

# Modelling of Dynamical Properties of a Resonant Converter under Step Frequency- and Loaded Converter Changes

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**Abstract**—Paper deals with investigation of two problems: frequency step changes, and step switching-off of the load of LCTL resonant inverter. Since the changes of frequency in the range from  $f_{\min}$  to  $f_{\max}$  are desirable to be the input current of the LCTL circuit not greater than nominal one, the load changes cause dangerous overvoltage spikes. Modelling, theoretical analysis and computer simulation are used for the solution of those phenomena. Simulation experiment results confirming theoretical assumptions are given in the paper.

**Keywords**—dynamical state-space model; modelling and simulation; resonant converter; LCTL inverter; voltage transfer characteristic; steady-state operation; transient phenomena.

## I. INTRODUCTION – RESONANT CONVERTER MATHEMATICAL MODEL

One of the novel types of converters is LCTL inverter [1] (Figure 1.), loaded either by simple RL load or bridge connected rectifier.

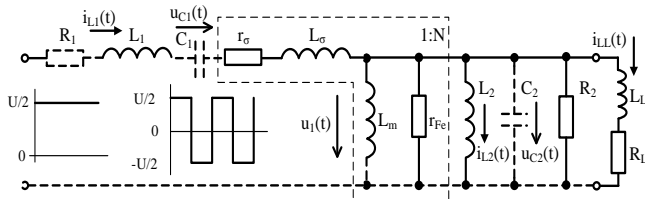


FIGURE 1. EQUIVALENT CIRCUIT OF LCTL INVERTER WITH RL LOAD

The dynamical model of resonant inverter can be created by using of different approaches [2], [3]. State-space operation of the LCLC filter is investigated in [4], [5]. The state-space equations for equivalent circuit with R-L load yield

$$\frac{di_{L1}}{dt} = -\frac{r_1}{L_1}i_{L1} - \frac{1}{L_1}u_{C1} - \frac{1}{L_1}u_{C2} + \frac{1}{L_1}u_1 \quad (1)$$

$$\frac{di_{L2}}{dt} = \frac{1}{L_1}u_{C2} \quad (1)$$

$$\frac{du_{C1}}{dt} = \frac{1}{C_1}i_{L1} \quad (1)$$

$$\frac{du_{C2}}{dt} = \frac{1}{C_2}i_{L1} - \frac{1}{C_2}i_{L2} - \frac{1}{r_2C_2}u_{C2} - \frac{1}{C_2}i_{LL} \quad (1)$$

$$\frac{di_{LL}}{dt} = \frac{1}{L_L}u_{C2} - \frac{R_L}{L_L}i_{LL} \quad (1)$$

where

$i_{L1}, i_{L2}$  - currents through the inductors  $L_1$  and  $L_2$ , respectively

$i_{LL}$  - current through the load  $R_{load}, L_{load}$

$u_{C1}, u_{C2}$  - capacitors voltages of  $C_1$  and  $C_2$ , respectively

$u_1(t)$  - output voltage of the converter (filter's input voltage).

Thus, we can rewrite system equation (1) into matrix form (2)

$$\frac{d}{dt} \begin{pmatrix} i_{L1} \\ i_{L2} \\ u_{C1} \\ u_{C2} \\ i_{LL} \end{pmatrix} = \begin{pmatrix} -r_1/L_1 - 1/L_1 - 1/L_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/L_1 & 0 \\ 1/C_1 & 0 & 0 & 0 & 0 \\ 1/C_2 & -1/C_2 & 0 & -1/r_2C_2 & -1/C_2 \\ 0 & 0 & 0 & 1/L_L & -R_L/L_L \end{pmatrix} \begin{pmatrix} i_{L1} \\ i_{L2} \\ u_{C1} \\ u_{C2} \\ i_{LL} \end{pmatrix} + \begin{pmatrix} 1/L_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} u_1(t) \quad (2)$$

