MIT MRAC Based Boost Converter with Modified 2\textsuperscript{nd}-Order Reference Model

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Abstract. To improve overall performance of Boost converter, MIT model reference adaptive control (MRAC) scheme was used to control the output voltage. Average model of the boost converter was built and the principle of the MIT MRAC was analyzed. Modified 2\textsuperscript{nd}-order model was used as reference model to improve dynamic performance of the system. A new parameter design method for the 2\textsuperscript{nd}-order reference model was proposed. Simulation results verified the voltage control scheme and the parameter design method.

Introduction

Boost converter is widely used in industry field\textsuperscript{[1]}, and the control scheme greatly effects its overall performance. Up to now, proportional-integral (PI) control scheme is the most popular scheme for this kind of converter\textsuperscript{[2]}. However, this traditional controller also has its own limitations, e.g. the system performance will change with the fluctuation of circuit parameters because controller parameters are fixed usually\textsuperscript{[3]}.

MIT MRAC is one of relatively early adaptive control methods. Ideally, with adaptive law, the system can achieve the same performance with that of reference model\textsuperscript{[4]}. Although MIT MRAC cannot make sure stability of the system absolutely, it is still a practical adaptive control method for its simplicity and convenience of algorithm implementation, specially, not like stability theory based MRAC methods, MIT MRAC is also suitable for non minimum phase system e.g. boost converter.

Based on above consideration, boost converter with MIT MRAC scheme for voltage control was studied. Modified 2\textsuperscript{nd}-order reference model was used to improve performance of the system. Parameter design method of the reference model was analyzed. Simulation was made to verify the control scheme in the end.

Modeling of Boost Converter

Fig. 1 Topology of boost converter
Topology of boost converter is shown in Fig. 1, $v_{in}$, $v_o$ are instantaneous voltage, $L$, $C$ are input and output side filter separately. Suppose switching frequency $f_s$ is high enough, the large signal average model\(^{[2]}\) of the converter can be yield in Eq.1 when the inductor current is continuous

$$
\begin{align*}
&L \frac{d(t)(\dot{i}_L(t))}{dt} = \left\{ v_{i_n}(t) \right\}_L - d(t) \left\{ v_o(t) \right\}_L \\
&C \frac{d(t)(\dot{i}_L(t))}{dt} = d(t) \left\{ \dot{i}_L(t) \right\}_L - \frac{\left\{ v_o(t) \right\}_L}{R}
\end{align*}
$$

(1)

Switching period $T_s = 1/f_s$, $d(t)$, $d'(t)$ are conduction and close-off duty cycle separately, $<i_L(t)>_{T_s}$ is the average inductor current during a switching period, other expressions are defined similarly. Based on Eq.1, time domain small signal model of the system can be deduced as shown in Eq.2, where $V_{in}$, $V_o$, $I_L$ and $D$ are steady state value at the moment of $t$.

$$
\begin{align*}
&L \frac{d\hat{i}_L(t)}{dt} = \left[ \hat{v}_{i_n}(t) - D \hat{v}_o(t) + V_o \dot{d}(t) \right] \\
&C \frac{d\hat{i}_L(t)}{dt} = \left[ D \hat{i}_L(t) - \hat{v}_o(t) - I_L \dot{d}(t) \right]
\end{align*}
$$

(2)

Frequency domain model can be obtained as shown in Eq.3 with Laplace transformation of Eq.2.

$$
F_p(s) = \left| \frac{v_o(s)}{v_{i_n}(s)} \right| = \frac{V_o (1 - L s / (D^2 R))}{L C s^2 + R D s + 0.25}
$$

(3)

Eq.3 is the key transfer function which will be used to control $v_o$ with MIT MRAC scheme. When $v_{in}$ is 200V, $v_o$ is 400V, $L$ is 1mH, $C$ is 2000uF, $R$ is 10Ω, $f_s$ is 20kHz, the above transfer function $F_p$ can be got shown in Eq.4.

$$
F_p(s) = \frac{-0.08s + 200}{0.000002s^2 + 0.0001s + 0.25}
$$

(4)

**Principle of MIT MRAC Controller**

Diagram of MIT MRAC is shown in Fig.2, $S_r$ is reference signal, $F_p$ is transfer function of the plant, that is the transfer function from output of controller to the output of the system, $F_m$ is reference model, $y_m$ and $y_p$ are ideal output and real output separately. The object of MRAC is to let $F_m$$\times$$F_p$ approach $F_m$ by adjusting controller parameters dynamically according to the adaptive law. $k_c(0)$ is the initial value of $k_c$. $\mu$ can be selected as a small constant.

