

The lateral effect research of traffic flow based on the modified car-following model

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Abstract. In this paper, an improved car-following model is proposed to suppress the traffic jams. Considering the relative velocity, relative optimal velocity and the difference between safety distance and headway, a comprehensive control scheme is constructed according to the feedback control theory. The stability condition for the modified model is obtained by using the linear stability theory. Numerical simulation is carried out to illustrate the advantage of our model with the new control signal, and the results are in good accordance with the theoretical analysis.

Introduction

In recent decades, traffic problems have influenced human's daily life, such as the traffic accident, fuel consumption and pollution. In order to solve those increasingly serious traffic problems, many traffic models have been put forward to study the complex phenomena of traffic jams. What's more, the scientific researchers have obtained many significant results [1–3].

To date, the problem of traffic jam has attracted much attention in the study of traffic flow. The concept of car-following was first described by Reuschel and Pipes [4], which assumed that the vehicle behind adjusts its behavior with respect to the preceding vehicle in the same lane. In 1961, Newell [5] developed a car-following model with a differential equation and gave a graphic description of the optimal velocity (OV) function. In 1995, Bando et al. [6] proposed a microscopic traffic model called optimal velocity model (OVM). Further, Konishi et al. [9] proposed a chaotic car-following model by setting the time delay feedback control schemes, and analyzed theoretically the traffic jam phenomena for open flow in 1999. Later, some scholars extended the model by introducing multiple information into relative velocity or headway [7].

So far, there is just little research of car-following from the viewpoint of control methods. In real traffic flow, on urban road without isolation belts, it is necessary to consider the effects of vehicles on adjacent lanes, and we need to taken into account the headway between the proceeding and following vehicles.

The rest of this paper is organized as follows. The car-following model considering the lateral effect is presented in section 2. In Section 3, the modified model including a new feedback control signal is constructed and the feedback control method is used to analyze the stability conditions. In Section 4, numerical simulation is carried out to confirm the theoretical results. Conclusions are given in Section 5.

Car-following model

To investigate the effects of the non-motor vehicles on adjacent lanes without isolation belts, the car-following model is depicted as follows:

$$\frac{d^2 x_n(t)}{dt^2} = a \left[V^{op}(\Delta x_n(t)) + \kappa \left(p \bar{V}^{op}(w_n(t)) + q \bar{V}^{op}(l_n(t)) \right) \right] - a v_n(t) \quad (1)$$

where $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$, $\Delta v_n(t) = v_{n+1}(t) - v_n(t)$ are the headway and the velocity difference between the n -th vehicle and the preceding vehicle, respectively; $x_n(t)$ is the n -th vehicle's position, a is the sensitivity of driver and κ is the influence coefficient between motor vehicle and non-motor vehicle; $V^{op}(\Delta x_n(t))$ is the optimal velocity function. The OV function is expressed as follows:

$$V^{op}(\Delta x_n(t)) = \frac{v_{\max}}{2} \left[\tanh(\Delta x_n(t) - h_c) + \tanh(h_c) \right] \quad (2) \quad \bar{V}^{op}(w_n(t)) = \begin{cases} 0, & \bar{x}_n \geq d_1, \\ V^{op}(w_n(t)), & 0 \leq \bar{x}_n < d_1, \end{cases}$$

$$\bar{V}^{op}(l_n(t)) = \begin{cases} 0, & \bar{x}_n > d_2, \\ V^{op}(l_n(t)), & 0 \leq \bar{x}_n < d_2, \end{cases} \quad (4)$$

where \bar{x}_n is the distance between the n -th motor vehicle and the non-motor vehicle, and \bar{x}_n is the minimum lateral distance between the n -th motor and non-motor vehicles; v_{\max} is the maximum velocity and h_c is the safety headway distance; $w_n = \bar{x}_n - d_1$, $l_n = \bar{x}_n - d_2$. d_1, d_2 are the lateral and longitudinal safety distances respectively; p, q ($p+q=1$) are the reaction coefficients of the lateral and longitudinal distances respectively.

In fact, according to the control theories, we have studied the stability condition for the system (1), which is

$$0 \leq K \leq \sqrt{\frac{a - 2\Lambda_1}{a \left[(p\Lambda_2)^2 + (q\Lambda_3)^2 \right]}}, (a - 2\Lambda_1 \geq 0). \quad (5)$$

$$\text{where } \Lambda_1 = \frac{dV^{op}(y_n(t))}{dy_n(t)} \Big|_{y_n(t) = V^{op-1}(v_0)};$$

$$\Lambda_2 = \frac{d\bar{V}^{op}(w_n(t))}{dw_n(t)} \Big|_{w_n(t) = d_1^*};$$

$$\Lambda_3 = \frac{d\bar{V}^{op}(l_n(t))}{dl_n(t)} \Big|_{l_n(t) = d_2^* \text{ and } y_n(t) = \Delta x_n(t)}.$$

