Knowledge Checking Service Selection Method in Pythagorean Fuzzy Environment

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Abstract— Knowledge checking service is very important for safeguarding the quality of the knowledge in the knowledge repository. There are many knowledge checking services with similar functions but different qualities in organizations, especially in the crossing organizations. It is not easy to measure the non-functional criteria because of the complexity and the involvement of user’s fuzzy perceptions of knowledge checking services. In order to help the evaluation and selection of the knowledge checking services, the knowledge checking services selection method in Pythagorean fuzzy environment is developed. Firstly, decision makers use linguistic terms to express their preferences on the alternative knowledge checking services with respect to each criterion. Afterwards, the linguistic terms are transformed into the Pythagorean fuzzy forms. Then the collective opinions are derived by aggregating the opinion given by each decision maker. After calculating the degree of linguistic grey relational coefficient of each alternative from PIS and NIS, the relative relational degree of each alternative from PIS are derived. Then all the alternatives are ranked in accordance with the relative relational degree in descending order. The illustrative example of knowledge service selection shows the proposed method is feasible and efficient.

Keywords— Knowledge Checking Service Selection; Multiple Attribute Group Decision Making; GRA; Pythagorean Fuzzy Sets

I. INTRODUCTION

In today’s rapid changing environment, knowledge plays the important role in organizations [1, 2]. It can keep and improve the core competition ability [3-5]. Since the knowledge within one organization is limited, more and more organizations try to build the alliance to use the knowledge outside the organizations [6-8]. The quality of knowledge in the knowledge repository determines the effects of knowledge sharing and using [9, 10]. Therefore, the knowledge needs to be checked before the using. The service oriented applications are developed and used more and more wildly. The level of knowledge checking service has great influence on the outcome of knowledge management. A knowledge checking service with superior quality can bring high value to organizations. In each organization, there are many knowledge checking services. These knowledge checking services may sometimes have the similar functions, especially in the alliance of many organizations. Measuring the quality of these services with similar functions but different qualities becomes an imperative concern for decision makers.

Therefore, these knowledge checking services with similar functions need to be evaluated and selected. In reality, the evaluation of knowledge checking services needs to be made from more than one aspect. With the increasing complexity, it is more and more difficult for single decision maker to give the accurate evaluation. In order to ease the bias of single decision maker and make the evaluation more accurate and objective, the evaluation should be made by more than one decision maker, which integrates various kinds of knowledge and experiences. Because of the complexity and the involvement of decision maker’s fuzzy perceptions of knowledge checking services, in the evaluation of knowledge checking services, it is more and more difficult to derive or give precise numerical opinions. On the contrary, decision makers prefer to use the linguistic terms, which facilitates the expression of opinions.

Pythagorean fuzzy set (PFS) was proposed by Yager to model the linguistic terms [11-15]. The background of the PFSs is that in the decision making process, the sum of the support degree and the opposite degree to which an alternative with respect to one criterion provided by the decision maker may be bigger than one, but the sum of their squares is not more than one [12]. PFS is characterized by a membership degree and a non-membership degree, which is similar to the intuitionistic fuzzy set [13-15]. The main differences between the Pythagorean fuzzy set and intuitionistic fuzzy set is that the square sum of PFS’s membership degree and PFS’s non-membership degree is not more than one, which can be deemed as the generalization of IFS. PFS release the restriction of the IFS and provides more loosing expression of fuzziness [16].
Grey relational analysis (GRA) is a commonly used tool to deal with the opinion given by a group of decision makers [17], which is an important part of grey system theory [18]. In grey system theory, the system with completely known information is called a white system; on the contrary, the system with completely unknown information is called a black system. If the information is partially known, the system is called a grey system [19, 20]. The calculation to deal with the opinions given by a group of decision makers based on GRA method is straightforward and the results are easier to be understood [21].

In this paper, a method for knowledge checking service selection method in Pythagorean fuzzy environment is developed. The remainder of this paper is organized as follows. In the next section, the Pythagorean fuzzy set is introduced. In the third Section, a method for knowledge checking service selection method in Pythagorean fuzzy environment is given. In the method, the opinions given by decision makers are expressed by the Pythagorean fuzzy sets. The GRA method is extended in the Pythagorean fuzzy environment to deal with the opinion given by a group of decision makers with respect to criteria. In Section 4, an illustrative example of knowledge service selection is given to verify the proposed approach. Finally, we conclude the paper.

