Modeling and Its Adaptive Fault-tolerant Control for UAV Two-axis Pan-title

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Abstract—According to the kinematics and dynamics characteristic of UAV Two-axis Pan-title, the dynamic equations of two-axis platform was erected up, and moreover by predigesting and linearizing the model in the balance point, the math model was gained which could be the same with Fault-tolerant controller design. Applying the adaptive fault-tolerant control method which was real time estimation of actuator faults; the adaptive fault-tolerant controller parameters are updating automatically to compensate the fault effects on systems. The disturbance tolerance ability of the adaptive controller is better than the fixed gain controller's, especially in the actuator saturation and actuator failures. The proposed algorithm was obtained in the form of linear matrix inequalities, and calculating and simulating used the aid of MATLAB LMI Control Toolbox. The results show that the disturbance tolerance ability of this controller is strong, and meeting the needs of stability and security control on UAV two-axis pan-title airborne system.

Keywords- Two-axis Pan-title ; Modeling ; linear matrix inequalities ; adaptive control; saturation and failures;

I. INTRODUCTION

UAV airborne stabilized pan-tilt system in the low altitude remote sensing technology plays an important role in the system .The system have to eliminate airborne disturbance and actuator failure problems to make high quality of low altitude remote sensing images. There are higher requirements of the fault- tolerance and the anti-jamming ability. The current modeling and stable control of three-axis tilt has been research more perfect, however, in order to reduce costs and improve efficiency in the use of components, low altitude remote sensing technology mainly uses two-axis table system, and two-axis pan-tilt system have more stringent requirements for stability and fault-tolerance technology.

In order to improve the reliability and safety of the system, using adaptive H_∞ fault-tolerant control algorithm which considers actuator saturation and failures from the point of design .

II. BRIEF INTRODUCTION OF THE TWO-AXIS PAN-TITLE

Two-axis pan-tilt has two rotational degrees of freedom, selecting the rectangular coordinate type in the low altitude remote sensing; the structure from the inside to the outside are rolling ring and the pitching loop. The roll frame is connected with the pitch frame. The roll frame can rotate around the pitch frame. The pitch is connected with the base. The base is fixed in UAV.

Figure.1 two-axis pan-title frame structure diagram

III. MODELING OF THE TWO-AXIS PAN-TITLE

According to the force and space rotation of the frame, we can gain the equation of kinematics and dynamics about the pan-title.

The actual motion of the frame is superimposed fixed-point rotation and horizontal motion in space ,including two rotational degrees .The paper did not consider the elastic deformation of the frame , in order to get a simplified mathematical model .

Pan-title mounted on the base of the UAV, the base relative to the inertial coordinate system rotation or translation together with UAV system. The definition of inertial coordinates is $O-X_aY_aZ_a$; the base is fixedly connected with the moving coordinate system: $O-X_bY_bZ_b$; pitch frame fixed moving coordinate system: $O-X_pY_pZ_p$; roll is fixed to a frame moving coordinate system: $O-X_rY_rZ_r$; which X_p is coincident with X_b . the relative

rotation angle which p system and b system is α ; the relative rotation angle which r system and p system is θ .

Figure.2 Frame coordinate diagram

The transformation matrix from the base coordinate system to the outer frame coordinate system:

$$T_{bp} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

The transformation matrix the outer frame coordinate system to the inner frame coordinate system:

$$T_{pr} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

Set the base for the relative angular velocity:

$$\omega_b = [p \quad q \quad r]^T$$

Angular velocity vector of the outer frame itself on the outer coordinate system described as: $\dot{\alpha} = [\dot{\alpha} \quad 0 \quad 0]^T$

The angular velocity of the carrier table change to frame coordinate system as: $\omega_{bp} = T_{bp} \cdot \omega_b$

Get on the outer frame coordinate angular velocity vector as follows:

$$\begin{aligned} \omega_p &= \omega_{bp} + \dot{\alpha} \\ &= [p + \dot{\alpha} \quad q \cos \alpha + r \sin \alpha \quad -q \sin \alpha + r \cos \alpha]^T \\ &= [\omega_{px} \quad \omega_{py} \quad \omega_{pz}]^T \end{aligned}$$

