Projective synchronization and parameter identification of a fractional-order chaotic system

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Keyword: fractional-order chaotic system, projective synchronization, parameter identification.

Abstract: In this paper, projective synchronization and parameter identification of a fractional-order chaotic system is studied. Based on the fractional-order stability theory, a universal projective synchronization controller and parameter identification rules are designed and proved by using Lyapunov stability theory. Finally, the numerical simulations verify the correction of the method.

Introduction

As fractional calculus played an important role for the nonlinear dynamical systems, studying dynamics of fractional-order nonlinear system has become an interesting topic[1-5]. Since the pioneering work of Pecore and Carro introduced a method about synchronization between identical and non identical systems with different initial conditional, chaos synchronization had a great variety of applications in physics[6], ecological system[7], secure communications[8], etc. In this paper projective synchronization and parameter identification of a fractional-order chaotic system are investigated. A reasonable controller and parameter identification rules are designed and proved by Lyapunov stability theory. Numerical simulation coincide with the theoretical analysis.

Problem Description.

Consider the following chaotic drive and response systems:

$$D_t^q x = F(x)\varphi + f(x) \tag{1}$$

and

$$D_t^q y = G(y)\psi + g(y) + U(x, y, \varphi, \psi) \tag{2}$$

where $x, y \in R^n$ are the state vectors. $\varphi, \psi \in R^m$ are the parameter vector. $f(x)$ and $g(y)$ are the $n \times 1$ matrices. $F(x)$ and $G(y)$ are the $n \times m$ matrix. $U(x, y, \varphi, \psi)$ is the a suitable controller. Besides $0 < q < 1$.

Theorem 1. From the definition, the projective synchronization between the system (1) and (2) is achieved, if $\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|y(t) - Cx(t)\| = 0$, where C is called the scaling matrix. $C = diag([\alpha_1, \alpha_2, ..., \alpha_n])$.

Theorem 2. Stability theorems of fractional-order system

Consider the following nonlinear system of fractional differential equation.

$$\frac{d^q x}{dt^q} = A(x)x \tag{3}$$

where $A(x) \in R^{n\times n}$, $0 < q < 1$, $x = (x_1, x_2, ..., x_n)^{\mathrm{T}}$ are state vectors. The fractional-order system (3) is asymptotically stable if and only if $|\arg(\lambda_i(A(x)))| > q\pi/2$, i =1,2,…,n, where $\arg(\lambda_i(A(x)))$ denotes the argument of the eigenvalue λ_i of A.

Projective Synchronization and Parameter Identification

In this section, projective synchronization and parameter identification of Qi fractional-order chaotic system will be studied.

The drive system described through (1) is given by

$$\begin{bmatrix} \frac{d^q x_1}{dt^q} \\ \frac{d^q x_2}{dt^q} \\ \frac{d^q x_3}{dt^q} \end{bmatrix} = \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & 0 & x_1 \\ 0 & -x_3 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} x_2 x_3 \\ -x_2 - x_1 x_3 \\ x_1 x_2 \end{bmatrix} \tag{4}$$

where x_1, x_2, x_3 are the state variables. The chaotic attractor of the system (4) for the order of derivative $q = 0.915$ are displayed through Fig 1 for the parameters' values $a = 35$, $b = 8/3$, $c = 80$.

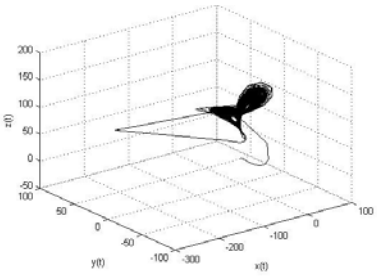


Fig.1 The phase portrait of the system (4) is shown in spaces x-y-z..

The response system described through (2) is given by

$$\begin{bmatrix} \frac{d^q y_1}{dt^q} \\ \frac{d^q y_2}{dt^q} \\ \frac{d^q y_3}{dt^q} \end{bmatrix} = \begin{bmatrix} y_2 - y_1 & 0 & 0 \\ 0 & 0 & y_1 \\ 0 & -y_3 & 0 \end{bmatrix} \begin{bmatrix} \bar{a} \\ \bar{b} \\ \bar{c} \end{bmatrix} + \begin{bmatrix} y_2 y_3 \\ -y_2 - y_1 y_3 \\ y_1 y_2 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{5}$$

where u_1, u_2, u_3 are the control parameters.