![Fig. 2 Control diagram of MIT MRAC](image-url)
Parameter Design of 2nd-Order Reference Model

Reference model has significant effect on MRAC system, modified 2nd-order model shown in Fig.3 was used to improve dynamic performance of the system with its differentiation element. Transfer function of the 2nd-order system is shown in Eq.5

$$G(s) = \frac{\omega_n^2}{s} \cdot \frac{s + z}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where

$$z = \frac{1}{T_d}$$

$$\xi_d = \xi + \omega_n/(2z)$$

Similarly as traditional 2nd-order model, rising time $t_r$, peak value time $t_p$, and overshoot coefficient $\sigma$ are key index of the system as shown in Eq.8.

$$\begin{cases} 
  t_r = \frac{-\phi}{\omega_n \sqrt{1 - \xi_d^2}} \\
  t_p = \frac{\beta - \phi}{\omega_n \sqrt{1 - \xi_d^2}} \\
  \sigma = r \sqrt{1 - \xi_d^2} e^{-\omega_n t} \times 100%
\end{cases}$$

where

$$\begin{align*}
  \phi &= -\pi + \arctan \left( \frac{\omega_n \sqrt{1 - \xi_d^2}}{(z - \xi_d \omega_n)} \right) + \arctan \left( \frac{\sqrt{1 - \xi_d^2}}{\xi_d} \right) \\
  \beta &= \arctan \left( \frac{\sqrt{1 - \xi_d^2}}{\xi_d} \right) \\
  r &= \sqrt{z^2 - 2\xi_d \omega_n z + \omega_n^2} / z \sqrt{1 - \xi_d^2}
\end{align*}$$

It can be seen from Eq.8 and Eq.9, $t_r$, $t_p$, and $\sigma$ are all transcendental function of $T_d$, $\xi$, $\omega_n$. Parameter design method of the modified 2nd-order system will be discussed as follows.

Firstly, $T_d$ should be set properly as shown in Eq.10. The reason for limitation of $T_d$ is that, too small $T_d$ cannot make full use of quick advantage of differentiation element, while the system may become instable if $T_d$ is too large.

$$z = k \xi \omega_n \quad k_i \in [2, 5]$$

Combined with Eq.7, the inequality group can be drawn as in Eq.11 (the derivation process can be referred to appendix A)
From Eq.8 it can be seen, $\xi_d$ has significant effect on $\sigma$, even though it is not the sole decisive factor. Exponential part dominates $\sigma$, which means that $\sigma$ can keep constant if $\xi_d$ increases meanwhile $\omega_n$ decreases. Based on the above analysis and the fact that $\omega_n$ is increasing function of $\xi$ (the derivation process can be referred to appendix B), decrease of $\xi$ and increase of $\xi_d$ meanwhile can keep the same $\sigma$.

From above analysis, parameter design method suitable for engineering application can be summarized as follows.

1) Select $\xi_d$. To achieve better dynamic performance, $\xi_d$ can be selected at a higher value.
2) Calculate the range of $\xi$ with Eq.11. Decrease of $\xi$ benefits for system dynamic performance.
3) Select $t_r$, and substitute $t_r$ into Eq.12, $\omega_n$ can be calculated out. The final rising time will be smaller than the selected value because of differential element.

$$t_r = \frac{\pi - \arccos \xi}{\omega_n \sqrt{1 - \xi^2}}$$ (12)

4) Substitute $\xi_d, \xi$ and $\omega_n$ into Eq.7 and calculate out $z$.
5) Substitute $\xi_d, \omega_n$ and $z$ into Eq.5 and get the 2nd-order model.