II. PRELIMINARIES

Pythagorean fuzzy set (PFS) is characterized by membership degree and non-membership degree, the square sum of which is not more than one [11-15]. In the following, the definition and operators of Pythagorean fuzzy set is reviewed [16].

Let a set $X$ be a universe of discourse. The PFS $P$ can be represented as

$$P = \{ x, P\mu (x), vP(x) \}$$

where $\muP$ denotes the degree of membership, and $vP$ denotes the degree of non-membership of the element $x$ to $P$.

$P(x, vP(x))$ can be represented by $\beta = P(\muP, vP)$, where $\muP, vP \in [0,1]$ and $\muP^2 + vP^2 \leq 1$.

Let $\beta_j = P(x, vP_j) (j = 1, 2, \ldots, n)$ be the set of PFNs and $\nu = (v_{11}, v_{12}, \ldots, v_{nm})$ be the corresponding weight of $\beta_j (j = 1, 2, \ldots, m)$, where $0 \leq v_{ij} \leq 1$ and $\sum_{j=1}^{m} v_{ij} = 1$, then the operators of the PFS can be defined as [16].

Definition 1 the sum of two PFNs can be defined as

$$\beta_1 \oplus \beta_2 = P\left( \mu_{\beta_1}^2 + \mu_{\beta_2}^2 - \mu_{\beta_1} \mu_{\beta_2}, v_{\beta_1}^2 + v_{\beta_2}^2 \right)$$

Definition 2 the product of two PFNs can be defined as

$$\beta_1 \otimes \beta_2 = P\left( \mu_{\beta_1} \mu_{\beta_2}, \sqrt{\mu_{\beta_1}^2 + v_{\beta_1}^2 - \mu_{\beta_1}^2 v_{\beta_1}^2} \right)$$

Definition 3 $\lambda \beta_1 = P\left( \lambda - (1 - \mu_{\beta_1}^2)^{\lambda}, v_{\beta_1}^2 \right), \lambda > 0$

Definition 4 the distances between two PFNs can be defined as

$$d(\beta_1, \beta_2) = \frac{1}{2} \left( \left| \left( \mu_{\beta_1}^2 - \mu_{\beta_2}^2 \right) \right| + \left| \left( v_{\beta_1}^2 - v_{\beta_2}^2 \right) \right| + \left( \sqrt{1 - \mu_{\beta_1}^2 - v_{\beta_1}^2} \right) \left( \sqrt{1 - \mu_{\beta_2}^2 - v_{\beta_2}^2} \right) \right)$$

III. KNOWLEDGE CHECKING SERVICE SELECTION METHOD IN PYTHAGOREAN FUZZY ENVIRONMENT

Let $A = \{A_1, A_2, \ldots, A_m\}$ be the set of alternatives, $D = \{D_1, D_2, \ldots, D_n\}$ be the set of decision makers, $C = \{C_1, C_2, \ldots, C_n\}$ be the set of criteria, $w = (w_1, w_2, \ldots, w_n)$ be the weight vector of criteria, where $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. Suppose $R = (r_{ij}^P)_{m \times n}$ is the group decision making matrix of the ratings to each alternative, where $r_{ij}^P$ is a preference value given by the decision maker $D_i \in D$.

To get the best knowledge checking service (s), by extending the GRA in Pythagorean fuzzy environment, the steps of the proposed method are given as follows:

Step 1 Transforming linguistic decision matrix of the ratings to each alternative into Pythagorean fuzzy linguistic decision matrix.

Step 2 Deriving the collective overall Pythagorean fuzzy decision matrix by aggregating the decision information given in matrix.

$$r_{ij} = \frac{1}{t} \sum_{k=1}^{t} r_{ij}^P = P\left( \frac{1}{t} \sum_{k=1}^{t} w_{ij}^P, \frac{1}{t} \sum_{k=1}^{t} v_{ij}^P \right)$$

where $t$ is the number of decision makers.

Step 3 Defining the positive ideal solution (PIS) and negative ideal solution (NIS) of the criteria as

$$r^+ = (r_{11}^+, r_{22}^+, \ldots, r_{nn}^+)$$

$$r^- = (r_{11}^-, r_{22}^-, \ldots, r_{nn}^-)$$

Where, the PIS and NIS of the criterion $C_j$ can be derived by

$$r_{ij}^+ = \max\{S(r_{ij})\}, j = 1, 2, \ldots, n.$$  \hspace{1cm} (9)

$$r_{ij}^- = \min\{S(r_{ij})\}, j = 1, 2, \ldots, n.$$  \hspace{1cm} (9)

Step 4 Calculating the grey relational coefficient of each alternative from PIS and NIS.