In order to facilitate the following analysis, the error functions through theorem 1 are defined as

$$e_1 = y_1 - \alpha_1 x_1, \quad e_2 = y_2 - \alpha_2 x_2, \quad e_3 = y_3 - \alpha_3 x_3 \tag{6}$$

And

$$e_a = \bar{a} - a, \quad e_b = \bar{b} - b, \quad e_c = \bar{c} - c \tag{7}$$

Now, we will choose suitable controllers and parameter identification rules to achieve projective synchronization according to theorem 2.

From Eq.(1), (2) , the error systems are obtained as

$$D_t^q e = G(y)\psi + g(y) + U(x, y, \varphi, \psi) - C[F(x)\varphi + f(x)] \tag{8}$$

Theorem 3. If the projective synchronization controllers are selected as

$$U = -g(y) + Cf(x) + Ke + D \tag{9}$$

Where

$$C = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}, \quad K = \begin{bmatrix} 0 & -a & 0 \\ -c & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ -y_2 + \alpha x_2 \\ 0 \end{bmatrix},$$

and identification laws of parameters are calculated as

$$\begin{cases} \dfrac{d^q e_a}{dt^q} = -y_2 e_1 + y_1 e_1 \\ \dfrac{d^q e_b}{dt^q} = y_3 e_3 \\ \dfrac{d^q e_c}{dt^q} = -y_1 e_2 \end{cases} \tag{10}$$

then, the response system (5) is synchronized with the drive system (4) globally and asymptotically, i.e. $\lim_{t\to\infty}\|e(t)\| = 0$.

proof: From Eq.(8), (9), the error systems are achieved as

$$D_t^q e = G(y)\psi - CF(x)\varphi + Ke + D \tag{11}$$

A Lyapunov function is defined as follows:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2)$$

the time derivative of V along the trajectory of the error system (11) leads to

$$\frac{d^q V}{dt^q} = e_1 \frac{d^q e_1}{dt^q} + e_2 \frac{d^q e_2}{dt^q} + e_3 \frac{d^q e_3}{dt^q} + e_a \frac{d^q e_a}{dt^q} + e_b \frac{d^q e_b}{dt^q} + e_c \frac{d^q e_c}{dt^q}$$

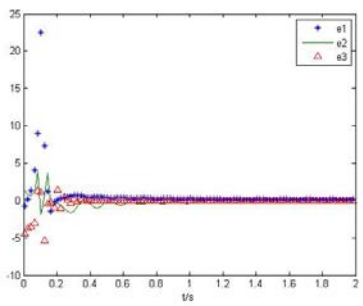
$$= e_1(y_2 e_a - y_1 e_a - a e_1) + e_2(y_1 e_c - e_2) + e_3(-y_3 e_b - b e_3) + e_a(-y_2 e_1 + y_1 e_1) + e_b(y_3 e_3) + e_c(-y_1 e_2)$$

$$= -a e_1^2 - e_2^2 - b e_3^2 \le 0$$

as $V \in R$ is positive definite function and $\dfrac{d^q V}{dt^q}$ is the negative definite function, so according to the Lyapunov stability theory, the response system (5) is synchronization to the drive system (4) asymptotically.

Numerical Simulation and Results

In this section, the initial conditions of the drive and response systems are $(x_1(0), x_2(0), x_3(0)) = (-1,-1,-2)$, and $(y_1(0), y_2(0), y_3(0)) = (-1,1,-5)$. $\alpha = 0.3$. The initial values of the estimated unknown parameter vectors of the systems are taken as $(\bar{a}, \bar{b}, \bar{c}) = (20,-5,100)$. From figure 2, it is seen the error vectors converge asymptotically to zero, and figure 3 show that the estimated parameter vectors converge to the original parameter vectors, respectively.



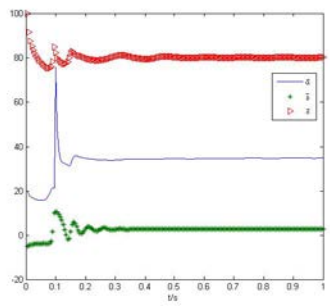


Fig.2 Trajectories of the errors function Fig.3 Trajectories of the estimated parameter vectors

Conclusion

In this manuscript, projective synchronization and parameter identification of a fractional-order chaotic system is presented. Moreover, suitable projective synchronization controller and parameter identification rules are given by using the fractional-order stability and proved by Lyapunov stability theory. Finally, numerical simulations are performed to verify these results.

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