Modelling of Stagnation-Point Flow and Diffusion of Chemically Reactive Species Past A Permeable Quadratically Stretching/Shrinking Sheet

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Abstract—**A numerical study has been conducted to investigate the modelling of steady two-dimensional stagnation-point flow and diffusion of chemically reactive species of a viscous and incompressible fluid past a permeable quadratically stretching/ shrinking sheet. Similarity transformation has been used to reduce the governing nonlinear partial differential equations into a system of nonlinear ordinary differential equations. The resulting equations are solved numerically using "bvp4c" function in MATLAB. Dual solutions are found for a certain range of the stretching/shrinking parameter. The effects of the governing parameters on the skin friction coefficient and the concentration gradient are discussed and illustrated graphically.**

Keywords-stagnation-point flow; diffusion; chemically reactive species; stretching/shrinking sheet; dual solutions

I. INTRODUCTION

The flow over a stretching surface has numerous applications in engineering and manufacturing processes in industries such as wire drawing and extrusion of plastic sheets. Sakiadis [1] was the first to consider a boundary-layer flow on continuous moving surfaces. Crane [2] considered a viscous incompressible fluid flow over a linearly stretching sheet. The study done in [2] was then extended for different types of fluid and various physical properties (see [3]–[7]). Miklavcic and Wang [8] were the first to study the flow induced by a shrinking sheet. Later, the study of viscous flow past a shrinking sheet/surface has been performed and extended by many researchers (see [9]-[16]).

The study of flow near a stagnation-point is a classical problem in fluid dynamics and has many applications in industry. Hiemenz [17] was the first to investigate the two-dimensional stagnation flow against a stationary semi-infinite wall. Homann [18] extended [17] by considering an axisymmetric case. The flow near the stagnation-point was studied by Howarth [19]. Later, Rott [20] investigated the two-dimensional normal stagnation flows towards a plate that is oscillating in its own plane, while Libby [21] extended [18] by considering the three-dimensional stagnation flow towards a moving plate. Iwan and Alexander [22] and Weidman and Mahalingam [23] studied a stagnation flow towards a porous plate by considering suction or injection. The study of axisymmetric stagnation flow on a moving plate and circular cylinder has been done by Wang [24] and Wang [25]. A review of the existing steady similarity stagnation flow solutions of the Navier-Stokes equations was thoroughly discussed by Wang [26].

The addition or diffusion of chemical reaction in the boundary layer flow has many applications in water and air pollutions, atmospheric flows, fibrous insulation and other chemical engineering problems. The diffusion of a chemically reactive species in a laminar boundary layer flow was discussed by Chambré and Young [27]. Andersson et al. [28] investigated the transfer of a chemically reactive species over a linearly stretching sheet for homogenous first and higher order reactions. Takhar et al. [29] studied the flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species, while Raptis and Perdikis [30] considered the viscous flow in the presence of a chemical reaction and magnetic field over a nonlinearly stretching sheet. Muhaimin et al. [31] studied the effect of chemical reaction, heat and mass transfer on nonlinear magnetohydrodynamic (MHD) boundary layer past a porous shrinking sheet in the presence of suction. The thermal-diffusion and diffusion-thermo effects on heat and mass transfer by MHD mixed convection stagnation-point flow of a power-law non-Newtonian fluid towards a stretching surface in the presence of a magnetic field, thermal radiation and homogenous chemical reaction effects has been studied by El-Kabeir et al. [32]. Further, Ziabakhsh et al. [33] studied the

analytical solution for the problem of flow and diffusion of chemically reactive species over a nonlinearly stretching sheet immersed in a porous medium, while Bhattacharyya [34] obtained dual solutions for the problem of boundary layer stagnation-point flow and mass transfer with chemical reaction past a stretching/shrinking sheet. Roşca et al. [35] investigated the steady forced convection stagnation-point flow and mass transfer past a permeable stretching/shrinking sheet in a nanofluid. The steady two-dimensional MHD stagnation-point flow towards a permeable stretching sheet with chemical reaction was studied by Rasekh et al. [36]. Recently, the unsteady two-dimensional boundary layer stagnation-point flow of a nanofluid over a heated stretching sheet is investigated numerically by Abd El-Aziz [37], while Najib et al. [38] discussed the stagnation-point flow and mass transfer with the chemical reaction past a stretching/shrinking cylinder.

