit time, then:

$$\mu_i = \lambda_i \sum_{j=1}^{\infty} j^2 f_{i,j},\ \sigma_i^2 = \lambda_i \sum_{j=1}^{\infty} L_0 \sigma_i^2$$

Thereby, the average demand and demand variance of retailer i in the lead time of distribution centers can be expressed as:

$$\mu_i(L_0) = L_0 \mu_i, V_i(L_0) = L_0 \sigma_i^2$$

Owing to the demand discussed here was Poisson demand, it could be known according to probability knowledge that if the sample capacity was great, Poisson distribution would be approximate to normal distribution. Thus, it was assumed that the demand of sub-warehouse in the lead time of central warehouse was close to normal distribution (as in [4]).

If the probability for placing k orders by sub-warehouse i to central warehouse in lead time L_0 was $p_{i,k}(L_0)$, the first order was placed after demand was up to x, where x obeyed uniform distribution on $(0, Q_i)$, so placing k orders by sub-warehouse i within L_0 means that the demand is between $x+(k-1)Q_i$ and $x+kQ_i$, then:

$$p_{i,k}(L_0) = \frac{1}{Q_i}\int_0^{Q_i} [\phi(\frac{x+kQ_i-\mu_i(L_o)}{\sigma_i(L_0)}) - \phi(\frac{x+(k-1)Q_i-\mu_i(L_o)}{\sigma_i(L_o)})]dx$$
$$= \frac{\sigma_i(L_0)}{Q_i}[\phi^{(1)}(\frac{(k-1)Q_i-\mu_i(L_0)}{\sigma_i(L_0)}) + \phi^{(1)}(\frac{(k+1)Q_i-\mu_i(L_0)}{\sigma_i(L_0)}) - 2\phi^{(1)}(\frac{kQ_i-\mu_i(L_0)}{\sigma_i(L_0)})]$$

where $\phi(x)$ is the standard normal density function, $\phi^{(1)}(x)$ is first-order loss function, then it can be obtained that:

$$\varphi^{(1)}(x) = \int_x^{\infty} (u-x)\varpi(u)du = \varpi(x) - x(1-\phi(x))$$

Thereby, the mean $\mu_i^0(L_0)$ and variance $V_i^0(L_0)$ of the demand of central warehouse obtained from sub-warehouse i in lead time L_0 can be calculated as:

$$\mu_i^0(L_0) = \mu(L_0),$$
$$V_i^0(L_0) = \sum (kQ_i - \mu_i^0(L_0))^2 p_{i,k}(L_0)$$

To sum up $\mu_i^0(L_0)$ and $V_i^0(L_0)$, the mean $\mu^0(L_0)$ and variance $V^0(L_0)$ of demand of central warehouse in lead time L_0 can be obtained, namely:

$$\mu^0(L_0) = \sum_{i=1}^{N} \mu_i^0(L_0),\ V^0(L_0) = \sum_{i=1}^{N} V_i^0(L_0)$$

I_0 and I_i are set as the stochastic inventory levels of central warehouse and sub-warehouse i respectively ($i = 1, 2, 3, \cdots, N$). Besides, it has been known that the inventory level of central warehouse is independent of the random variable of demand in lead time and the stationary distribution at arbitrary time t in the interval $[R_0+1, R_0+Q_0]$ is uniform distribution (as in [7]).

The total demand of central warehouse is made up of the random demand of each sub-warehouse. In addition, based on central limit theorem, when sample capacity N is large enough, it can be considered that the demand of central warehouse in lead time almost obeys normal distribution. Therefore, the average stock-out quantity of central warehouse can be acquired as:

$$E(I_0)^- = \frac{1}{Q_0 - 1}\int_1^{Q_0}\int_{R_0+y}^{\infty} (x - R_0 - y) d\phi(\frac{x-\mu^0(L_0)}{\sigma^0(L_0)})dy$$

$$=\frac{(\sigma^0(L_0))^2}{Q_0-1}[\phi^{(2)}(\frac{R_0+1-\mu^0(L_0)}{\sigma^0(L_0)})-\phi^{(2)}(\frac{R_0+Q_0-\mu^0(L_0)}{\sigma^0(L_0)})]$$

where $\phi^{(2)}(x)$ is the quadratic loss function of standard normal distribution, so:

$$\varphi^{(2)}(x)=\int_x^{\infty}\varphi^{(1)}(u)du=\frac{1}{2}\left[(x^2+1)(1-\varphi(x))-x\varpi(x)\right]$$

Then the average existing inventory of central warehouse is:

$$E(I_0)^+=E(I_0)+E(I_0)^-=R_0+(Q_0+1)/2-\mu^0(I_0)+E(I_0)^-$$

Finally, the average holding cost and shortage cost of distribution center can be expressed as:

$$C_0(R_0,Q_0)=h_0E(I_0)^++p_0E(I_0)^-$$

B. The Inventory Model of Sub-Warehouse

The randomness of the lead time of sub-warehouse is caused by the random delay of central warehouse and the uncertainty of transportation time. Random delay means the random waiting time led by the stock-outs of central warehouse when sub-warehouses place orders to central warehouse.