Control scheme

Considering the positive and negative velocity differences and optimal velocity differences into account, in the meantime, in order to avoid collision, the difference between the safety headway and the headway is considered. The new feedback control signal $u_n(t)$ including all these factors is added into system (1), so we have

$$\frac{d^2 x_n(t)}{dt^2} = a \left[V^{op}(\Delta x_n(t)) + \kappa \left(p \bar{V}^{op}(w_n(t)) + q \bar{V}^{op}(l_n(t)) \right) \right] - a v_n(t) + u_n(t) \quad (6)$$

$$u_n(t) = \gamma \left[\Delta v_n(t) + \Delta V_n^{op}(t) \right] - \lambda^2 (h_c - y_n(t)). \quad (7)$$

where γ ($\gamma > 0$) is the reaction coefficient for the relative velocity $\Delta v_n(t)$ and the relative optimal velocity $\Delta V_n^{op}(t)$; λ is another reaction coefficient for the $H(y_n(t) - h_c)(h_c - y_n(t))$.

Then, Eq. (6) can be described as

$$\begin{cases} \frac{dv_n(t)}{dt} = a \left[V^{op}(\Delta y_n(t)) + \kappa \left(p \bar{V}^{op}(w_n(t)) + q \bar{V}^{op}(l_n(t)) \right) \right] \\ -av_n(t) + \gamma \left[v_{n+1}(t) - v_n(t) + \left(V^{op}(y_{n+1}(t)) - V^{op}(y_n(t)) \right) \right] \\ -\lambda^2 (h_c - y_n(t)) \\ \frac{dv_n(t)}{dt} = v_{n+1}(t) - v_n(t). \end{cases} \quad (8) \quad \text{Suppose that the leading vehicle travels}$$

at a constant velocity v_0 , so we can get the the steady state for the following vehicles, that is

$$\begin{bmatrix} v_n^*(t), y_n^*(t) \end{bmatrix}^T = \begin{bmatrix} v_0, V^{op-1}(v_0) \end{bmatrix}^T. \quad (9)$$

The traffic system (8) can be linearized at the steady state (9), that is

$$\begin{cases} \frac{d\delta v_n(t)}{dt} = a \left[\delta y_n(t) \Lambda_1 + \kappa \left(p \delta w_n(t) \Lambda_2 + q \delta l_n(t) \Lambda_3 \right) \right] \\ -\delta v_n(t) + \gamma \left[\delta v_{n+1}(t) - \delta v_n(t) + \left(\delta(y_{n+1}(t) \Lambda_4 - \delta y_n(t) \Lambda_1) \right) \right] \\ +\lambda^2 \delta y_n(t), \\ \frac{d\delta v_n(t)}{dt} = \delta v_{n+1}(t) - \delta v_n(t). \end{cases} \quad (10) \quad \text{where}$$

$$\delta v_n(t) = v_n(t) - v_0, \delta y_n(t) = y_n(t) - V^{op-1}(v_0);$$

$$\delta w_n(t) = \delta \bar{x}_n - d_1^*, \delta l_n = \delta \bar{x}_n - d_2^*;$$

$$\Lambda_4 = \frac{dV^{op}(y_{n+1}(t))}{dy_{n+1}(t)} \Big|_{y_{n+1}(t) = V^{op-1}(v_0)}.$$

After the Laplace transform for Eq.(12), we can get the transfer function $\bar{G}(s)$ as follows:

$$\begin{aligned} \bar{G}(S) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{p(s)} \begin{bmatrix} s & a\Lambda_1 - \kappa\Lambda_1 + \lambda \\ -1 & s + a + \lambda \end{bmatrix} \\ &\times \begin{bmatrix} a\kappa p\Lambda_2 & a\kappa p\Lambda_3 & \gamma & \gamma\Lambda_4 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\ &= \frac{1}{p(s)} \begin{bmatrix} s\kappa p\Lambda_2 & s\kappa q\Lambda_2 & s\gamma + \beta & s\gamma\Lambda_4 \end{bmatrix} \quad (11) \end{aligned}$$

$$\bar{p}(s) = s^2 + \alpha s + \beta, \quad (12)$$

$$\alpha = a + \gamma, \beta = (a - \kappa)\Lambda_1 + \lambda^2 \quad (13)$$

where s is a complex variable.