The aim of the present study is to extend the work done by Ziabakhsh et al. [33] by considering the stagnation-point flow past a permeable quadratically stretching/shrinking sheet. The governing partial differential equations are reduced to a system of nonlinear ordinary differential equations by using similarity transformations and solved numerically. The effects of the governing parameters on the skin friction coefficient and concentration gradient are investigated and discussed.

II. GOVERNING EQUATIONS

Consider the steady two-dimensional stagnation-point flow and diffusion of chemically reactive species of a viscous and incompressible fluid over a permeable quadratically stretching/ shrinking sheet, where x and y are the Cartesian coordinates measured along and normal to the surface of the sheet, respectively. It is assumed that the surface is stretched/shrunk in the x-direction with the velocity $u_w(x)$, the mass transfer velocity is $v_w(x)$ and the velocity of the far flow (inviscid flow) is $u_e(x)$.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{d u_e}{d x} + v\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_1 C. \tag{3}$$

We shall solve (1)-(3) subject to the following boundary conditions:

$$\left. \begin{array}{l} u = u_w(x) = a\lambda x + b\lambda x^2, \\ v = v_w(x) = v_0 + v_0\dfrac{x}{c}, C = C_w, \\ u = u_e(x) \to ax + bx^2, C \to 0 \end{array} \right\} \begin{array}{l} \text{at } y = 0, \\ \\ \text{as } y \to \infty, \end{array} \tag{4}$$

where u and v are the velocity components along the x and y axes, respectively, C is the species concentration in the fluid, v is the kinematic viscosity, k_1 is the chemical reaction rate, D is

the mass diffusion coefficient, a, b and c are positive constants with c ($\neq 0$), v_0 is the constant mass velocity with $v_0 < 0$ for suction and $v_0 > 0$ for injection, and λ is the constant stretching/shrinking parameter, with $\lambda > 0$ for a stretching sheet and $\lambda < 0$ for a shrinking sheet, respectively. It is assumed that the diffusing species is destroyed, i.e. destructive chemical reaction $k_1 > 0$ in the homogeneous reaction. We notice that when $b = 0$, it corresponds to the linearly stretching/shrinking sheet. The other case is $b \neq 0$, which corresponds to a quadratic stretching/shrinking sheet.

Introducing the following similarity solutions (see [33])

$$u = axf'(\eta) + bx^2 g'(\eta), v = -\sqrt{av}f(\eta) - \frac{2xb}{\sqrt{a/v}}g(\eta),$$

$$C = C_w\left[F(\eta) + \frac{2bx}{a}h(\eta)\right], \eta = \sqrt{\frac{a}{v}}y, \tag{5}$$

where primes denote differentiation with respect to η.

Substituting (5) into (2) and (3), the following set of ordinary differential equations results in

$$f''' + ff'' + 1 - f'^2 = 0, \tag{6}$$

$$g''' + fg'' - 3f'g' + 2f''g + 3 = 0, \tag{7}$$

$$F'' + Sc\,fF' - KF = 0, \tag{8}$$

$$h'' + Sc\left(fh' - f'h + F'g\right) - Kh = 0, \tag{9}$$

subject to the boundary condition

$$f(0) = S, f'(0) = \lambda, g(0) = S_1, g'(0) = \lambda, F(0) = 1, h(0) = 0,$$

$$f'(\eta) \to 1, g'(\eta) \to 1, F(\eta) \to 0, h(\eta) \to 0 \text{ as } \eta \to \infty, \tag{10}$$

where Sc is the Schmidt number, K is the dimensionless homogeneous reaction parameter and S and S_1 (where $S = S_1$) are the dimensionless velocity mass flux parameters with $(S, S_1) > 0$ for suction and $(S, S_1) < 0$ for injection, which are defined as

$$Sc = \frac{v}{D}, K = \frac{k_1 Sc}{a}, S = -\frac{v_0}{\sqrt{av}}, S_1 = -\frac{v_0}{2bc}\sqrt{\frac{a}{v}}. \tag{11}$$

