Observer-based Guaranteed Cost Control for a Class of Singular Time-delay Systems with Uncertainties

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Abstract- The problem of guaranteed cost observer-based controller design for a class of singular systems time-delay with uncertainties is investigated. The design method of guaranteed cost observer-based controller is given. Based on linear matrix inequality (LMI) approach, sufficient conditions for the existence of guaranteed cost observer-based state feedback control law and corresponding guaranteed cost performance index are obtained by constructing generalized Lyapunov function. It makes that the closed-loop system is robust stable.

Keywords-Observer-based controller; guaranteed cost control; time-delay systems; linear matrix inequality

I. INTRODUCTION

The control of singular systems has been extensively studied in the past years[1-2]. In [3-4], guaranteed cost control for uncertain systems are discussed. In [5-6], observer-based guaranteed cost control for singular time-delay systems with uncertainties are discussed, but the state matrices and the control input matrices are consistent with the original system.

In this paper, we consider observer-based guaranteed cost control for a class of singular time-delay systems with uncertainties. The state matrices and the control input matrices are not consistent with the original system, and are uncertain matrices representing time-varying parameter uncertainties in the system model, $d_i(t)$ and $d_j(t)$ are unknown constant matrices representing the number of delay units in the state and input respectively, which satisfy $0 \leq d_i(t) < d_i < \infty$, $0 \leq d_j(t) < d_j < \infty$, $d = \max[d_i, d_j]$. The parameter uncertainties considered in this paper are assumed to be norm-bounded and of the form

$\Delta = F_1(t) H_1, \Delta = F_2(t) H_2, \Delta = F_3(t) H_3, \Delta = F_4(t) H_4, \Delta = J_1(t) H_6, \Delta = J_2(t) H_7, \Delta = J_3(t) H_8, \Delta = J_4(t) H_9, \Delta = J_5(t) H_5, \Delta = J_6(t) H_6, \Delta = J_7(t) H_7, \Delta = J_8(t) H_8, \Delta = J_9(t) H_9, \Delta = J_{10}(t) H_{10}, \Delta = J_{11}(t) H_{11}, \Delta = J_{12}(t) H_{12}$

Where $E_i$, $H_i$ ($i = 1, 2, \cdots, 10$) are unknown real constant matrices with appropriate dimensions, and $F_i(t)$ ($i = 1, 2, \cdots, 8$) are unknown real matrices with Lebesgue-measurable elements and satisfy

$F_i(t) F_i(t) T \leq \tilde{I}_e, \tilde{I}_e = 1, 2, \cdots, 8.$

We define the cost function:

$J = \int \left[ x^T(t) Q x(t) + u^T(t) R u(t) \right] dt$

where $Q > 0$ and $R > 0$ are given matrices.

The objective is to design an observer-based controller of the form:

$\begin{align*}
\dot{\hat{x}}(t) &= F_1 x(t) + F_2 \dot{x}(t) + F_3 u(t) + F_4 \dot{u}(t) + F_5 \dot{y}(t) + F_6 \dot{\eta}(t), \\
\dot{\hat{y}}(t) &= C_1 \hat{x}(t) + C_2 \dot{x}(t) + C_3 \dot{u}(t) + C_4 \dot{\eta}(t) + C_5 \dot{y}(t), \\
\dot{\hat{\eta}}(t) &= K \hat{x}(t), \quad t \in [-d, 0]
\end{align*}$

(5)

Which $L \in R^{\infty}$ is the observer gain and $K \in R^{\infty}$ is the feedback control gain.

Let the error vector be such $e(t) = x(t) - \hat{x}(t)$.

We combine (1) and (5) to produce the closed-loop system:
Theorem 1 The closed-loop system (5) is robust stable and a guaranteed cost controller if there exist invertible matrices \(P_1, P_2\) and symmetric positive-definite matrices \(S_1, S_2, R_1, R_2\), he following matrix inequality holds

\[
E^T P_1 = P_1^T E \geq 0, \quad E^T R_1 = R_1^T E \geq 0
\] (7)

An upper bound on the cost \(J^*\) is given by

\[
J^* = \begin{bmatrix} \phi(0)^T & \phi(0) \\ E^T P_1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ E^T P_2 & e(0) \end{bmatrix}
+ \int_0^\infty \begin{bmatrix} \phi(\sigma)^T \\ e(\sigma) \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} \phi(\sigma) \\ e(\sigma) \end{bmatrix} d\sigma
\] (8)

where

\[
N_1 = \begin{bmatrix} \Sigma_1 & -P_1 B_1 & 0 \\ -P_1 B_1^T & (A - A_1)^T P_1 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\quad N_2 = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[
\Sigma_i = 2A - BK + S_i + \Sigma R_i K + Q + \Sigma R_i K + \sum_{i=1}^{\infty} \eta_i E_i E_i^T P_i
\]

\[
\Sigma_i = \eta_i E_i B_i + (A_i - A_i - BK) P_i + \sum_{i=1}^{\infty} \eta_i E_i E_i^T P_i
\]

\[
\Sigma_i = 2A_i + BK + S_i + \Sigma R_i K + \sum_{i=1}^{\infty} \eta_i E_i E_i^T P_i + \sum_{i=1}^{\infty} \eta_i E_i E_i^T \eta_i E_i^T P_i
\]