Angular velocity vector of the inner frame itself on the inner coordinate system described as: $\dot{\theta} = [0 \quad 0 \quad \dot{\theta}]^T$

Similarly, the inner frame with respect to the inertial system angular velocity:

$$\begin{aligned} \omega_r &= \omega_p + \dot{\theta} \\ &= \begin{bmatrix} p \cos \theta + q \cos \alpha \sin \theta + r \sin \alpha \sin \theta + \dot{\alpha} \cos \theta \\ -p \sin \theta + q \cos \alpha \cos \theta + r \sin \alpha \cos \theta - \dot{\alpha} \sin \theta \\ -q \sin \alpha + r \cos \alpha + \dot{\theta} \end{bmatrix} \\ &= [\omega_{rx} \quad \omega_{ry} \quad \omega_{rz}]^T \end{aligned}$$

Because of the frame relative to the shaft is symmetrical, by the theorem of moment $\frac{d\vec{H}}{dt} = \vec{M}$ get:

The outer frame dynamics equation:

$$\begin{aligned} M_p &= [M_{px} \quad M_{py} \quad M_{pz}]^T \\ M_{px} &= J_{px} \dot{\omega}_{px} + (J_{pz} - J_{py}) \omega_{pz} \omega_{py} \\ M_{py} &= J_{py} \dot{\omega}_{py} + (J_{px} - J_{pz}) \omega_{px} \omega_{pz} \\ M_{pz} &= J_{pz} \dot{\omega}_{pz} + (J_{py} - J_{px}) \omega_{py} \omega_{px} \end{aligned}$$

Inner and outer frame are both square. The moment of inertia as follows:

$$J_{px} = J_{py} = J_{pz} = 0.0125 \cdot N \cdot m / s^2 / rad$$

$$J_{rx} = J_{ry} = J_{rz} = 0.0171 \cdot N \cdot m / s^2 / rad$$

The inner frame dynamics equation:

$$\begin{aligned} M_r &= [M_{rx} \quad M_{ry} \quad M_{rz}]^T \\ M_{rx} &= J_{rx} (-\dot{\theta} \omega_{px} \sin \theta + \dot{\omega}_{px} \cos \theta - \dot{\theta} \omega_{pz} \cos \theta - \dot{\omega}_{pz} \sin \theta) \\ M_{ry} &= J_{ry} (\dot{\omega}_{py} + \ddot{\theta}) \\ M_{rz} &= J_{rz} (\dot{\theta} \omega_{px} \cos \theta + \dot{\omega}_{px} \sin \theta - \dot{\theta} \omega_{pz} \sin \theta - \dot{\omega}_{pz} \cos \theta) \end{aligned}$$

The torque expressions of internal and external frame:

$$M_{in} = J_{ry} \ddot{\theta} + J_{ry} \dot{\alpha} (r \cdot \cos \alpha - q \cdot \sin \alpha) + J_{ry} \dot{q} \cdot \cos \alpha + J_{ry} \dot{r} \cdot \sin \alpha$$

$$M_{out} = (J_{rz} + J_{px}) \ddot{\alpha} - J_{rz} \dot{\theta} (\cos \alpha \cdot p - q \cdot \sin \alpha) + (J_{px} + J_{rz}) \dot{p}$$

Because of the above mathematical model is too complex and non-linear, not easy to design and Simulation of the controller. Simplification and linearization needs to be done.

