Models of the Transportation Problem under Carbon Emissions Policies
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Abstract. The paper investigates the classical transportation problem under carbon emissions policies (carbon cap, carbon tax, and cap-and-trade). We develop a firm’s optimal total transportation cost formulation which helps the firm to consider the impact of policies and calculate the total transport cost. We show that under carbon cap the firm must control the carbon emissions, whereas the tax imposed on the firm can effectively reduce carbon emissions by increasing the transportation cost. The firms may decide the carbon emissions regarding the true value of the freight, for high-profit goods the firm would like to absorb high tax. We also derive the conditions under which, the firm’s total transportation problem is similar under the carbon cap and the cap-and-trade policies, and the transportation cost can be effectively adjusted under cap-and-trade policy. We show that, in order to reduce the transportation cost under carbon emissions policies, the firm have to choose the reasonable policies. Furthermore, more variables and ingredients should be integrated into the linear programming formulation mode for the further research.

Introduction
The carbon dioxide concentrated in the atmosphere has been increasing significantly in the recent years. In practice, various actions contributed to control carbon emissions have been taken, in U.S.-China Joint Announcement on Climate Change, the two countries work together to deal with the serious challenge. As reported, China intends to achieve the peaking of CO₂ emissions around 2030 [1]. To meet the target, many regulations have been implemented to help control the carbon emissions. Considering the prospect that the carbon market will be launched in 2016 and the current situation of carbon emissions, we analysis three carbon emissions policies: carbon cap, carbon tax, cap-and-trade, the first two policies can be named as “hard constraints”, by adjusting the production or taking new technologies to reduce the carbon emissions. Contrarily, the last one can be seen as a “soft constraints” policy, allowing free trade among the firms on carbon emissions rights.

In this paper we modify the classical transportation problem to investigate the impact of the three most common carbon policies on transportation problem. We note that many scholars have observed the relationship between carbon emissions policies and the transportation cost. We also learn that companies have actively modeled their supply chain footprints and chosen the appropriate transport trait and mode to reduce the carbon emissions. Mingzhou Jin et al. [2] used the Integrated Assessment Models to study freight transportation patterns under various carbon emissions policies.

For our analysis, we focus on the transportation problem under carbon emissions policies. We know the distribution of the freights contributed mostly to the carbon emissions. For example, carbon dioxide emissions from transportation accounted for about three quarters of transport emissions in 2012 [3]. In addition, many researchers have paid attention to environmental issues and carbon emissions, Yang, B., Yang, Y., & Wijngaard, J. [4] focused on the impact of emissions from transportation. Nieuwenhuis, Beresford, and Choi [5] compared the carbon emissions for different transport modes. Following the practice and other papers (Jingpu Song; Mingzhou Jin; Benjaafar; Hoen) [6//2/7/8], this paper investigates the three most common carbon policies and
solve the following problem: 1) deduce the firm’s transportation cost equation under carbon emissions policies; 2) analysis the impact of carbon emissions on transportation problem.

The organization of the paper is as follows: In Section 2, we briefly review the related literature. In Section 3, we discuss the carbon emissions policies and present the linear programming formulation model, deduce the optimal transportation cost under carbon emissions policies. In section 4, we conduct a numerical study with sensitivity analysis. In section 5, we conclude the paper and provide future research directions.

**Literature Reviews**

In this section, we briefly review the major literature on carbon emissions policies. Ekins and Barker [9] make a detailed conclusion about the literature on carbon tax and emissions trading. Harrison and Smith, and Baranzini et al. [10/11] make a detailed description and contrast between the carbon tax and the cap-and-trade. Miao Fu et al. [12] evaluates the impacts of carbon related taxation policies (vehicle registration tax, motor taxation, and fuel tax) in Ireland and announced the combination of these policies can significantly improve the environment. In addition, Sujeetha Selvakumaran [13] analyses carbon mitigation possible under carbon cap and carbon tax.


A number of studies have extended the transportation problem. Hoen et al. [8] and Gökçe Palak et al. [18] investigated the impact of carbon policies on various transport modes selection. Hongqi Li. et al. [19] estimated the carbon emissions in road freight and discussed the effect of related regulations. Petra Stelling [20] discussed the relationship between various policies and carbon emissions from freight transports. Dincer Konur et al. [21] analyzed the inventory and transportation problem under various carbon emissions regulations.