Discrete incremental model has been created using Euler's implicit method given by formula (in matrix form)

$$\mathbf{x}_{n+1} = [\mathbf{E} - \Delta\mathbf{A}]^{-1}\mathbf{x}_n + [\mathbf{E} - \Delta\mathbf{A}]^{-1}\Delta\mathbf{B}\{\mathbf{u}_n\} \quad (3)$$

where

$\mathbf{x} = (i_{L1}, i_{L2}, u_{C1}, u_{C2}, i_{LL})^T$  – vector of state variables

$\mathbf{E}$  is unity matrix

$\mathbf{A}, \mathbf{B}$  are matrices of system parameters

$\Delta$  is integration step

$\{\mathbf{u}_n\}$  is sequence of values of input voltage

Then, discrete model suitable for numerical computing will be

$$\begin{aligned} & \begin{pmatrix} i_{L1} \\ i_{L2} \\ u_{C1} \\ u_{C2} \\ i_{LL} \end{pmatrix}_{n+1} \\ &= \left[ \mathbf{E} - \Delta \begin{pmatrix} -r_1/L_1 - 1/L_1 - 1/L_1 & 0 & 0 \\ 0 & 0 & 1/L_1 & 0 \\ 1/C_1 & 0 & 0 & 0 \\ 1/C_2 & -1/C_2 & 0 & -1/r_2 C_2 - 1/C_2 \\ 0 & 0 & 0 & 1/L_L - R_L/L_L \end{pmatrix} \right] \mathbf{x} \\ & \quad \times \begin{bmatrix} \begin{pmatrix} i_{L1} \\ i_{L2} \\ u_{C1} \\ u_{C2} \\ i_{LL} \end{pmatrix}_n + \Delta \begin{pmatrix} 1/L_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \{\mathbf{u}_n\} \end{bmatrix} \end{aligned} \quad (4)$$

This model has been programed in Matlab environment.

## II. INPUT IMPEDANCE AND VOLTAGE TRANSFER CHARACTERISTICS

Input impedance of the equivalent LCTL circuit in the frequency domain yields [6]

$$Z_{in}(\omega) = Z_1(\omega) + Z_2(\omega); \quad (5)$$

where:

$$Z_1(\omega) = R_{11} + j\omega L_{11} + \frac{1}{j\omega C_1} + R_\sigma + j\omega L_\sigma; \quad (6)$$

$$Z_2(\omega) = \frac{1}{Y_2(\omega)}; \quad (7)$$

and

$$\begin{aligned} Y_2(\omega) &= \frac{1}{j\omega L_m} + \frac{1}{R_{Fe}} + \frac{1}{j\omega L_{22}} + j\omega C_2 + \\ & \quad + \frac{1}{R_{22}} + 1/R_{load} + j\omega L_{load} \end{aligned} \quad (8)$$

Then the normalized impedance characteristic referred to the nominal load impedance is

$$\frac{Z_{in}(\omega)}{Z_N(\omega)} = \frac{Z_1(\omega)}{Z_N(\omega)} + \frac{Z_2(\omega)}{Z_N(\omega)} \quad (9)$$

Normalized impedance log-characteristic depending on relative frequency is shown in Figure 2

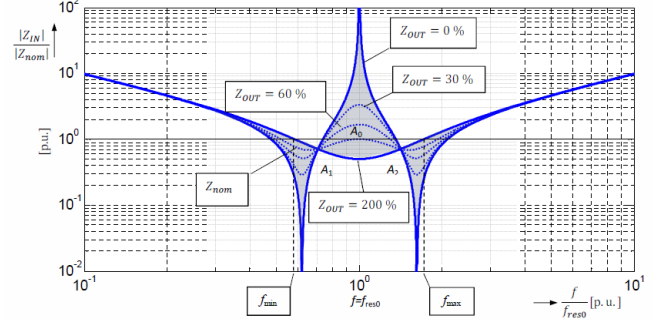


FIGURE II. INPUT IMPEDANCE TRANSFER FREQUENCY LOG-CHARACTERISTICS FOR 0; 30; 60; 100 AND 200 % OF THE LOAD

Similarly, the output voltage of the equivalent LCTL circuit in the frequency domain yields

$$U_2(\omega) = U_1(\omega) \times F_U(\omega); \quad (10)$$

where:

$$F_U(\omega) = \frac{U_2(\omega)}{U_1(\omega)} = \frac{Z_2(\omega)}{Z_{in}(\omega)} = \frac{Z_2(\omega)}{Z_1(\omega) + Z_2(\omega)} \quad (11)$$

is voltage transfer or voltage gain of the LCTL circuit.