The quantities of physical interest are the skin friction coefficient C_f and the local Sherwood number Sh_x, which are defined as

$$C_f = \frac{\tau_w}{\rho(ax)^2}, Sh_x = \frac{xq_m}{DC_w}, \tag{12}$$

where τ_w is the skin friction or the shear stress along the surface, ρ is the fluid density and q_m is the surface concentration of the reactant, which are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_m = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}. \tag{13}$$

Using (5) into (12) and (13), we obtain

$$\text{Re}_x^{1/2} C_f = f''(0) + \frac{b}{a} x\, g''(0),$$
$$\text{Re}_x^{-1/2} Sh_x = -F'(0) - \frac{2bx}{a} h'(0), \tag{14}$$

where $\text{Re}_x = (a\,x)\,x/\nu$ is the local Reynolds number.

III. RESULTS AND DISCUSSION

The nonlinear ordinary differential equations (6)-(9) with boundary conditions (10) are solved numerically by using the "bvp4c" function from MATLAB (see Kierzenka and Shampine [39]). To validate the accuracy of the present numerical method, the values of the reduced skin friction coefficient $f''(0)$ are compared with [34], which can be observed in Table 1. The comparisons are found to be in excellent agreement, and thus we are confident that the present method is accurate.

Figure 1 displays the variations of the reduced skin friction coefficients $f''(0)$ with λ for some values of S when $K = 0.5$ and $Sc = 0.6$. The values of $f''(0)$ are seen to increase with the increase of S. It can be seen that there exists more than one solution for a certain range of the stretching/shrinking parameter λ. The dual solutions are obtained by setting two different initial guesses for the missing values of $f''(0)$, $g''(0)$, $-F'(0)$ and $-h'(0)$, where all profiles satisfy the far field boundary conditions (10) asymptotically. It appears that the solution is unique for $\lambda > -1$, while the dual solutions exist for $\lambda_c \le \lambda \le -1$, and no solution for $\lambda < \lambda_c$, where λ_c is the critical values of λ. Beyond these critical points, the boundary layer separates from the surface, hence the solutions based upon the boundary layer approximations are not possible. Figure 1 also shows that the values of $|\lambda_c|$ increase with the increase of S. This indicates that mass flux parameter S widen the range of λ for which solutions exist.

TABLE I. COMPARISON OF THE VALUES OF $f''(0)$ WITH [34] FOR SOME VALUES OF λ WHEN $S = 0$

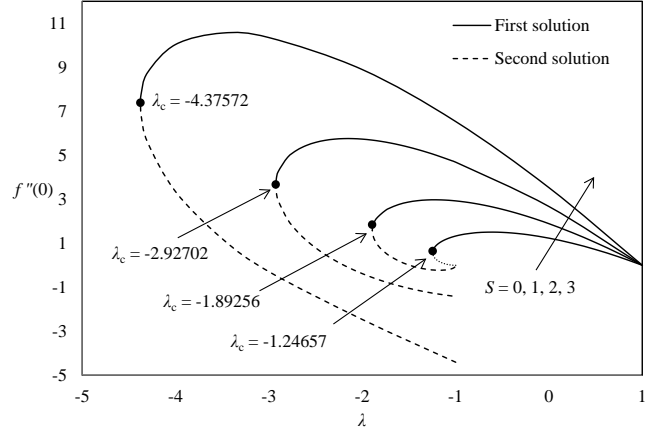|	[34]		Present results	
λ	First solution	Second solution	First solution	Second solution
-1	1.3288169	0	1.3288169	0
-1.2	0.9324728	0.2336491	0.9324734	0.2336497
-1.2465	0.5842915	0.5542856	0.5842979	0.5542962
-1.24657	0.5745268	0.5639987	0.5745599	0.5640126

FIGURE I. VARIATIONS OF $f''(0)$ WITH λ FOR DIFFERENT S WHEN $K = 0.5$, $Sc = 0.6$

Figure 2 illustrates the variations of the concentration gradients $-h'(0)$ with λ for various values of reaction parameter K when $S = 3$ and $Sc = 0.6$. It can be observed that the values of $-h'(0)$ are decreasing with the increase of K. In general, increasing reaction parameter K is to increase the concentration gradient at the surface. The critical point appears to be the same for all values of K, which happened because the changes in K does not affect the stretching/shrinking parameter λ.