Our modeling framework stems from transportation program problem and we only consider the transportation cost under carbon policies, which significantly distinguished from the study by Mingzhou Jin [2]. Specifically, they redesign the supply chains under carbon policies and deduce the corresponding transport cost of different transportation modes. While, we present the linear programming formulation model and deduce the optimal transportation cost and make analysis.

**Models of the transportation Problem under Carbon Emissions Policies**

In this section, we present the classical linear programming formulation model under three carbon emissions policies. Now, we start with no constraint on carbon emissions. For our analysis, we let Z be the total cost and \( x_{ij} (i =1, 2, \ldots, m; j =1, 2, \ldots, n) \) be the number of units to be distributed from source \( i \) to destination \( j \), in addition, \( c_{ij} \) is the unit distribution cost; \( s_i \) is the number of units in source \( i \); \( d_j \) is the number of units to destination \( j \). So the transportation problem is

Minimize

\[
Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]

Subject to

\[\sum_{j=1}^{n} x_{ij} = s_i \quad \text{for all} \quad i = 1, 2, \ldots, m\]

\[\sum_{i=1}^{m} x_{ij} = d_j \quad \text{for all} \quad j = 1, 2, \ldots, n\]

\[x_{ij} \geq 0 \quad \text{for all} \quad i = 1, 2, \ldots, m \quad \text{and} \quad j = 1, 2, \ldots, n\]
\[
\sum_{j=1}^{n} x_{ij} = s_i \quad \text{for } i = 1, 2, \ldots, m, \tag{2}
\]

\[
\sum_{i=1}^{m} x_{ij} = d_j \quad \text{for } j = 1, 2, \ldots, n, \tag{3}
\]

And

\[x_{ij} \geq 0, \text{ for all } i \text{ and } j. \tag{4}\]

The classical linear programming formulation is widely used in the distribution problem. Considering the carbon emissions during the distribution, we modify the classical transportation problem to get the following distribution problems.

**The Transportation Problem under Policy 1 (Carbon Cap)**

Under the policy, the firm has to adopt a limited capacity \( C \) on the carbon emissions and cannot transport more products that produce more carbon emissions. We let \( e_{ij} \) to be the average carbon emissions per unit of the product from source \( i \) to destination \( j \), and \( C_0 \) to be the actual carbon capacity. To simplify our analysis and address the impact of carbon emissions policies, we assume there is a linear relationship between \( e_{ij} \) and \( c_{ij} \) (that is \( e_{ij} = k \times c_{ij} \)). So we have the transportation cost:

Minimize

\[
Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{5}
\]

Subject to (2), (3), (4) and

\[
C_0 = \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} x_{ij} \leq C \tag{6}
\]

Above all indicated that the firm has to adhere to a fixed carbon capacity \( C \), if carbon emissions generated during transportation is below the carbon cap regulation, the firm should arrange the transportation as usual. That's because the regulation has no limitation on the firm. In reality, carbon emissions per unit of the product from \( i \) to \( j \) vary from different transportation modes. We should let \( k_{ij} \) instead of \( k \) indicated different modes have different impact on carbon emissions.

**The Transportation Problem under Policy 2 (Carbon Tax)**

Under the policy, the firm needs to pay the tax \( \$\alpha \) for each unit product that emit carbon, this means the firm should add carbon emissions expense to its transportation cost. Thus, we can easily write the firm’s corresponding linear programming formulation by adding the cost of carbon tax.

Minimize

\[
Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \alpha \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} x_{ij} \tag{7}
\]

Subject to (2), (3) and (4)

Compared with the first regulation, we find this doesn't have the limit of carbon cap, but add the cost of tax on carbon emissions. More specially, if the policy is adopted, the firm should bear the carbon tax. In addition, if the firm comprehensively considers the revenue of the product and the
unit consumed cost of the carbon emissions, it may pay more tax to acquire higher profit.

**The Transportation Problem under Policy 3 (Cap-and-Trade)**

Under the cap-and-trade policy, the firm can sell or buy carbon quota in trade market. In detail, the firm can buy (sell) carbon quota at per unit price $\beta$ ($\gamma$) if its carbon emissions level is higher (lower) than the carbon cap. We thus calculate the cost of buying carbon credits as $\beta(C_0-C)$ and correspondingly the revenue as $\gamma(C-C_0)$ by selling unused carbon quota.

$$(C_0-C) = \max(C_0-C, 0) = \begin{cases} C_0-C, & \text{if } C_0 > C \\ 0, & \text{otherwise.} \end{cases}$$

Therefore, the firm’s transportation cost under the cap-and-trade can be described as,