Based on the control theories, traffic jams will never

occur in the traffic flow system as long as the characteristic function $\bar{p}(s)$ is stable and $\|\bar{G}(s)\|_\infty \leq 1$. According to the Hurwitz stability criterion, it is easy to obtain that the characteristic function $\bar{p}(s)$ is stable. Then, consider $\|\bar{G}(s)\|_\infty \leq 1$, that is

$$\|\bar{G}(s)\|_\infty = \sup_{\omega \in [0, \infty)} |\bar{G}(j\omega)| \leq 1$$

$$\begin{aligned} |\overline{G}(j\omega)|^2 &= |\overline{G}(-j\omega)\overline{G}(j\omega)| \\ &= \frac{(\alpha\kappa p\omega)^2 + (\alpha\kappa q\omega)^2 + (\lambda\omega)^2 + \beta^2 + (\gamma\Lambda_4\omega)^2}{(\beta - \omega^2)^2 + (\alpha\omega)^2} \leq 1. \end{aligned} \quad (14)$$

Then, we can obtain the sufficient condition through the analysis above. We have the region for the reaction κ

$$0 \leq \kappa \leq \sqrt{\frac{a^2 + 2a\gamma - 2a\Lambda_1 - 2\lambda^2 - (\lambda\Lambda_4)^2}{a[(p\Lambda_2)^2 + (q\Lambda_3)^2]}}, \quad (15)$$

where $a^2 + 2a\gamma - 2a\Lambda_1 - 2\lambda^2 - (\lambda\Lambda_4)^2 \geq 0$.

Numerical simulation

In the section, the parameters for the modified car-following model are set as $h_c = 7.02\text{m}$, $a = 2\text{s}^{-1}$, $d_1 = 5.5\text{m}$, $d_2 = 6.5\text{m}$, $v_0 = 20\text{m/s}$ and $T = 0.1\text{s}$. It is assumed that all vehicles have the same parameters. The initial condition is the steady state for the model, and the initial positions and speeds are set as $x_n(0) = \sum y_n^*$, $y_n(0) = y_n^*(t)$, $v_n(0) = v_n^*(t)$ $n = 1, 2, 3, \dots, N$, and $N = 120$ is the total number of vehicles. We consider a case where the leading vehicle stops suddenly for $v_n(0) = 0$, $t = nT = 100 - 103$.

Figure 1 shows the velocity-time patterns of the 1th, the 25th and the 50th vehicles with different parameter values of γ . It can be seen from Fig. 1 that with the control signal, as the reaction coefficient γ increases from 0.15 to 0.75, the stability of the traffic system is strengthened. And we can find that vehicles can reach steady running state in relatively short time with the increasing of reaction coefficient γ .

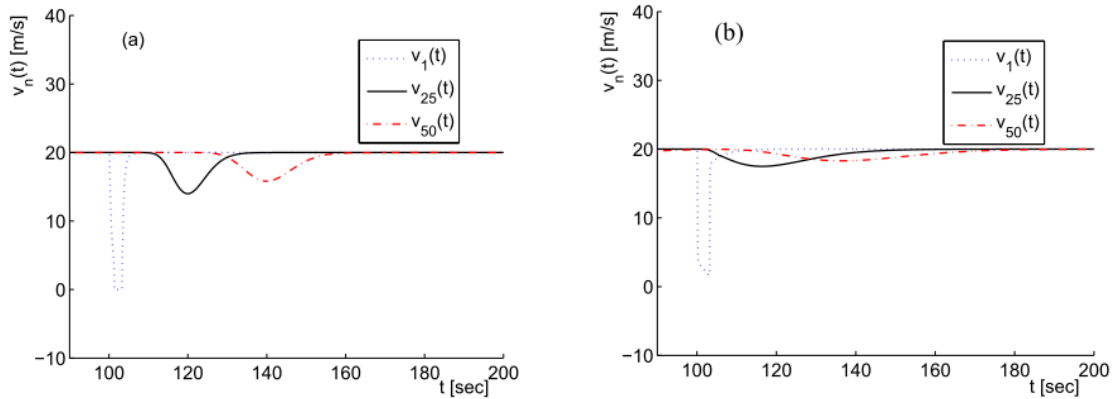
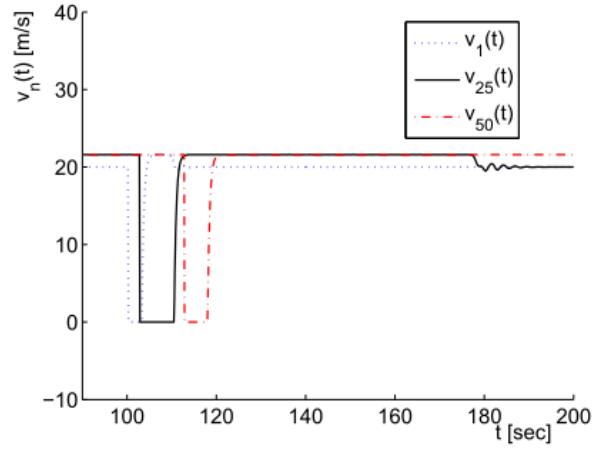
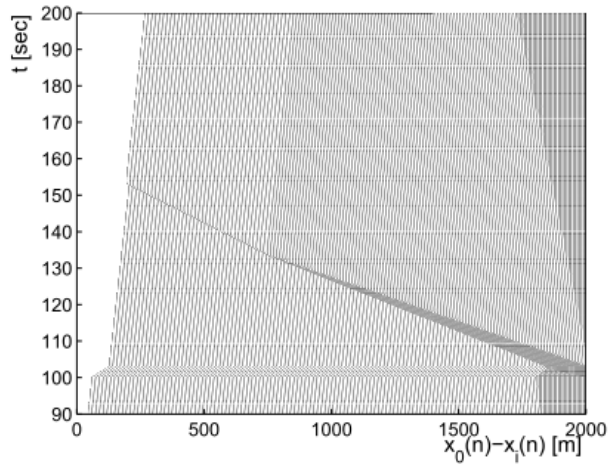


