Dynamic Modelling and Analysis of Performance for the Electromagnetic-driven Spherical Robot

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Abstract—This paper analyses the dynamic performances of the electromagnetic-driven spherical robot including rolling, obstacle overrunning, uphill climbing and like, as well as discusses the influence factors on the dynamic performance of the spherical robot, which comprise the equivalent pendulum mass, the spherical robot mass and the pendulum length to the spherical robot. Form the above discussions, it becomes obvious that the conclusion can be reached that the larger the ratio defined as \( \lambda \), the better of the dynamic performance is. Obviously, a same conclusion can be drawn with regard to the ratio defined as \( k \).

Keywords—spherical robot; dynamic modeling; electromagnetic driven; obstacle overrunning; uphill climbing.

I. INTRODUCTION

Spherical mobile robot, which has a ball-shaped outer shell to accommodate all its mechanism and control devices in it, is characterized as walking mainly in a rolling way, simple, small friction ,compact, well-sealed structure, never turning over and so on. These advantages provide the spherical robots with strong viability and applications in many fields, such as military, transportation, surveillance, search and rescue, toys, entertainment etc. It has attracted the interest of many researchers.

In fact, as early as in 1893, Tate J.L. invented a spherical toy, and applied for a patent [1]. In 1909, Robert E. Cecil designed an amusing spherical toy rolling along a track of zig-zig, and then he applied for a patent [2]. In the subsequent nearly a century, more than 80 patents related to the ball-shaped object were produced. However, few patents came into use, for neither mathematical studies nor experimental researches were carried out until about twenty years ago.

In 1906, Brown H. Benjamin, Jr. and Xu Yangsheng at the Carnegie Mellon University designed a disk-shaped rolling robot named as Gyrover with a single wheel moving forwards and backwards, which is not strictly a spherical robot [3]. The static analysis were developed for the robot, yet neither the dynamic study nor the control method was performed. However, the findings has attracted many scientists from many countries all over the world [8-14].

So far, the studies concerning the kinematics, dynamics model and control strategy seem insufficient. Thus, a great of further studies are required to consummate the kinematics and dynamics models as well as to perfect control methods for the spherical robot. This paper focus on rolling conditions, uphill climbing, obstacle overrunning of an electromagnetic-driven spherical robot in order to lay the foundation for effective control strategies in the future.

II. MECHANISM OF ELECTROMAGNETIC-DRIVEN SPHERICAL ROBOT

The present spherical robot is composed of a left hemispherical shell, a right hemispherical shell, a magnetic steel ring and an inner driving mechanism, which is called electromagnetic-driven spherical robot as shown in Figure.1. The left hemispherical shell and the right hemispherical shell form a spherical shell assembled through the fixing screws coaxially with the magnetic steel ring on which the permanent magnetic steel pieces are uniformly disposed. N-electrodes of two adjacent permanent magnetic steel pieces are opposite in direction.

The inner driving mechanism positioned in the spherical shell mainly comprises an inner driving bracket, a main shaft, a flywheel, a steering motor, a motor support, a flywheel shaft, an electricity power supply, a controller, electromagnetic coils and the like. The main shaft, on which the other components are hung up through the inner driving bracket, is fixedly connected with the spherical shell through a couple of bears respectively fixed on inner surface of the left and right shell in opposite direction. The flywheel is hung on the inner driving bracket through a bearing. The steering motor shaft is connected with the flywheel through a coupling in order to drive the lower part of the motor to rotate together with the flywheel shaft vertical to the main shaft and the flywheel. The electromagnetic coils disposed on the bracket are arranged symmetrically relative to the steering motor shaft.
III. OBSTACLE OVERRUNNING

The spherical robot can also overrun an obstacle of certain maximum height. The configuration of a spherical robot encountering an obstacle is shown as Figure 2.

For simplicity, we have some assumptions as follows,

1. The spherical robot is in contact with the ground or the obstacles without deformation;
2. The spherical robot travels in rolling with no slippage;
3. The initial velocity of spherical robot is zero;
4. The analysis is based on a two-dimensional plane shown in Figure 2.

In order to overrun the obstacle, the driving torque must be greater than the counter torque by the gravity. We assume that \( h \) is the height of the obstacle and \( \theta \) is the driving angle of the equivalent pendulum. The maximum value of the height of the obstacle can be calculated in the following equations.

\[
mg(r \sin \theta - \sqrt{R^2 - (R - h)^2}) = Mg \sqrt{R^2 - (R - h)^2}
\]

(1)

Where, \( m \) is the mass of the equivalent pendulum, \( M \) is the mass of the shell of the spherical robot, \( g \) is the gravitational acceleration, \( R \) is the radius of the sphere, and \( r \) is the distance between the centers of equivalent pendulum and the spherical shell.

According to the Eq. (1), we have,

\[
h = R - \sqrt{R^2 - (nr \sin \theta / (M + m))^2} = R(1 - \sqrt{1 - (nr \sin \theta / (M + m))^2})
\]

(2)

Assuming that \( \kappa \) is the ratio of \( r \) to \( R \), that is

\[
\kappa = r / R
\]

Hence,

\[
h = R(1 - \sqrt{1 - (\sin \theta)^2 \lambda^2 \kappa^2})
\]

(4)

The fact revealed here is that without slippage, the bigger the driving angle of the equivalent pendulum, the larger the height of the obstacle overrun for the spherical robot would be.

Considering \( \theta \) can be close to \( \pi / 2 \), that is \( \sin \theta = 1 \). Thus, Eq. (3) can be written as

\[
h_{\text{max}} \leq R(1 - \sqrt{1 - \lambda^2 \kappa^2})
\]

(5)

Eq. (5) shows that the overrunning ability of the spherical robot depends mainly on the two factors without considering slippage, one is the ratio of \( r \) to \( R \), and another is the ratio of \( m \) to \( (m + M) \).