For example, if $\xi_d$ is set to 0.95, the range of $\xi$ will be $[0.12, 0.829]$. When $\xi$ is selected at 0.2, and $t_r$ is selected at 0.5, $\omega_n$ and $z$ will be 3.6 and 2.4 separately. The final 2nd-order model is shown as Eq.13

$$G(s) = \frac{(5.4s + 12.96)}{(s^2 + 6.84s + 12.96)}$$ (13)

**Simulation Results**

**Frequency Domain Simulation.** Frequency domain simulation was made according to the control diagram shown in Fig.2, where $k_c(0)=0$, $\mu=3 \times 10^6$, plant model and reference model are shown in Eq.4 and Eq.13 separately. Simulation results of $y_m$ and $y_p$ shown in Fig.4 and Fig.5 prove that the reference model and control plant has approximately the same dynamic performance.

**Time Domain Simulation.** Time domain simulation results are shown in Fig.6, $v_m$ is the reference of $v_{out}$. Load resistor $R$ change from 10Ω to 20Ω at 2s. Fig.7 and Fig.8 are partial enlarged detail of $v_m$ and $v_{out}$. Simulation results show that there is no steady error between $v_{out}$ and $v_m$ although saturation of controller effect tracking at the beginning time. Waveform of $v_{out}$ shown in Fig.8 shows that 50% load decrease only leads to 5% voltage ripple, which verifies MIT MRAC scheme.
Fig. 6 Waveform of $v_m$ and $v_{out}$

Fig. 7 Partial enlarged detail of $v_m$ and $v_{out}$

Fig. 8 Partial enlarged detail of $v_{out}$

Conclusions

1) MIT MRAC scheme was used for voltage control of Boost converter, which was verified by simulation results.

2) Modified 2nd-order model was used in the control scheme, and parameter design method for the reference model was proposed.

Appendix A

Substitute Eq. 10 into Eq. 7, Eq. 14 can be got

$$\xi_d = \xi + 1/(2k_z \xi)$$  \hspace{1cm} (14)

Arrange Eq. 14, we can get Eq. 15

$$k_z = 1/(2\xi_d \xi - 2\xi^2)$$  \hspace{1cm} (15)

When $k_z \in [2,5]$, inequality of Eq. 16 can be got

$$0.1 \leq \xi_d \xi - \xi^2 \leq 0.25$$  \hspace{1cm} (16)

Rearrange Eq. 16, can we get

$$\begin{cases} \xi^2 - \xi_d \xi + 0.1 \leq 0 \\ \xi^2 - \xi_d \xi + 0.25 \geq 0 \end{cases}$$  \hspace{1cm} (17)

Take $\xi$ as variable, to ensure establishment of the 1st inequality of Eq. 17, Eq. 18 should hold

$$\Delta = \xi^2 - 0.4 \geq 0$$  \hspace{1cm} (18)

From Eq. 18, Eq. 19 holds
\[ \xi_j \geq 0.633 \]  

Substitute Eq.19 into the 2\textsuperscript{nd} inequality of Eq.17, inequality of \( \xi > 0 \) can be got, which means Eq.19 gives the final range of \( \xi_j \). Take Eq.19 into 1\textsuperscript{st} inequality of Eq.17, Eq.20 can be got.

\[ \xi \in \left[ \frac{\xi_j - \sqrt{\xi_j^2 - 0.4}}{2}, \frac{\xi_j + \sqrt{\xi_j^2 - 0.4}}{2} \right] \]  

\section*{Appendix B}

From Eq.12, Eq.21 can be got

\[ \omega_n = \frac{\pi - \arccos \xi}{t_r \sqrt{1 - \xi^2}} \]  

In Eq.21, \( t_r \) is constant and \( \xi \) is variable, take derivation of Eq.21, can we get Eq.22

\[ \frac{d\omega_n}{d\xi} = \frac{1 + \xi (\pi - \arccos \xi)}{\sqrt{1 - \xi^2}} \left[ \frac{1}{t_r (1 - \xi^2)} \right] \]  

When \( 0 < \xi < 1 \), Eq.22 >0 always holds, which proves that \( \omega_n \) is increasing function of \( \xi \).

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