Then the normalized voltage transfer log-characteristic  $F_U(\omega)$  depending on relative frequency is shown in Figure 3

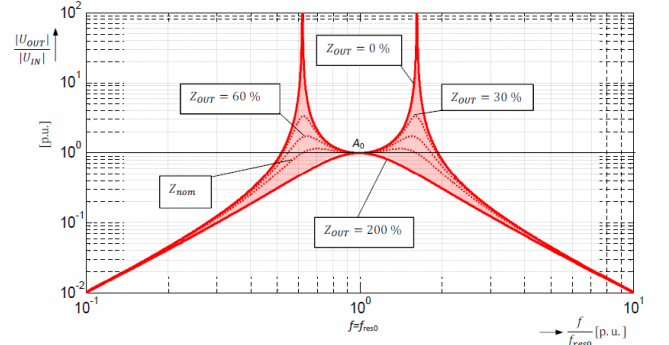


FIGURE III. OUTPUT VOLTAGE TRANSFER FREQUENCY LOG-CHARACTERISTICS FOR 0; 30; 60; 100 AND 200 % OF THE LOAD

Design of filter elements is given in [7], harmonic distortion can be calculated using [8].

## III. DYNAMICAL PROPERTIES OF THE SYSTEM UNDER STEP FREQUENCY CHANGE

It is clear from the Figure 2. and 3. that impedance and voltage transfer characteristics for 200 % overloading are crossing at minimal (or maximal, respectively) frequency  $f_{min}$  (or  $f_{max}$ ) whereby the input current of the circuit will be nominal one due to input impedance equal nominal impedance.

By similar way is possible to determine the optimal operation frequencies for other value of overloading and functional relation is

$$|f_{\min}|_{\text{overload}} = f(Z_{\text{overload}}) \text{ or } |f_{\max}|_{\text{overload}} f(Z_{\text{overload}}) \quad (12)$$

to input current was the same as nominal one.

Simulation results under step- change of inverter switching frequency during constant overload (200 %) are shown in Figure 4. for both changes: from nominal to 70 % frequency, and from nominal to 140 % frequency.

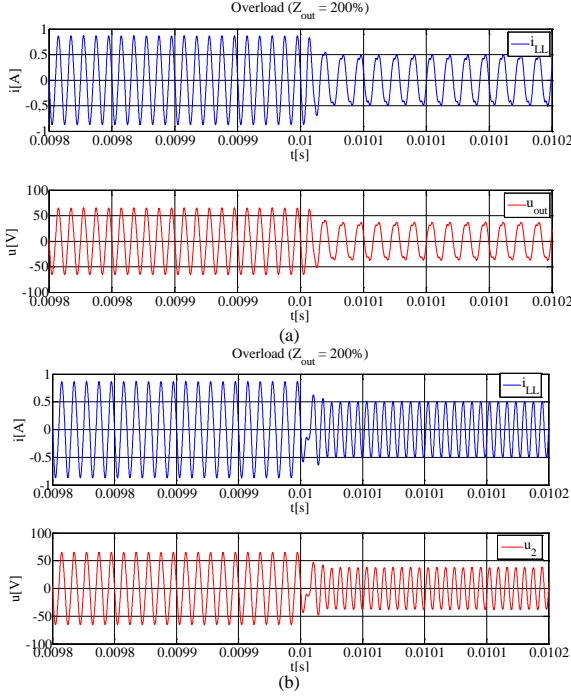


FIGURE IV. LCTL C CONVERTER RELATIVE FREQUENCY STEP CHANGE: (A) MINIMAL FREQUENCY, (B) MAXIMAL FREQUENCY

Based on input impedance frequency characteristic and voltage transfer characteristic is possible to choose minimal (or maximal, respectively) frequency  $f_{\min}$  (or  $f_{\max}$ ) whereby the input current of the LCTL C circuit will be nominal one.