Figure 3 displays the velocity profiles $f'(\eta)$ for different stretching/shrinking parameter λ when $S = 3$, $K = 0.5$ and $Sc = 0.6$. The velocity is shown to decrease with the increase of λ. We notice that the boundary layer thickness for the second solution is larger than the first solution. It is worth mentioning that the first solutions are stable and physically realizable, while the second solutions are not, as discussed via the stability analysis by Roşca and Pop [15], Weidman et al. [40] and Harris et al. [41]. The profiles displayed in Figure 3 satisfy the far field boundary conditions (10) asymptotically, thus supporting the validity of the dual solutions obtained in this study.
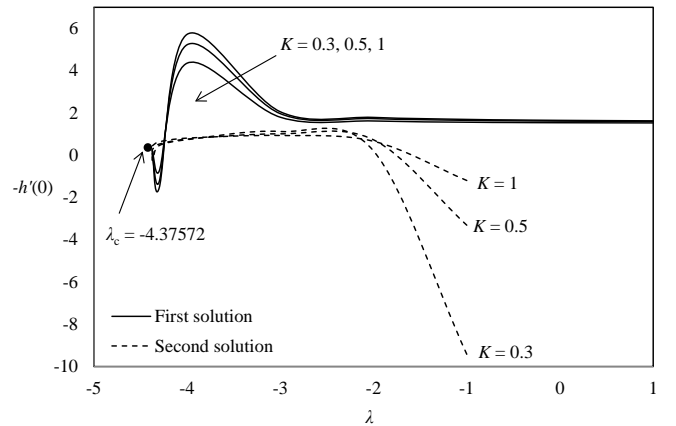

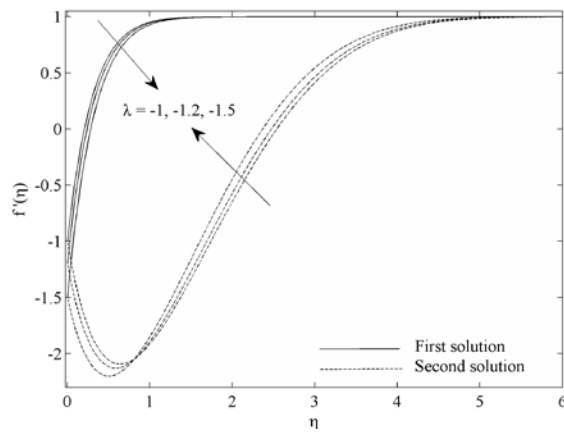FIGURE II. VARIATIONS OF $-h'(0)$ WITH λ FOR DIFFERENT K WHEN $S = 3$, $Sc = 0.6$

FIGURE III. VELOCITY PROFILES $f'(\eta)$ FOR SEVERAL VALUES OF λ WHEN $S = 3$, $K = 0.5$, $Sc = 0.6$

IV. CONCLUSIONS

The problem of steady two-dimensional stagnation-point flow and diffusion of chemically reactive species of a viscous and incompressible fluid past a permeable quadratically stretching/shrinking sheet has been studied. This problem is solved numerically by the bvp4c function from MATLAB. The influence of the velocity mass flux parameter S, stretching/shrinking parameter λ, reaction parameter K and Schmidt number Sc have been analyzed and presented graphically. Dual solutions are found for a certain range of the stretching/shrinking parameter. The mass flux parameter S widens the range of λ for which similarity solutions exist. The values of the skin friction coefficients increase with the increase of velocity mass flux parameters S. The concentration gradient $-h'(0)$ decreases with the increase of reaction parameter K. The velocity profiles $f'(\eta)$ decrease with the increase of the stretching/shrinking parameter λ.

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