1. The bigger the ratio of \( r \) to \( R \) is, the larger the height of the obstacle overrun with no slippage for the spherical robot would be. If \( r \) can be made close to \( R \), \( h_{\text{max}} \) can be expressed as

\[
h_{\text{max}} \leq R(1 - \sqrt{1 - \lambda^2 \kappa^2}) = R(1 - \sqrt{1 - (m / (m + M))^2})
\]

(6)

2. The bigger the ratio of \( m \) to \( (m + M) \) is, the larger the height of the obstacle would be. If \( M \) can be close to 0 compared with \( m \), \( h_{\text{max}} \) will only be influenced by the ratio of \( r \) to \( R \), that is,

\[
h_{\text{max}} \leq R(1 - \sqrt{1 - \kappa^2}) = R(1 - \sqrt{1 - (r / R)^2})
\]

(7)

IV. UPHILL CLIMBING

The robot is capable to roll uphill in certain inclination as shown in Figure 3.

The equation for uphill climbing of the spherical robot is given as follows:

\[
mg(r \sin \theta - R \sin \Phi) = MgR \sin \Phi
\]

(8)

Where,

- \( M \): the mass of the robot,
- \( m \): the mass of the equivalent pendulum,
- \( R \): the radius of the robot,
- \( r \): the distance between centers of the robot and the pendulum,
the gravitational acceleration, $\theta$: the rotation angle of the equivalent pendulum with respect to the robot.

$\Phi$: the slope angle of the inclination.

Assuming the robot rolls without slipping, the slope angle of the inclination can be expressed as

$$\Phi = \sin^{-1}\left(\frac{m \times r}{(M + m) \times R}\right)$$

$$= \sin^{-1}\left(\frac{\sin \theta \times m}{(M + m) \times (r / R)}\right)$$

(9)

As a consequence of the driving angle of the equivalent pendulum increasing, the inclination slope angle which the robot is cable of climbing would be greatnessen proportionately.

Considering $\theta$ can be close to $\pi / 2$, that is $\sin \theta = 1$, the maximum slope angle of the inclination $\Phi_{\text{max}}$ can be written as

$$\Phi_{\text{max}} = \sin^{-1}\left(\frac{m r}{(M + m) R}\right) = \sin^{-1}\left(\frac{m}{(M + m)} \times \frac{r}{R}\right)$$

(10)

Hence,

$$\Phi_{\text{max}} = \sin^{-1}(\lambda \kappa)$$

(11)

Where $\lambda = m / (M + m), \kappa = r / R$.

Note that if $r$ can be made close to $R$ or $\lambda$ can be made close to 1, $\Phi$ will increase considerably. It seems that the situation becomes unrealistic.

Eq. (11) demonstrates that the inclination slope angle which the robot is cable of climbing is mainly conditioned by two factors, which comprise the ratio of $r$ to $R$ and the ratio of $m$ to $(m + M)$. A short discussions are given as bellow.

(1) The bigger the ratio of $r$ to $R$, the larger the inclination slope angle for the spherical robot would be. If $r$ can be made close to $R$, $\Phi_{\text{max}}$ can be rewritten as

$$\Phi_{\text{max}} \leq \sin^{-1}(\lambda) = \sin^{-1}\left(\frac{m}{(m + M)}\right)$$

(12)

(2) The bigger the ratio of $m$ to $(m + M)$, the larger the inclination slope angle for the spherical robot. If $M$ can be close to 0 compared with $m$, $\Phi_{\text{max}}$ will be only influenced by the ratio of $r$ to $R$, thus, the equation of the maximum of the inclination slope angle becomes,

$$\Phi_{\text{max}} \leq \sin^{-1}(\kappa) = \sin^{-1}\left(\frac{r}{R}\right)$$

(13)

Taking the ratios including $r$ to $R$ and $m$ to $(m + M)$ as the horizontal coordinate axes, $h_{\text{max}}$ and $\Phi_{\text{max}}$ as the vertical coordinate axes respectively, A diagram can be obtained, which as shown in Figure. 4.

Figure 4. Maximum height of obstacle and inclination angle of slope

A most important fact revealed in Eq. (5) and Eq. (11) is that the variable of $\lambda$ has the same behavior influence with $\kappa$ on $h_{\text{max}}$ and $\Phi_{\text{max}}$, so the variable of $\lambda$ marked in the horizontal coordinate axis are consistent with $\kappa$. As shown in Fig. 6, the red dashed line represented the teensy of $\lambda$ or $\kappa$ versus the maximum height $h_{\text{max}}$, while the black thin lines indicated teensy of $\lambda$ or $\kappa$ versus the maximum slope angle of the inclination $\Phi_{\text{max}}$ respectively. Therefore, it can be concluded that the electromagnetic-driven spherical robot have the better performance with the increase of the ratios including the equivalent pendulum mass to the spherical robot mass and the length of the pendulum to the spherical robot radius.

V. CONCLUSIONS

Based on an electromagnetic-driven spherical robot, we focused on analyses of the dynamic abilities including rolling, uphill climbing, obstacle overrunning and the like in this paper. It follows from what has been said that under the consistent environmental conditions such as materials, rolling friction, sliding friction, etc., the dynamic performance of the spherical robot are mainly conditioned by two factors: one is the radio of the equivalent pendulum mass to the spherical robot mass defined as $\lambda$ and another is the radio of the length of the pendulum to the spherical robot radius defined as $\kappa$. In conclusion, we can say that hat the electromagnetic-driven spherical robot have the better performance with the increase of the ratios including the equivalent pendulum mass to the spherical robot mass and the length of the pendulum to the spherical robot radius.

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