

# Research on Job Shop Scheduling Method based on Genetic Algorithm under Uncertainty

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**Abstract**—The job shop scheduling problem with different due date windows is researched. A mathematical model is presented with the consideration of uncertain processing time which is presented by triangular fuzzy number. Furthermore, a genetic algorithm is developed to solve this problem. The algorithm generates the initial population by the method of integer coding, combining roulette method and elitist strategy is extended in the selection operation. Precedence operation crossover (POX) is used to ensure the feasibility of the offspring. In the end, an example is given to guarantee the validity of the model and algorithm.

**Keywords**- Job Shop Scheduling; Uncertainty; Earliness / Tardiness; Different Due Date Window; Genetic Algorithm

## I. INTRODUCTION

Job Shop Scheduling Problem (JSSP) is a well-known scheduling problem, while the production scheduling optimization is the core of modern management techniques as well as advanced manufacturing technologies, which is a key factor to achieve efficient production of enterprise. With the continuous development of economic globalization, the customer needs is entering a new phase of diversification, the manufacturing industry is facing the problem of how to effectively organize many varieties of small batch production. JIT (Just in time) production method arises as a high-quality, low-cost mode of production under the condition of multi-species and small batch, it requires machining parts at the stipulated time, on-time delivery. Therefore, delivery of the goods is getting more attention as the evaluation index of scheduling problem, and the delivery window scheduling is more general expression of the scheduling problem in advance/tardiness, it rules that the completion time will not be punishment if it is between a certain range, otherwise it will be punished [1].

For previous researches on shop scheduling, less consideration is taken into about the uncertainty factors of the production, while there are various uncertainties during the actual production process such as the uncertainty of processing time, uncertain product handling capacity and the emergency uncertainty [2]. The processing time is the core data of scheduling problem, Yang Hongan [3] proposed a new interval model based on selecting E/T penalty interval as optimization goal, using interval indicates the processing time. Lei [4] used a population-based neighborhood search (PNS) to optimize the interval makespan of the problem. For stochastic job shop scheduling problem, Azadeh [5] presented a heuristic algorithm based on computer simulation and artificial neural networks, and achieved optimal makespan by

selecting the optimal scheduling rules. Hu [6] researched the fuzzy processing time and fuzzy due date of the job shop scheduling problem, and proposed a differential evaluation algorithm (DE) for multi-objective optimization. Li Junqing [7] proposed a hybrid algorithm (CROTS) which was combined with chemical reaction optimization and tabu search for fuzzy Job Shop scheduling. Yang Jianbin [8] extended the model to maximize customer satisfaction as the minimum scheduling objective under the fuzzy circumstances. The uncertainties described in above studies use the interval number, random variables and fuzzy mathematics method.

Job Shop Scheduling problem with different delivery window is studied, considering uncertainty of the product processing time, triangular fuzzy numbers is used to describe the uncertain processing time. Most previous research on fuzzy JSP problem, just set the target on processing time and delivery time, but in reality, policy makers are more concerned of the problems such as the production efficiency, cost, profit and others, therefore in this paper reducing the total cost is targeted, considering both the idle cost of the machine, and also the early and tardiness penalty value. A mathematical model is built with the consideration of uncertain processing time which is presented by triangular fuzzy number. An improved genetic algorithm is developed to solve this problem. Finally, a numerical example is given to show the convergence and effectiveness of the algorithm.

## II. DESCRIPTION OF THE PROBLEM

### A. General Description of Scheduling problem

Uncertain Job Shop Scheduling can be defined as: Suppose there are  $m$  machines,  $n$  jobs to be processed, each job  $j$  composed by  $d_j$  processes, in this paper  $O_{ij}$  expresses the  $i$ th operation on the job  $j$ . In each process the fuzzy processing time and delivery time are given in advance. The process needs to satisfy the following constraints:

- (1) In the same time the same machine can only process one part;
- (2) Each job can only be processed on a single machine at the same time, and each operation cannot be interrupted;
- (3) The operations of different jobs do not have constraints;
- (4) The different jobs have the same priority.

### B. Fuzzy numbers and its operations

In this paper triangular fuzzy number is used to describe the uncertainty of the processing time, triangular

fuzzy number is recorded as  $\tilde{d} = (d^L, d^M, d^U)$ . Its membership function is:

$$u(x) = \begin{cases} 0 & x \leq d^L \\ \frac{x - d^L}{d^M - d^L} & d^L < x \leq d^M \\ \frac{d^U - x}{d^U - d^M} & d^M < x \leq d^U \\ 0 & x > d^U \end{cases}$$

Membership function is showed in Fig. 1:

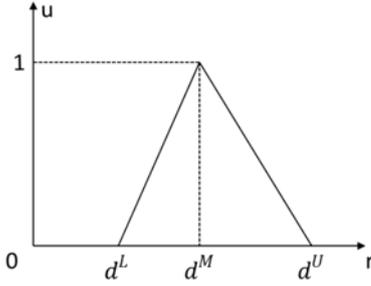


Figure 1. Membership function

Solving fuzzy shop scheduling problems, there are mainly three operations related to the triangular fuzzy number, including addition, the V (max) and sorting operations. Given two triangular fuzzy numbers:  $\tilde{s} = (s_1, s_2, s_3)$ ,  $\tilde{t} = (t_1, t_2, t_3)$ .