Written in matrix form:

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = A_1^{-1} M - A_1^{-1} B \begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \end{bmatrix} - A_1^{-1} C$$

When Two-axes frame stability, the balance point:

$$\alpha = \theta = 0, \quad \dot{\alpha} = \dot{\theta} = 0$$

The definition of state variables: $X = [\alpha \quad \theta \quad \dot{\alpha} \quad \dot{\theta}]^T$

Gain the Equation of state:

$$\dot{X} = AX + B_2 u + B_1 \omega$$

$$Y = C_2 X = [\dot{\theta} \quad \dot{\alpha}]^T \quad (1)$$

Among them,

$$u = [M_{in} \quad M_{out}]^T, \quad \omega = [\dot{p} \quad \dot{q} \quad \dot{r}]^T,$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -r \\ 0 & 0 & \frac{p \cdot J_{rz}}{J_{rz} + J_{px}} & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{J_{ry}} & 0 \\ 0 & \frac{1}{J_{rz} + J_{px}} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For a two-axis pan-tilt system:

$$p = 0.0875 \text{ rad/s}, \quad r = 0.0525 \text{ rad/s};$$

After calculation, we obtained state matrices:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -r \\ 0 & 0 & \frac{p \cdot J_{rz}}{J_{rz} + J_{px}} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0525 \\ 0 & 0 & 0.0505 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{J_{ry}} & 0 \\ 0 & \frac{1}{J_{rz} + J_{px}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 58.4795 & 0 \\ 0 & 33.7838 \end{bmatrix}$$

IV. ADAPTIVE FAULT-TOLERANT CONTROLLER DESIGN

UAV pan-tilt stability frame system is the key to obtain high quality low altitude remote sensing images. The physical characteristics of the frame and servo motor can produce a variable range which is a actuator saturation phenomenon. When the attitude of the UAV changes, the system will produce certain disturbance and fault. So the pan-tilt system have higher demanding requirements on the stability of fault-tolerant control technology.

The state equation of the two-axis pan-tilt frame system is transformed the general equations of state.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 \sigma(u) \\ Z(t) &= Cx(t) + D\sigma(u) \end{aligned} \quad (2)$$

Among them, $x(t)$ is state of the pan-tilt stability system, $\sigma(u) \in R^m$ is the input of the PTZ frame system with saturation phenomenon, $\omega(t)$ is generated by the system disturbance input $L_2[0, \infty]$. The pan-tilt system (1) of the coefficient matrix corresponding to in the parameter matrix (2).

For the two-axis table system, when the input of the system reaches the saturated situation, the large disturbance may cause the system state and the output divergence and the system work abnormally. So, this paper assumes that the system is bounded external disturbance. The system only make the pan-tilt stability system work normally in practical work if the bounded external disturbances.

Two-axis table system can be transformed into the system with actuator failures and actuator saturation phenomenon:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 \omega + B_2 (1 - \rho) \sigma(u(t)) \\ z(t) &= Cx(t) + D(1 - \rho) \sigma(u(t)) \end{aligned}$$

The controller can be set to a structure with adaptive parameter, as follows:

$$u(t) = K(\hat{\rho}(t))x(t) = (K_0 + K_a(\hat{\rho}(t)) + K_b(\hat{\rho}(t)))x(t) \quad (3)$$

$\hat{\rho}(t)$ is a estimated value for ρ ; In that,

$$K_a(\hat{\rho}(t)) = \sum_{j=1}^m K_{aj} \hat{\rho}_j(t)$$

$$K_b(\hat{\rho}(t)) = \sum_{j=1}^m K_{bj} \hat{\rho}_j(t)$$

Description: Through (3) shows, compared with $u(t) = K_0 x(t)$ in the fixed gain controller, structure of the system is added $K_a(\hat{\rho}(t))$ and $K_b(\hat{\rho}(t))$ which is the function of $\hat{\rho}$. Because $\hat{\rho}$ is the estimated value, and the system can real-time update the value of the $\hat{\rho}$. By real-time estimating the ρ , the gain of the controller can be adjusted online. In this way, so that better performance of control system.

The marking variables η_i is the function which is composed of x and $\hat{\rho}$. The values of the parameters can be real-time measured. η_i is the response of the actuator saturation. Under the same conditions, η_i have the different expression. In the reference [3] gives details of the method.