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \beta \left( \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} x_{ij} - C \right) + \gamma \left( C - \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} x_{ij} \right)$$

Subject to (2), (3) and (4)

The most straightforward approach to solving a cap-and-trade transportation problem is to transform it to an equivalent problem of a standard type for which effective solution procedures already are available. To illustrate, suppose that $C_0 \leq C$ or $C_0 \geq C$, (8) can transform to (5) and

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \beta \left( \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} x_{ij} - C \right)$$

Subject to (2), (3), (4) and

$$\sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} x_{ij} \geq C$$

Under Policy 3 (cap-and-trade), the firm should minimize $Z$ in (8) to calculate the firm’s optimal transportation cost. As supposed above, the carbon generated during transportation below the carton cap, the linear programming formulation is similar to (5). Furthermore, the firm can reduce the transportation cost by getting extra income under the cap-and-trade policy. Contrarily, the firm also can fulfill its demand by transferring carbon credit, correspondingly increase the transportation cost.

**Numerical Examples**

In this section we conduct a set of numerical computations to illustrate our analysis in Section 3. For simplicity, we only consider one transportation mode. The problem as follows, units of freights distributed from source A to destination B, corresponding statistic is given in Table 1.

<table>
<thead>
<tr>
<th>Destination</th>
<th>Source</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>6</td>
<td>4</td>
<td>6</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>Allocation</td>
<td>150</td>
<td>150</td>
<td>200</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the first scenario, we assume that there is no capacity constraint and can respectively arrange the delivery schedule. So we calculate the result as follows: $x_{11}=50$, $x_{12}=150$, $x_{13}=0$, $x_{21}=100$, $x_{22}=0$, $x_{23}=0$.
\( x_{23} = 200, Z = 2500 \). Now, we consider the carbon emissions, just as supposed in 3.1, we let \( k = 0.2 \) as a constant \( (c_{ij} = 0.2c_{ij}) \), we infer the carbon emissions is \( C_0 = 500 \). As a result, if the carbon emissions generated by transporting below the cap \( (C_0 < C) \), the firm can make arrangement as usual. However, if the carbon consumed above the cap \( (C_0 > C) \), the firm must change the plan to meet the constraint. Especially we should consider different \( k_{ij} \) for various transport modes, which need further research.

In the second scenario, we assume the carbon tax \( \alpha \) is equal to 0.5 and 0.6, correspondingly, we get the same arrangement and cost \( Z = 2750 \) and \( Z = 2800 \). Compared \( Z \) we find the transportation cost increased, which means under carbon tax the firm can design transportation plan as the actual demand and have to shoulder the extra carbon tax \( (C_0 \ast \alpha) \), so the total transportation cost increased. In a word, contrary to the carbon cap policy, the firm does not need to limit the carbon emissions, but it must pay for the extra carbon emissions. Furthermore, we derive that under the same carbon emissions, the high-profit firm prefer to accept high tax in order to obtain high profit.

Under cap-and-trade, we assume \( \beta = 0.6, \gamma = 0.7 \) to represent the difference between the selling and buying prices in a market after considering transactional costs. As above mentioned, we know the carbon emissions are 500. Here we have two scenarios to discuss: First, if the carbon cap regulated by the government is 400, the firm should buy carbon credit in carbon market to satisfy the emissions, correspondingly we get the new cost \( Z = 2560 \) besides the transportation plan as usual. Furthermore, if the carbon cap is 600, the firm can not only arrange the transportation as usual but also earn extra income by selling its carbon credit surplus, and get the optimal cost \( Z = 2430 \). From the analysis we find the cap-and-trade can effectively adjust the transportation cost. Especially, if the profit margin is lower, the income of selling carbon credit is vital for the firm’s expected profit, the firm may choose to sell some carbon credit rather than running out of the carbon credit.

**Conclusions and Future Research**

In this paper we investigate the models of the transportation problem under three carbon policies (carbon cap, carbon tax, and cap-and-trade). Under each policy we obtained the linear programming formulation and analysis the impact to the transportation cost. Such as, the carbon cap can limit the carbon emissions effectively without increasing the transportation cost. For the carbon tax policy, the firm must bear extra cost for carbon emissions. We also find under cap-and-trade, the firm’s transportation cost and carbon emissions are effectively adjusted. Furthermore, the transportation problem in this paper could be extended in several directions. Firstly, we will consider the transportation cost on various transportation modes under the carbon emissions policies. In addition, we may also consider the firm’s optimal profit on supply chain included transportation.

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**Reference**