(1) Addition operation. Addition operation is used to calculate the sum of the fuzzy start time and the processing time which is the completion time, just formula (1) as follows:

$$\tilde{s} + \tilde{t} = (s_1 + t_1, s_2 + t_2, s_3 + t_3) \quad (1)$$

(2) V(max) operation. V(max) operation is used to compare the maximum of the release time and machine idle time, and then calculate the fuzzy start time of the job. The calculation formula (2) as follows:

$$\tilde{s} \vee \tilde{t} = (s_1 \vee t_1, s_2 \vee t_2, s_3 \vee t_3) \quad (2)$$

(3) Evaluation of fuzzy numbers. Guidelines which is proposed by Sakawa [9] is conducted in this paper.

(4) The defuzzification of fuzzy number

For the ambiguity resolution the expected demand method [10] is available. Such as triangular fuzzy number  $\tilde{q} = (a_1, a_2, a_3)$ , so  $E_1, E_2$  represent the bounds of Expect interval, respectively, then:

$$E_1 = a_2 - \int_{a_1}^{a_2} f(x) dx$$

$$E_2 = a_2 + \int_{a_2}^{a_3} g(x) dx$$

Where:

$$f(x) = \frac{x - a_1}{a_2 - a_1};$$

$$g(x) = \frac{a_3 - x}{a_3 - a_2}$$

So the expectation is:

$$E_{\tilde{q}} = \frac{1}{2}(E_1 + E_2) = \frac{a_1 + 2a_2 + a_3}{4} \quad (3)$$

### III. MATHEMATICAL MODEL OF SCHEDULING PROBLEM

The notations used in this study are listed as follows:

- $n$  total number of jobs
- $m$  total number of machines
- $I, e$  index of machines,  $1, 2, \dots, m$
- $J, k$  index of jobs,  $1, 2, \dots, n$
- $d_j$  total number of operation3.33s of job  $j$
- $O_{jh}$  the  $h$ th operation of job  $j$
- $\tilde{p}_{jh}$  processing time of operation  $O_{jh}$ , represented by triangular fuzzy numbers  $\tilde{p}_{jh} = (a, b, c)$ ,  $a$  represents the most pessimistic processing time,  $b$  represents the most likely processing time,  $c$  represents the most optimistic processing time
- $\tilde{s}_{jh}$  start time of operation  $O_{jh}$
- $\tilde{c}_{jh}$  completion time of operation  $O_{jh}$
- $L$  a large positive number
- $\tilde{c}_j$  completion time of job  $j$
- $\tilde{C}_{\max}$  production makespan
- $[E_j, T_j]$  due date window, where  $E_j, T_j$  stand for the earliest and latest due date, when  $\tilde{c}_j < E_j$ , Job  $j$  is early, else Job  $j$  is tardy.
- $IC$  cost of idle unit time
- $TC$  cost of tardy unit time
- $EC$  cost of early unit time, generally  $EC \leq TC$
- $\tilde{g}_j(\tilde{c}_j)$  early /tardiness penalties value of Job  $j$
- $y_{jhkl} = \begin{cases} 1 & \text{if } O_{jh} \text{ process prior to } O_{kl} \\ 0 & \text{else} \end{cases}$

Objective function is to minimize the total cost, the total cost including the idle cost of the machine and the early/tardiness penalty value. Idle cost of the machine ensures that product makespan should be reduced. For early and tardiness penalties it requires the jobs should be on-time delivery, early or tardiness will increase costs.

The objective function can be defined as:

$$\text{Min } Z = IC \left( m \cdot \tilde{C}_{\max} - \sum_{j=1}^n \sum_{h=1}^{d_j} \tilde{p}_{jh} \right) + \sum_{j=1}^n \tilde{g}_j(\tilde{c}_j) \quad (4)$$

Where:

$$\tilde{g}_j(\tilde{c}_j) = EC \cdot \max(0, E_j - \tilde{c}_j) + TC \cdot \max(0, \tilde{c}_j - T_j)$$

