Hybrid Techniques for Blind Source Separation

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Abstract. This paper proposes a parametric density model under a maximum likelihood framework. One may set different exponential parameters, based on the kurtosis properties of the desired signal, to match different possible signal distributions. In contrast to traditional techniques, the proposed signal separation method can provide more freedom and low computation load.

Introduction

Blind source separation (BSS), emerging from the neural network field, is a statistical and computational technique for revealing hidden factors that underlie sets of measurements [1]. This technique aims to recover the original sources from all kinds of their mixtures without the knowledge of the mixing process and the sources themselves [2,3]. Recently, BSS has received much research attention due to its wide application in the domain of biomedical engineering, sonar, speech enhancement and so on [4,5].

Generally speaking, there are two principal approaches to deal with the BSS problem. One approach aims to separate all sources from signal mixtures simultaneously. Another approach tries to separate sources one by one sequentially, called blind source extraction (BSE). In many practical applications such as biological signal processing, the observed mixture is composed of a lot of source signals while only one or a few are desired. For example, in EEG or MEG analysis, one obtains typically more than 64 sensor signals but only several source signals (e.g. the periodically evoked brain potentials) are desired while the rest are considered to be interfering noise [6,7]. These components are statistically independent as they are from different sources. Therefore, one can utilize BSS technique to separate desired signal from observed mixture. The simultaneous BSS approach tries to recover all source signals from their mixture, thus introducing heavy computational load and costing time. In such applications, it is necessary to present improved techniques which enable us to extract only one or a few source signals that are potentially interesting and contain useful information.

Until now, a few BSE methods have been proposed based on different assumptions, such as independent sources, spatial decorrelation, and temporal correlation of the sources[6-8]. Higher order statistics (HOS) based methods require few assumptions aside from the statistical independence of the original sources, so they generally represent the preferred technique for BSS/BSE problem[1-3]. Maximum likelihood estimation (MLE) is a fundamental technique of higher order statistics estimation. MLE has become an increasingly popular data analysis technique for blind signal separation problem. This paper presents an adaptive source extraction method under a maximum likelihood framework by introducing a parametric density model. The model is exploited with a family of exponential power density functions. Subsequently, an explicit algorithm for the adaptation of the non-linearity to various marginal densities is presented. As a result, the signal extraction can be conducted without any precise knowledge of the probability distribution. Computer simulations on biomedical data confirm the validity and robustness of the proposed method.

Proposed algorithm

We observe sensor signals \( x(k) = [x_1(k), \cdots, x_n(k)]^T \) described in the matrix notation:
where $A$ is an $n \times n$ unknown mixing matrix, $k$ is the discrete time index, and $s(k)=[s_1(k),\cdots,s_n(k)]^T$ is a vector of unknown zero-mean and unit-variance primary sources. Here only $x(k)$ can be observed while everything else is unknown. Source extraction is to estimate a specific source signal by using the observed data $x(k)$ and some statistical properties of original sources.

An efficient technique is to introduce an iterative process to find a weight vector $w$ so that $y(k) = w^T x(k) = w^T A s(k)$ is a good approximation to the desired source signal [4,5]. Fundamentally, BSE methods are developed by exploiting independence of the original sources. Signal independence may be measured with the minimum mutual information (MMI), MLE and the cumulants [6-8]. As a fundamental technique of HOS estimation, MLE can be efficiently utilized to develop BSE methods. The basic idea of BSE in terms of MLE is to find the gradient of the log likelihood through a stochastic gradient optimization [8,9]. The likelihood of the data set in model (1) is a function of the model’s parameters. The maximum likelihood function for extraction of one source signal exclusively can be described as [8,9]

$$\left\{ \begin{array}{l}
\max \quad L(w) = E\{\log p(w^T x(k))\} \\
\text{s.t.} \quad \|w\| = 1
\end{array} \right.,$$

where $p$ is the probability density function (PDF) of the source signal, which is unknown and should be estimated.

According to the gradient ascent optimization rule [1-3], we can maximize the cost function (2) and develop a gradient BSE algorithm:

$$m + 1 = m + \mu(m) E\{g(w(m)^T x)\} / E\{g'(w(m)^T x)\}$$

where $m$ is an iteration index, $\mu(m)$ is a step size, and $g(y) = (\log p(y))' = p(y)' / p(y)$. After algorithm (3) converges to a specific weight vector $\tilde{w}$, the desired signal can be estimated by $y(k) = (\tilde{w})^T x(k)$. Since function $p$ is unknown in practice, we should specify it in advance. When one calculates the maximum likelihood, a difficult task is how to choose an appropriate density function to match the input’s probability density [1-3].