τ was set as the random delay of the orders from sub-warehouses in central warehouse. For the sake of simplicity, assuming that it is the same to each sub-warehouse, the mean and variance of random delay are $E[\tau]$ and $V[\tau]$ respectively. The time for central warehouse demanding a unit goods is t_1, while the arrival time of the goods to central warehouse is t_2. If $t_1\geq t_2$, there is no delay;

while if $t_1<t_2$, there is a delay, where,

$$\tau=\max(\delta,0)(\delta=t_2-t_1)$$

According to calculation, it can be obtained the following relationship:

$$E[\tau]=E(I_0)^-/\sum_{i=1}^{N}\mu_i$$

If δ obeys normal distribution, then

$$P(\delta\leq 0)=(-\mu_\delta\sigma_\delta)=P(\tau=0)$$

Where

$$\begin{cases}P(\tau=0)=\dfrac{1}{Q_0-1}\displaystyle\int_1^{Q_0}\varphi(\frac{R_0+y-\mu^0(L_0)}{\sigma_0(L_0)})dy\\ E[\tau]=\displaystyle\int_0^{\infty}[1-\varphi(\frac{x-\mu_\delta}{\sigma_\delta})]dx=\sigma_\delta\phi^{(1)}(-\mu_\delta\sigma_\delta)\end{cases}$$

So it can be obtained that:

$$\sigma_\delta=E[\tau]/\phi^{(1)}(-\mu_\delta\sigma_\delta)$$

$$\mu_\delta=-\sigma_\delta{}^{-1}(P(\tau=0))$$

$$V[\tau]=E(\tau^2)-(E[\tau])^2$$

$$=\int_0^{\infty}\frac{x^2}{\sigma^\delta}\varpi(\frac{x-\mu_\delta}{\sigma^\delta})dx-(E[\tau])^2$$

$$=\sigma_\delta^2(1-P(\tau=0))+E[\tau]\mu_\delta-(E[\tau])^2$$

If the transportation time T_i of sub-warehouse i, obeys the Gamma distribution with α_i and β_i as parameters, and the mean and variance of random lead time of sub-warehouse i are $E_i(LT)$ and $V_i(LT)$, then:

$$\begin{cases}E_i(LT)=E(T_i)+E[\tau]=\alpha_i/\beta_i+E[\tau]\\ V_i(LT)=V[T_i]+V[\tau]=\alpha_i^2/V[\tau]\end{cases}$$

Through further analysis, it can be known that the mean $E_i(LTD)$ and variance $V_i(LTD)$ of demand of sub-warehouse i in random lead time LT are:

$$\begin{cases}E_i(LTD)=\mu_iE_i(LT)\\ V_i(LTD)=\sigma_i^2E_i(LT)+(\mu_i)^2V_i(LT)\end{cases}$$

In general inventory model, if the demand process is Poisson process, and the lead time obeys gamma distribution, most of the demand during lead time is approximated by using Poisson distribution. However, if there are many sub-warehouses and the mean demand of some sub-warehouses in lead time is small, using normal distribution for approximation will produce quite a number of negative values, which is unreasonable. Owing to its mean and variance values are the same, the change of demand during lead time cannot be well reflected by approximating through Poisson distribution. Besides, the characteristics of negative binomial distribution can preferably conform to the practical situation of the demand of retailers during lead time. In addition, it takes Poisson distribution as limiting distribution. As a result, it is more reasonable to utilize negative binomial distribution for approximating the demand distribution of retailers during lead time (as in [8]).If the demand of retailer i in lead time can obey the negative binomial distribution with n_i and P_i as parameters, distribution density function and distribution function are respectively $g_i(x)$ and $G_i(x)$, $(i=1,2,\cdots,N)$ then:

$$\begin{cases}P_i=[V_i(LTD)-E_i(LTD)]/V_i(LTD)\\ n_i=E_i^2(LTD)/[V_i(LTD)-E_i(LTD)]\end{cases}$$

Setting $G_i^{(0)}(x)=1-G_i(x)$,Owing to n is not a integer, it is impossible to calculate accurately $g_i(x)$ and

$G_i(x)$ .Thus, the first-order loss function $G^{(1)}(x)$ and quadratic loss function $G^{(2)}(x)(x \geq 0)$ for LTD are following:

$$G^{(1)}(x) = E\{[X-x]^+\} = \sum_{y \geq x} G_i^{(0)}(y)$$
$$= [\frac{n_i P_i}{1-P_i} - x] G_i^{(0)}(x) + (x+n_i)\frac{P_i}{1-P_i} g_i(x)$$
$$G^{(2)}(x) = \frac{1}{2} E\{[X-x]^+[X-x-1]^+\}$$
$$= \frac{1}{2} E\{X[X-1]\} - \sum_{0<y\leq x} G^{(1)}(y)$$
$$= \frac{1}{2}\{n_i(n_i+1)[\frac{P_i}{1-P_2}]^2 - 2\frac{n_i P_i}{1-P_i} x + x(x+1)G^{(1)}(x)$$
$$+[\frac{(n_i+1)P_i}{1-P_i} - x](x+n_i)\frac{P_i}{1-P_i} g_i(x)\}$$