Firstly, the distances of each alternative to PIS and NIS are defined as

$$\gamma_{ij} = \frac{1}{t} \sum_{k=1}^{t} \frac{u_{ij}^P}{v_{ij}^P}$$

$$\gamma_{ij}^+ = \max\{S(\gamma_{ij})\}, j = 1, 2, \ldots, n.$$  \hspace{1cm} (9)

$$\gamma_{ij}^- = \min\{S(\gamma_{ij})\}, j = 1, 2, \ldots, n.$$  \hspace{1cm} (9)

Finally, the grey relational coefficient of each alternative is calculated and the best alternative is determined.
\[ d_{ij} = d(r_i, r_j^*) = \frac{1}{2} \left[ \left( u_{ij}^2 - u_j^* \right)^2 \right] + \left( v_{ij}^2 - v_j^* \right)^2 + \left( 1 - u_{ij}^2 - v_{ij}^2 \right)^2 - \left( 1 - u_j^* - v_j^* \right)^2 \] 

(10)

\[ d_{ij} = d(r_i, r_j^+) = \frac{1}{2} \left[ \left( u_{ij}^2 - u_j^* \right)^2 \right] + \left( v_{ij}^2 - v_j^* \right)^2 + \left( 1 - u_{ij}^2 - v_{ij}^2 \right)^2 - \left( 1 - u_j^* - v_j^* \right)^2 \] 

(11)

Then grey relational coefficient of each alternative from PIS and NIS can be obtained by

\[ \zeta_{ij}^* = \frac{\min_{i} \min_{j} d_{ij}^* + \lambda \max_{i} \max_{j} d_{ij}^*}{d_{ij}^* + \lambda \max_{i} \max_{j} d_{ij}^*} \] 

(12)

\[ \zeta_{ij}^- = \frac{\min_{i} \min_{j} d_{ij}^- + \lambda \max_{i} \max_{j} d_{ij}^-}{d_{ij}^- + \lambda \max_{i} \max_{j} d_{ij}^-} \] 

(13)

Where, \( \lambda \) is the identification coefficient.

**Step 5** Weighted grey relational coefficient of each alternative from PIS and NIS is derived by using the following equation, respectively:

\[ \rho_i^* = \sum_{j=1}^{m} w_j \times \zeta_{ij}^*, \quad i = 1, 2, \ldots, m \] 

(14)

\[ \rho_i^- = \sum_{j=1}^{m} w_j \times \zeta_{ij}^-, \quad i = 1, 2, \ldots, m \] 

(15)

**Step 6**. The relative relational degree of each alternative from PIS can be obtained by

\[ RCD_i = \frac{\rho_i^*}{\rho_i^* + \rho_i^-}, \quad i = 1, 2, 3, \ldots, m \] 

(16)

Then the alternatives can be ranked in the descending order according to the relative relational degrees.

IV. NUMERICAL EXAMPLE

Suppose there are five knowledge checking services denoted by \( A = \{A_1, A_2, A_3, A_4, A_5\} \) which have similar functions but different qualities are to be evaluated. In order to find the best knowledge checking service, five criteria are identified [22], which include ‘Security’ (C1), ‘Transaction’ (C2), ‘Cost’ (C3), ‘Runtime’ (C4) and ‘Network’ (C5). The weights of the criteria are 0.2, 0.3, 0.3, 0.1 and 0.1. The decision makers denoted by \( D = \{D_1, D_2, D_3, D_4\} \) use the linguistic terms in Table 1 to express their preferences.

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>PNFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definitely low (DL)</td>
<td>[0.1, 0.9]</td>
</tr>
<tr>
<td>Very low (VL)</td>
<td>[0.2, 0.8]</td>
</tr>
<tr>
<td>Low (L)</td>
<td>[0.4, 0.7]</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>[0.5, 0.6]</td>
</tr>
<tr>
<td>High (H)</td>
<td>[0.6, 0.5]</td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>[0.7, 0.3]</td>
</tr>
<tr>
<td>Definitely high (DH)</td>
<td>[0.9, 0.1]</td>
</tr>
</tbody>
</table>

The linguistic rating of the five alternative knowledge checking services with respect to the five criteria given by the four decision makers are shown in Tables 2-5, respectively.
Step 2 Utilizing the decision information given in Table 6-9 to derive the collective overall Pythagorean fuzzy linguistic decision information, the results of which are shown in Table 10.