Figure 1. Numerical simulations for the modified car-following model with (a) $\gamma = 0.15$, (b) $\gamma = 0.75$.

Now consider the system with the new control signal. As the stability condition in Eq. (5) and Eq. (15) is met, the comparisons between the results in Fig. 2 and Fig. 3 are carried out. When we choose the appropriate parameters ($\kappa = 0.75$, $\gamma = 0.75$, $\lambda = 0.48$, $v_{\max} = 33.6\text{m/s}$), we can find that vehicles can reach more steady running state in relatively short time, although the maximal speed is larger compared with Fig. 2. The amplitude of the velocity for the 25th vehicle decreases and the 50th vehicle runs more smoothly. That is to say, as we add a feedback control signal to system (1), the traffic flow can reach steady state quickly.



(2a)



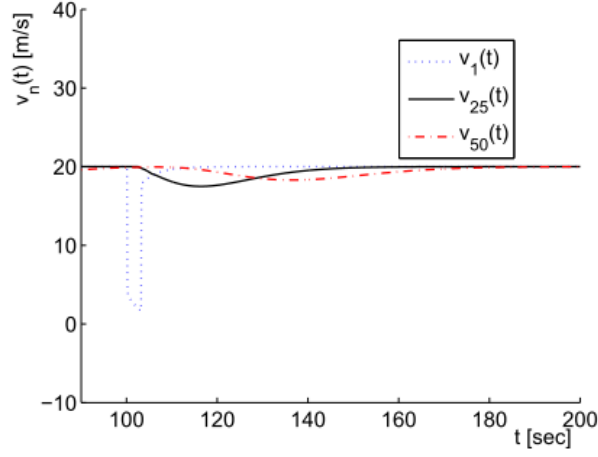
(2b)

Figure 2. (2a) Space-time plot of the traffic system . (2b) Temporal velocity behavior of the first, 25th and 50th vehicles. ($\kappa=0.5, \lambda=0, \gamma=0, v_{\max}=21.5\text{m/s}$)

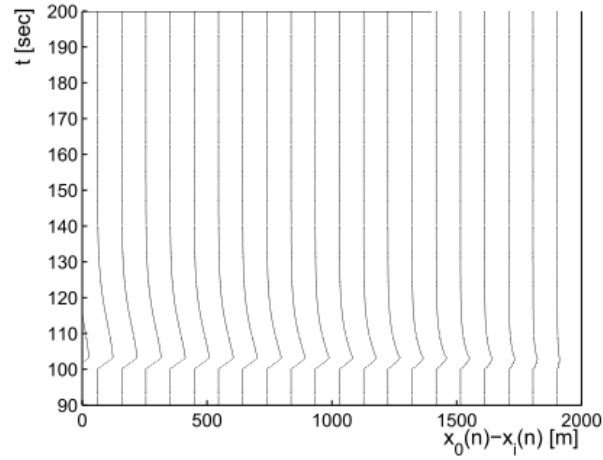
From the above simulation, it can be concluded that the proposed feedback control scheme is useful for suppressing the traffic congestion problems.

Conclusions

In this study, a improved car-following model considering the lateral effect has been proposed. The other effects of some important comprehensive information on the traffic current and the jamming transition have been investigated. Moreover, the stability condition has been obtained for the model by using control method. The new model has been tested by using numerical simulations and the results are consistent with theoretical ones.



(3a)



(3b)

Figure 3. (3a) Space-time plot of the traffic system . (3b) Temporal velocity behavior of the first, 25th and 50th vehicles. ($\kappa=0.75, \lambda=0.48, \gamma=0.75, v_{\max}=33.6\text{m/s}$)

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