In the same way of central warehouse, it can be obtained that sub-warehouse i are equally distributed on $[R_i+1, R_i+Q_i]$ at arbitrary time. Hence, it can be acquired that the average stock-out quantity of sub-warehouse i is as follows:

$$E(I_i)^- = \frac{1}{Q_i - 1}\int_1^Q \int_{R_i+y}^{\infty} (x - R_i - y) g_i(x) dx dy$$
$$= \frac{1}{Q_i - 1}[G^{(2)}(R_i+1) - G^{(2)}(R_i+Q_i)]$$

Then the average existing inventory of sub-warehouse i is:

$$E(I_i)^+ = E(I_i)^+ E(I_i)^- = R_i + (Q_i+1)/2 - E_i(LTD) + E(I_i)^-$$

Therefore, the average holding cost and shortage cost of sub-warehouse i can be obtained as:

$$C_i(R_i, Q_i) = h_i E(I_i)^+ + P_i E(I_i)^-$$

Through calculation and derivation, the expected total cost function of two-echelon inventory system can be expressed as:

$$TC = C_0(R_0, Q_0) + \sum_{i=1}^{N} C_i(R_i, Q_i)$$

C. Optimizing and Control Inventory Cost

Through analyzing the cost structure of two-echelon inventory system, the inventory control model based on cost optimization can be obtained as follows:

$$\min TC = \min[C_0(R_0, Q_0) + \sum_{i=1}^{N} C_i(R_i, Q_i)]$$

The key to solving the model is to select appropriate order point R_i^* and order quantity $Q_i^*(i = 0, 1, \cdots, N)$ to make the objective function achieve the optimum value.

Firstly, R_0, Q_0 is given to determine the order point $R_i^*(R_0, Q_0)$ and order quantity $Q_i^*(R_0, Q_0)$ of sub-warehouse i. According to the analysis, it can be known that $E(I_i)^-$ is a convex function of R_i & Q_i, and $E(I_i)^+$ is a linear equation of (R_i, Q_i). The non-linear combination $C_i(R_i, Q_i)$ is still a convex function based on (R_i, Q_i). It can be known that there must be an optimal R_i, Q_i to minimize $C_i(R_i, Q_i)$ through the properties of convex function.

After further simplifying the objective function, it can be obtained that:

$$TC(R_0, R_i) = C_0(R_0) + \sum_{i=1}^{N} C_i(R_0, R_i)$$

V. CONCLUSION

Considering supply-demand balance, the research started from demand analysis. On the assumption that the demand process obeyed mutually independent compound Poisson process, a two-echelon inventory system consisting of a central warehouse and several sub-warehouses was constructed through simplifying the research object. Besides, aiming at minimizing the mean total cost of inventory system in unit time, the optimization and control model for inventory of the system was established. In addition, the optimal algorithm for computing the inventory cost was provided. In this paper, the researchers explored the issue about the optimization and control of inventory system and established the corresponding optimization and control model for inventory to optimize the allocation of resources and utilize effectively by managing the limited inventory resources quantitatively. The modeling idea and method presented in this paper are excepted to be further developed to the higher-level multi-echelon inventory structure.

ACKNOWLEDGMENT

This research is supported by Youth special funds for scientific and technological innovation of the Xinjiang production and Construction Corps (2011CB001), the doctor foundation of the Xinjiang production and Construction Corps(2013BB013),the doctor foundation of Tarim university(TDZKBS201205) and the national Spark Program Project(2011GA891010).Here, thanks to the relevant company and organizations.

REFERENCES

[1] R.C. Baker, T.L.Urban, "A deterministic inventory system with an inventory level dependent demand rate", Vol.39, pp. 823-831, 1988.

[2] B.N. Mandal, S. Phaujdar, "An inventory model for deteriorating items and stock-dependent consumption rate", Journal of the Operational Research Society, Vol.40, pp.483-488, 1989.

[3] J. Ray, K.S. Chaudhuri, "An EOQ model with stock-dependent demand, shortage, inflation and time discounting", International Journal of Production Economics, Vol.53, pp.171-180,1997.

[4] C.Y. Dye, L.Y. Ouyang, "An EOQ model for perishable items under stock-dependent selling rate and time-dependent partial backlogging", European Journal of Operational Research, Vol.163, pp.776-783, 2005.

[5] C. Chang, J. Teng, S.K. Goyal, "Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand", International Journal of Production Economics, Vol.123, pp. 62-68, 2010.

[6] Y. Duan, G.Li, J. Tien, J. Huo. "Inventory models for perishable items with inventory level dependent demand rate", Applied Mathematical Modelling, Vol. 36, pp.5015-5028,2012.

[7] M.J.Ji, Z.H.Jin, "Inventory optimization model with controllable lead time", Journal of Dalian Maritime University, Vol.37, pp.87-90, 2011.

[8] F.Z.Li,"A research into multi-echelon inventory control and optimization model in the supply chain", Southwest Jiaotong University ,2009.