The adaptive laws $\dot{\hat{\rho}}_j(t)$ in the system can be adjusted online:

$$\begin{aligned} \dot{\hat{\rho}}_j &= Proj_{[\min_q \{\underline{\rho}_j^q\}, \max_q \{\bar{\rho}_j^q\}]} \{L_{1j}\} \\ &= \begin{cases} \hat{\rho}_j = \min_q \{\underline{\rho}_j^q\} & \& L_{1i} \leq 0 \\ 0 & \\ \hat{\rho}_j = \max_j \{\bar{\rho}_j^q\} & \& L_{1j} \leq 0 \\ L_{1j} & other \end{cases} \quad (4) \end{aligned}$$

$L_{1i} = -l_i x^T(t) [PB_2 (\sum_{i=0}^{2^n-1} \eta_i D_i) K_{aj} + PB_2^j (\sum_{i=0}^{2^n-1} \eta_i D_i) K_b(\hat{\rho})]$
 $P = \delta X^{-1}$, $K_0 = Y_0 X^{-1}$, $K_{aj} = Y_{aj} X^{-1}$, $K_{bj} = Y_{bj} X^{-1}$,
 $H_{aj} = O_{aj} X^{-1}$, $H_{bj} = O_{bj} X^{-1}$. Among them,
 $l_j > 0$ ($j \in I[1, m]$) is the adaptive gain law. The gain structure of system controller:

$$K(\hat{\rho}) = K_0 + \sum_{j=1}^m \hat{\rho}_j K_{aj} + \sum_{j=1}^m \hat{\rho}_j K_{bj} \quad (5)$$

Note: γ_n is adaptive H_∞ performance index on the trouble-free condition. γ_f is adaptive H_∞ performance index on the trouble condition. δ is the disturbance tolerance level of the system.

Through the optimization algorithm to get the minimization γ_f and γ_n , the maximum δ :

$$\min \eta = \alpha \eta_n + \beta \eta_f + \gamma \eta_\delta, \quad s.t. (a), \quad (b)$$

$$\varepsilon^*(P, \delta^*) \subset \wp(H(\hat{\rho})) \quad , \quad \eta_n = \gamma_n^2 \quad , \quad \eta_f = \gamma_f^2 \quad \text{and}$$

$$\eta_\delta = \frac{1}{\delta^*} = \frac{1}{\delta + \max \left\{ \sum_{j=1}^m \frac{\bar{\rho}_j^2(t)}{l_j} \right\}}$$

Among them, α , β , γ is the weighted coefficient.

V. EXPERIMENT AND CONCLUSIONS

There are two modes of actuator pan-tilt stabilization system: normal mode and fault mode.

Normal mode: the pitch and roll box frame motor is in normal state, i.e. $\rho_1^1 = \rho_2^1 = 0$;

Failure mode: one motor is in failure, i.e. $\rho_1^2 = 1, 0 \leq \rho_2^2 \leq a$; $a = 0.8$,

Among them, $a = 0.8$ is the biggest failure rate of actuator.

Set the weighting parameter $\alpha = 0.5$, $\beta = 0.01$, $\gamma = 0.5$. Calling the Matlab of the LMI toolbox to solve this problem. As the fixed gain controller design method $\eta_n = 16.3668$, $\eta_f = 490.4561$,

$\eta_\delta = 26.1502$ is obtained; and, as the adaptive control algorithm obtained $\eta_n = 11.3668$, $\eta_f = 100.6399$, $\eta_\delta = 1.4438$; As is shown above, the adaptive fault-controller is better than the fixed gain controller.

Considering the problem of disturbance tolerance, in the following simulation selection disturbance:

$$\omega(t) = \begin{cases} 200, & 4 \leq t \leq 5 \\ 0 & \text{other} \end{cases} \quad (6)$$

Fig. 3 shows the state curve of the closed-loop system in the adaptive controller of normal model. Fig. 4 shows the state curve of the closed-loop system with fixed gain controller in normal mode. Obviously, on the disturbance (6), in the role of the adaptive controller for the closed loop system remains asymptotically stable, but in the fixed gain controller is divergent. This phenomenon indicates that the adaptive method in this paper has better effect.

Figure. 3 Trajectories of closed-loop systems with adaptive controller in normal case

Figure. 4 Trajectories of closed-loop systems with fixed gains controller in normal case

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