By such a way overloading operation of the LCTL C inverter can be without any overloading currents. That way, of course, can be combined with the asymmetrical control [9] of input voltage of the inverter.

By comparison of the simulation results (Figure 5.) and impedance/voltage transfer characteristic (Figure 2. Figure 3) we can confirm accuracy of the mathematical model. The output voltage during overloading and step change of the switching frequency decreases and output current reaches less than nominal value same as in the characteristics.

#### IV. DYNAMICAL PROPERTIES OF THE SYSTEM UNDER STEP LOAD CHANGE OF RECTIFIER OUTPUT

The LCTL C inverter can be loaded by rectifier unlike the [10] with resistive or resistive-inductive load, Figure 4

Depending on type of rectifier switches (uni- or bi-directional ones) the rectifier can be operated in direct or regenerative modes. The mathematical model for rectified mode and dynamic states is very similar to the system

equation given by (1) with the difference that initial conditions in time instant of the dynamical changes are not equal zero. The initial conditions will be given by actual values of steady state of the system.

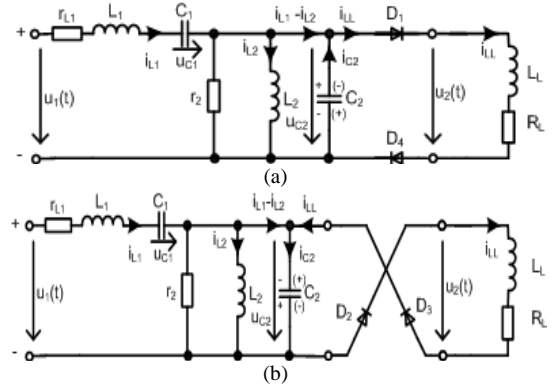


FIGURE V. EQUIVALENT CIRCUIT OF LCTL C INVERTER WITH RECTIFIER LOAD –UPPER: POSITIVE HALF PERIOD, BOTTOM: NEGATIVE HALF PERIOD

Since in positive half-period the state-space model is the same as described by (1), in the second i.e. negative half-period last two equations have to be changed. It is due to over-polarity of the load circuit

$$\frac{di_{L1}}{dt} = -\frac{r_1}{L_1} i_{L1} - \frac{1}{L_1} u_{C1} - \frac{1}{L_1} u_{C2} + \frac{1}{L_1} u_1 \quad (13)$$

$$\frac{di_{L2}}{dt} = \frac{1}{L_1} u_{C2} \quad (13)$$

$$\frac{du_{C1}}{dt} = \frac{1}{C_1} i_{L1} \quad (13)$$

$$\frac{du_{C2}}{dt} = \frac{1}{C_2} i_{L1} - \frac{1}{C_2} i_{L2} - \frac{1}{r_2 C_2} u_{C2} + \frac{1}{C_2} i_{LL} \quad (13)$$

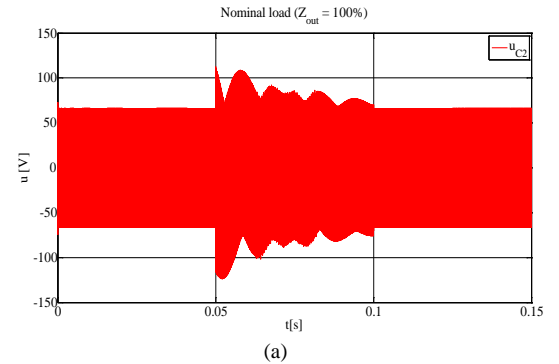
$$\frac{di_{LL}}{dt} = -\frac{1}{L_L} u_{C2} + \frac{R_L}{L_L} i_{LL} \quad (13)$$

Initial conditions in time instant of the dynamical changes are as following

$$i_{L1} = 0 \text{ V}; i_{L2} = 0.377 \text{ A}; u_{C1} = 193 \text{ V}; u_{C2} = 0 \text{ V}.$$

Discrete dynamical state-space model is creating by the same way as in Chapter I.