Step 3 Defining the PIS and NIS

The S function is used to compare the aggregated rating values. The value of S function of each alternative with respect to the criteria is shown in Table 11.

Step 4 Calculating the grey relational coefficient of each alternative from PIS and NIS

With (10) and (11), the distances of each alternative with respect to each criterion from PIS and NIS are obtained, which are shown in Table 13 and 14, respectively.

Step 5 Calculating weighted grey relational coefficient of each alternative from PIS and NIS.

By using (14) and (15), firstly, the weighted grey relational coefficient of each alternative from PIS and NIS can be obtained, which are shown in Table 15 and 16, respectively.


TABLE 17. WEIGHTED GREY RELATIONAL COEFFICIENT OF EACH ALTERNATIVE WITH RESPECT TO EACH CRITERIA FROM PIS

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.097</td>
<td>0.133</td>
<td>0.131</td>
<td>0.055</td>
<td>0.047</td>
</tr>
<tr>
<td>A2</td>
<td>0.100</td>
<td>0.141</td>
<td>0.118</td>
<td>0.054</td>
<td>0.079</td>
</tr>
<tr>
<td>A3</td>
<td>0.099</td>
<td>0.148</td>
<td>0.157</td>
<td>0.056</td>
<td>0.092</td>
</tr>
<tr>
<td>A4</td>
<td>0.100</td>
<td>0.126</td>
<td>0.134</td>
<td>0.054</td>
<td>0.082</td>
</tr>
<tr>
<td>A5</td>
<td>0.100</td>
<td>0.138</td>
<td>0.132</td>
<td>0.055</td>
<td>0.064</td>
</tr>
</tbody>
</table>

TABLE 18. WEIGHTED GREY RELATIONAL COEFFICIENT OF EACH ALTERNATIVE WITH RESPECT TO EACH CRITERIA FROM NIS

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.144</td>
<td>0.170</td>
<td>0.129</td>
<td>0.066</td>
<td>0.039</td>
</tr>
<tr>
<td>A2</td>
<td>0.181</td>
<td>0.159</td>
<td>0.115</td>
<td>0.067</td>
<td>0.052</td>
</tr>
<tr>
<td>A3</td>
<td>0.166</td>
<td>0.126</td>
<td>0.112</td>
<td>0.054</td>
<td>0.057</td>
</tr>
<tr>
<td>A4</td>
<td>0.112</td>
<td>0.135</td>
<td>0.127</td>
<td>0.055</td>
<td>0.080</td>
</tr>
<tr>
<td>A5</td>
<td>0.181</td>
<td>0.145</td>
<td>0.128</td>
<td>0.065</td>
<td>0.045</td>
</tr>
</tbody>
</table>

TABLE 19. GREY RELATIONAL COEFFICIENT OF EACH ALTERNATIVE FROM PIS AND NIS

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIS</td>
<td>0.464</td>
<td>0.492</td>
<td>0.552</td>
<td>0.495</td>
<td>0.489</td>
</tr>
<tr>
<td>NIS</td>
<td>0.549</td>
<td>0.573</td>
<td>0.514</td>
<td>0.510</td>
<td>0.565</td>
</tr>
</tbody>
</table>

Step 6 Calculating the relative closeness degree of each alternative from PIS.

The relative relational degrees of each alternative are derived by (16), the results of which are shown in Table 20.

TABLE 20. RELATIVE RELATIONAL DEGREES

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.458</td>
<td>0.462</td>
<td>0.518</td>
<td>0.493</td>
<td>0.464</td>
</tr>
</tbody>
</table>

Then the alternatives can be ranked in the descending order according to the relative relational degrees and the ranking is: A3 > A4 > A5 > A2 > A1.

V. CONCLUSION

This paper proposed the method for the evaluation and selection of the knowledge checking services. In the method, the linguistic terms are provided to decision makers to express their opinions on the alternatives with respect to more than one criterion. Then the opinions which are in the form of linguistic terms are transformed into the Pythagorean fuzzy forms. After the aggregation of opinion of each user, the collective opinions are derived. Then based on the idea of GRA, the relative relational degrees of each alternative from PIS are derived. All the alternatives are ranked in accordance with the relative relational degree in the descending order. The alternative that has the highest relative relational degree is the best knowledge checking service. The illustrative example of knowledge service selection shows the proposed method is feasible and efficient. With the method, when facing many knowledge checking services with similar functions but different qualities, the best knowledge checking service can be find more objectively and accurately.

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