Subject to:

$$\tilde{c}_j = \tilde{s}_{jd_j} + \tilde{p}_{jd_j} \quad (5)$$

$$\tilde{c}_{jh} \leq \tilde{s}_{j(h+1)} \quad (6)$$

$$\tilde{c}_{jd_j} \leq \tilde{C}_{\max} \quad (7)$$

$$\tilde{s}_{jh} + \tilde{p}_{ijh} \leq \tilde{s}_{kl} + L(1 - y_{jhkl}) \quad (8)$$

$$\tilde{c}_{jh} \leq \tilde{s}_{j(h+1)} + L(1 - y_{klj(h+1)}) \quad (9)$$

$$s_{jh} \geq 0, c_{jh} \geq 0 \quad (10)$$

Constraint (5) defines the completion time of each production. Constraint (6) ensures the precedence constraints based on each production. Constraint (7) represents the job completion time constraint. Constraints (8) (9) required that two operations cannot be processed simultaneously. Constraint (10) each parameter variables must be positive.

#### IV. SCHEDULING ALGORITHMS BASED ON GENETIC ALGORITHMS

##### A. Population initialization

Population initialization gives an initial solution by way of chromosome encoding. Using genetic algorithms to optimize production scheduling problem, it will be complicated using binary encoding or floating point encoding. It is not good for programming and computing, so integer coding method is used here, each chromosome represents all the job processing sequences. Randomly generated initial population according to the jobs and processing sequence, and these chromosomes is used to iterate as the initial point.

##### B. Selection

Select operation is a process to form a new population which is selected a strong vitality individual from an older population. This article researchers takes the roulette wheel selection and elitist approach to select operation. The greater the fitness is, the higher probability the individual is selected.

##### C. Crossover

Crossover recombines certain genes randomly in two parents' chromosomes to make two offspring, in order to generate a better solution. The aim is to combine good genes, so that the offspring can inherit better parents' genes. This paper is based on the POX (precedence operation crossover) operator, it is good to inherit parents' excellent characteristics and all the children are feasible [11].

For example, chromosome C1 = [1,3,1,2,4,4,3,2,1,3,2,4], C2 = [4,1,3,4,2,3,1,3,4,2,1,2], divided the Job set{1,2,3,4} randomly into J1 = {1,4} and J2 = {2,3}. The offspring chromosome after crossing were C3 = [1,3,1,2,4,4,3,3,1,2,2,4], C4 = [4,1,3,4,2,3,1,2,4,3,1,2]. The crossover process is as shown in Fig. 2. In the chromosomes crossover phase, whether each chromosome can be crossed is determined by crossover probability Pc.

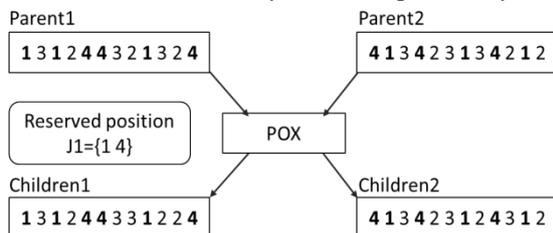


Figure 2. POX crossover operation

##### D. Mutation

Mutation operation aimed at improving the ability of local search algorithm, which helps maintain the diversity of evolution population, prevent premature local optimum. In this paper, exchange mutation method is used for this operation. Firstly the mutation operator randomly selected individuals from the population. Then the two mutation positions are randomly selected and swapped. The process of mutation is as shown in Fig. 3.

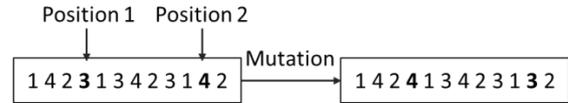


Figure 3. REM mutation operation

#### V. NUMERICAL EXAMPLE

In order to verify the efficiency of proposed genetic algorithm for solving fuzzy job shop scheduling, researchers use the same parameters in literature [12], 6 × 6 and 10 × 10 examples were solved in this paper. The idle costs per unit of time, tardiness and early penalty values were 5, 4 and 3.

The algorithm was implemented in MATLAB and the tests were run on a computer with a 2.1 GHz Intel CPU on the Windows 7 operating system. Job due date window is shown below in Table 1, 2.

TABLE I. DUE DATE WINDOW OF 6×6 NUMERICAL EXAMPLE

Jobs	1	2	3	4	5	6
Due date window	[30, 40]	[35, 40]	[20, 28]	[32, 40]	[30, 35]	[40, 45]

TABLE II. DUE DATE WINDOW OF 10×10 NUMERICAL EXAMPLE

Jobs	1	2	3	4	5
Due date window	[45, 60]	[50, 60]	[50, 65]	[50, 65]	[50, 65]
Jobs	6	7	8	9	10
Due date window	[45, 60]	[45, 60]	[50, 60]	[45, 60]	[50, 60]

With the above method, taking the number of population Popsiz = 40, evolution generation Gennum = 200, crossover probability Pc = 0.8, mutation probability Pm = 0.6, 20 times simulation of the problem is conducted and taking the best result.