Denote the assumed densities of the independent component (IC) by $\tilde{p}$, and

$$g_i(s_i) = \frac{\partial}{\partial s_i} \log \tilde{p}(s_i) = \frac{\tilde{p}'(s_i)}{\tilde{p}(s_i)}.$$

Constrain the estimates of the ICs to be uncorrelated and to have unit variance. The maximum likelihood estimator is locally consistent if $\tilde{p}$ satisfies

$$E\{s_i g_i(s_i) - g_i'(s_i)\} > 0.$$  

It is worth mentioning that sufficiently small changes cannot change the sign in (5), so small misspecifications of $\tilde{p}$ do not affect the local consistency of the maximum likelihood estimator. We introduce the parameterized generalized Gaussian distributions to express the probability distribution of the desired signal [1-3]:

$$p(y; \alpha) = \frac{\alpha}{2\theta(\alpha)} e^{\alpha|y|^\alpha}$$

where $\alpha$ is used to control the peak of the probability distribution, and $\xi(x)$ is the Gamma function which can be expressed as $\xi(x) = \int_0^\infty t^{x-1} e^{-t} \, dt$. Since $E\{y^2\} = 1$, we can deduce $\theta = \sqrt{\xi(1/\alpha) / \xi(3/\alpha)}$. In practice, the desired source signal may have various distributions such as super-Gaussian (the kurtosis is positive) and sub-Gaussian (the kurtosis is negative). When $\alpha$ is set to different values, we can obtain different densities which range from super-Gaussian to...
sub-Gaussian. We find that when the desired signal is super-Gaussian, $\alpha = 1.5$ is suitable; when the desired signal is sub-Gaussian, $\alpha = 4.0$ is suitable.

**Computer Simulations**

We adopted four typical biomedical signals, shown in Fig.1. Each signal had 5000 samples with zero means and unit variances. From top to down, they were, respectively, a colored Gaussian signal, an ECG, an electrode artifact and a respiratory artifact. A $4 \times 4$ mixed matrix $A$ was randomly generated with rows $A_1 = [0.7902, 0.0732, 0.1198, 0.3846]$, $A_2 = [0.4919, 0.1523, 0.2017, 0.4015]$, $A_3 = [0.1509, 0.5978, 0.4956, 0.267]$ and $A_4 = [0.3916, 0.0487, 0.4987, 0.8789]$.

These source signals were mixed by matrix $A$. The mixed results were shown in Fig.2. We aim to extract the desired ECG from the synthetic data exclusively. To make comparison, we ran the following BSE algorithms: BCBSE [2], EVBSE [5, 6] and our algorithm (NEWBSE). As ECG is super-Gaussian, here we set $\alpha = 1, 1.5, 3.0$, respectively. All algorithms' parameters were adjusted so as to obtain the best average performance. The extraction results are shown in Fig.3, which are in accordance with the sequence of these BSE algorithms. Obviously, the BCBSE algorithm shows the poorest performance, due to the fact that the extracted $y_1$ is always contaminated by other signals or noise. The NEWBSE algorithm performs best. Specifically, when $\alpha = 1.5$, the extracted $y_4$ by the
NEWBSE algorithm is very similar to the source signal s2. In general, we note that $\alpha = 1.5$ is the best choice to extract the super-Gaussian signal.

![Fig.3 Extraction results on synthetic data by various algorithms.](image)

To further measure the accuracy of extraction, we adopted the index of signal-to-noise ratio (SNR) in dB given by $\text{SNR} = 10 \log_{10}(S2/MSE)$, where S2 denotes the variance of the source and MSE denotes the mean square error between the original signal and recovered signal. Obviously, the higher SNR is, the better the performance is.

To exactly compare their performance of these algorithms, we repeated these simulations 100 times independently so as to get the average and highest SNR. The simulation results are shown in Table 1. Here we set $\alpha = 1.5$ in the NEWBSE algorithm. Obviously, the NEWBSE algorithm outperforms the other two algorithms, as expected. In contrast, the BCBSE algorithm performs poorly.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>BCBSE</th>
<th>EVBSE</th>
<th>NEWBSE</th>
</tr>
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<tbody>
<tr>
<td>average SNR</td>
<td>13.8732</td>
<td>26.6543</td>
<td>30.7653</td>
</tr>
<tr>
<td>highest SNR</td>
<td>15.1923</td>
<td>28.7654</td>
<td>31.9825</td>
</tr>
</tbody>
</table>

**Conclusions**

Recently, the BSS/BSE problem has received much research attention due to its wide range of potential application fields such as speech processing, data communication, sonar array processing, seismic exploration, and biological signal processing. In many applications such as biomedical signal processing, one is only interested in one or a few source signals. To improve signal extraction performance, an improved method is presented in this paper. Compared with the general techniques, the proposed method uses all higher-order statistics of the signals instead of only using the fourth-order statistic or kurtosis. Its validity and robustness are demonstrated by computer simulations on biomedical signals.
References