Dynamical properties for switching on and switching off of the rectifier are shown in Figure 6



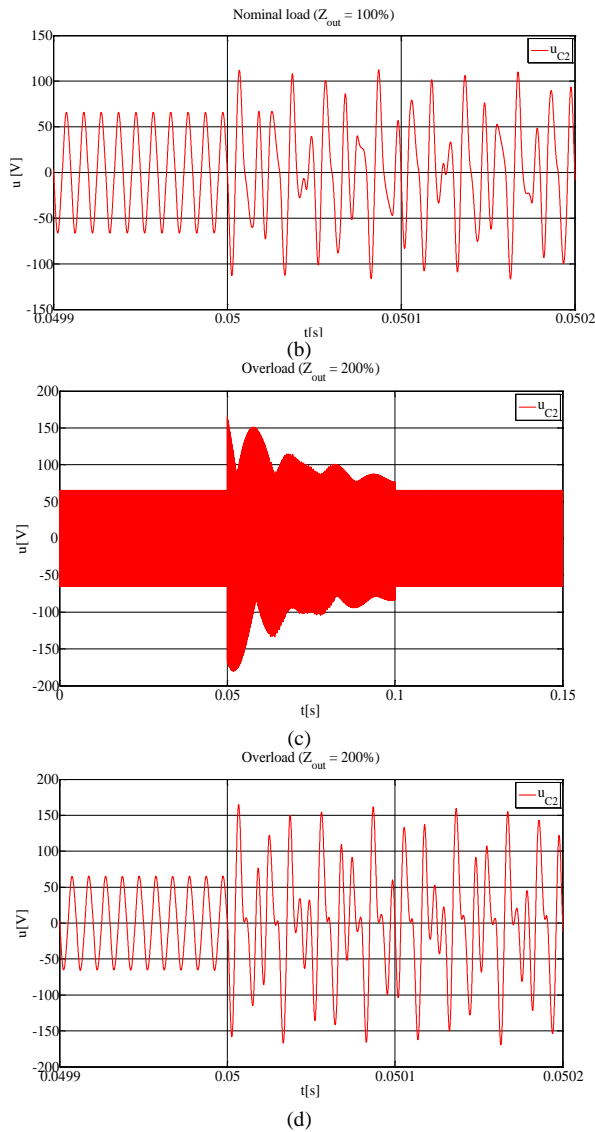


FIGURE VI. CONVERTER OUTPUT VOLTAGE DURING RECTIFIER LOAD SWITCH OFF AND SWITCH ON: (A) NOMINAL LOAD, (B) DETAIL OF OUTPUT VOLTAGE, (C) OVERLOAD, (D) DETAIL OF OUTPUT VOLTAGE

From the Figure 6. can be seen that amplitude of the voltage  $u_{C2}$  is dependent on the accumulated energy in the storage elements of the filter (the bigger energy the higher overvoltage) during rectifier load switch off.

## V. CONCLUSION

Continuous and discrete dynamical state-space model LCLC type resonant converter was introduced with both simply resistive-inductive load and output loaded rectifier. The model was used to presenting of solution of frequency step change as well as step change of the loaded rectifier switching-off.

Using input impedance and voltage transfer characteristics was shown that during frequency step changes under even 200 % overloading of the LCLC type resonant the input current will not be overcoming of nominal value.

Other dynamical state, i.e. step change of the loaded rectifier switch-off, is much more dangerous. During switching-off of the nominal rectified load the overvoltage spikes reach two-multiply of the nominal voltage, and during switching-off of the 200 % rectified load the overvoltage spikes reach three-multiply of the nominal voltage value.

The simulation experiment results confirm theoretical assumptions, and presented techniques are suitable for both transient and steady-state behavior of investigated system mainly in electrical engineering, and it can be effectively combined with classical solution used non-symmetrical control technique.

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