Proof. Consider the following Lyapunov function:

\[
V(x(t), e(t)) = \begin{bmatrix} x^T(t) & 0 \end{bmatrix} \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix} \begin{bmatrix} x(t) & e(t) \end{bmatrix}
+ \int_0^t \begin{bmatrix} x^T(\sigma) & 0 \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} x(\sigma) & e(\sigma) \end{bmatrix} d\sigma
\]

Then, the time-derivative of \(V(x(t), e(t))\) gives

\[
\dot{V} = x^T(t)Qx(t) + u^T(t)Ru(t) \leq \eta^T(t) \Omega \eta(t)
\]

where

\[
\Omega = \begin{bmatrix} \Sigma_1 & -P_1 A_1 & 0 \\ -P_1 A_1^T & (A - A_1)^T P_1 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\quad \Sigma_1 = 2A - BK + S + \Sigma R K + Q + \Sigma R K + \sum_{i=1}^{\infty} \eta_i E_i E_i^T P_i
\]

\[
\Omega_2 = \begin{bmatrix} P_1 E_1 & P_1 E_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\quad \Omega_3 = N_2
\]
According to Lemma 1 and Schur Complement, it holds for any $F(t)$ satisfying (2), if and only if there exists a scalar $\varepsilon > 0$ such that

$$
\begin{bmatrix}
N_1 & N_2 \\
N_3 & -\varepsilon I
\end{bmatrix} \preceq 0.
$$

Hence, $\dot{V} < -[x^T(t)Qx(t) + u^T(t)Ru(t)] < 0$. By (7)-(8) and (10) implies that system (5) is asymptotically stable. On the other hand, using (10), $\int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)] dt < V(0)$, let $T \to \infty$, we get (9).

**Theorem 2** The closed-loop system (5) is asymptotically stable if there exist invertible matrices $X$ and symmetric positive-definite matrices $T, Y, M, N$, the following matrix inequality holds

$$
\begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
* & \varepsilon I & 0 \\
* & * & T_{23}
\end{bmatrix} < 0
$$

An upper bound on the cost $J_f$ is given by (9). Where

$$
T = \begin{bmatrix}
\Pi_{11} & \Pi_{12} & \Pi_{13} \\
\Pi_{21} & 0 & B^T_Y \\
\Pi_{22} & 0 & 0
\end{bmatrix}
$$

$$
T_2 = \begin{bmatrix}
M & H_5 & 0 \\
-\varepsilon M^T & H_4 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
T_3 = \begin{bmatrix}
M^T & H_5^T & 0 \\
-\varepsilon M & H_4^T & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
T_4 = \begin{bmatrix}
\varepsilon M & H_5 & 0 \\
-\varepsilon M^T & H_4 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
T_5 = \begin{bmatrix}
\varepsilon M^T & H_5^T & 0 \\
-\varepsilon M & H_4^T & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
T_6 = \begin{bmatrix}
\varepsilon M & H_5 & 0 \\
-\varepsilon M^T & H_4 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
T_7 = \begin{bmatrix}
\varepsilon M^T & H_5^T & 0 \\
-\varepsilon M & H_4^T & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
T_8 = \begin{bmatrix}
\varepsilon M & H_5 & 0 \\
-\varepsilon M^T & H_4 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
T_9 = \begin{bmatrix}
\varepsilon M^T & H_5^T & 0 \\
-\varepsilon M & H_4^T & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
T_{10} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
\Pi_1 = X^T A X + X^T B \dot{X} + X^T C T + \frac{1}{2} \sum_{i=1}^n \varepsilon_i E_i E_i^T;
$$

$$
\Pi_2 = \dot{X}^T A X + X^T B \dot{X} + X^T C T + \dot{X}^T C T + \frac{1}{2} \sum_{i=1}^n \varepsilon_i E_i E_i^T;
$$

**Proof.** Let $L = P_2 + C^T, \ P_2 = P$. Pre-multiplying and post-multiplying matrix inequality (8) by diag$(P^T, S_1^{-1}, R_1^{-T}, P^{-T}, S_2^{-T}, R_2^{-T}, I)$ and

$$
\dot{X}^T A X + X^T B \dot{X} + X^T C T + \frac{1}{2} \sum_{i=1}^n \varepsilon_i E_i E_i^T = 0;
$$

hence, $X = W^T, \ L = X^T C^T$, then (12) is obtained by Schur Complement.

**IV. CONCLUSION**

In this paper, the problem of observer-based guaranteed cost control for a class of singular time-delay systems with uncertainties has been studied. Sufficient conditions for the existence of guaranteed cost observer-based state feedback control law and corresponding guaranteed cost performance index are obtained. The results in this paper are much more desirable and less conservative than the existing results.

**REFERENCES**