For the 6 × 6 problem, the minimum cost to get the optimal schedule is 518.75. Maximum fuzzy completion time is (32, 39, 46). The minimum cost convergence graph of 6 × 6 is shown in Fig. 4.

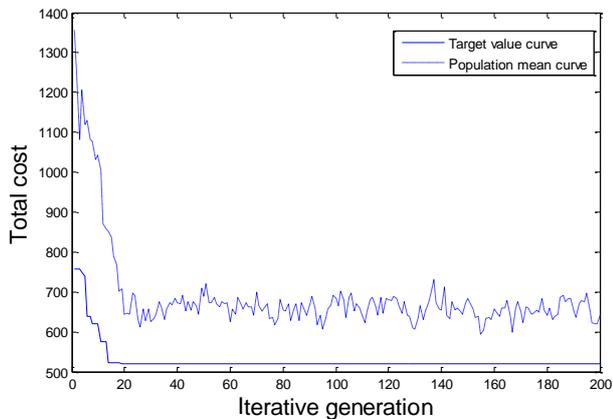


Figure 4. The minimum cost convergence graph of 6x6

For the  $10 \times 10$  problem, the minimum cost to get the optimal schedule is 926.25. Maximum fuzzy completion time is (28, 49, 63). The minimum cost convergence graph of  $10 \times 10$  is shown in Fig. 5.

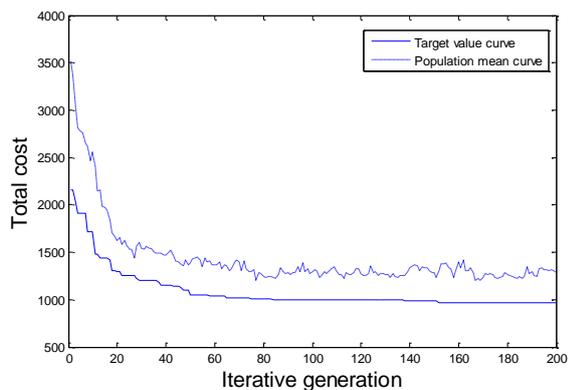


Figure 5. The minimum cost convergence graph of 10x10

Curve represents the optimal value evolution curve of each generation. With the constant evolution of the algorithm, the optimal objective function value tends to become more and more stable, indicating the correctness of the algorithm and convergence. The optimal processing sequence Gantt charts are shown in Fig. 6 and Fig. 7 (most likely processing time of triangular fuzzy numbers taken as processing time).

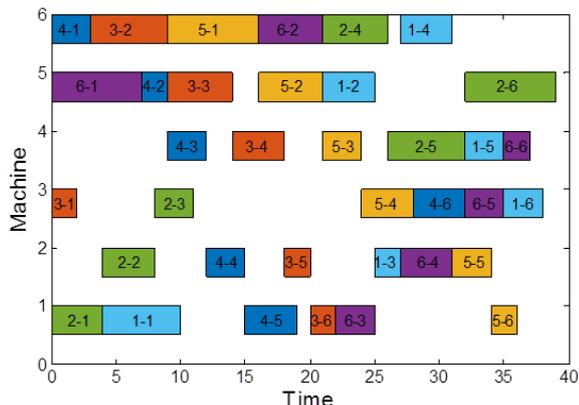


Figure 6. The optimal processing sequence Gantt chart of 6x6

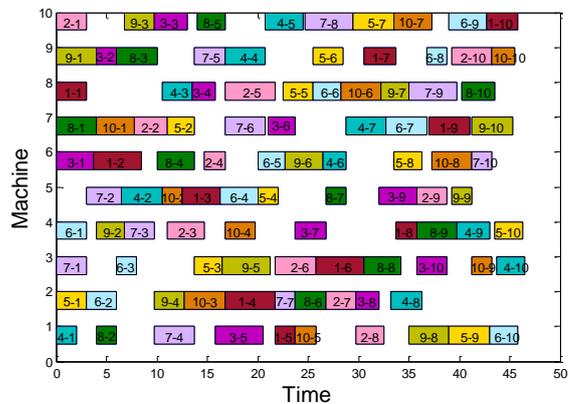


Figure 7. The optimal processing sequence Gantt chart of 10x10

## VI. CONCLUSION

In this paper, Job Shop problem with different due date windows was studied, considering the uncertainty of processing time, researchers use triangular fuzzy number to represent the processing time, a mathematical model was built to reduce the total cost of the production process and proposed an improved genetic algorithms to solve this problem. Finally, a numerical example was used to show the effectiveness of the algorithm. However, in real production systems, there are many uncertain factors, solving the more complex problems of uncertainty and how to achieve trade-offs between multiple objectives will be the future research directions.

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