The Study of Environmental Air Quality Assessment Method Based on Interval Number

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Abstract. We select and which are two of environmental air pollution indexes, and which is one of respirable particulate matters as three main indicators. Using interval number theory, we conduct a comprehensive assessment of the air quality from 2004 to 2010 of five cities (Chongqing, Yichang, Jinzhou, Wuhan, and Wuhu) along the Yangtze River. Firstly, this paper converts the annual average concentration raw data of urban air pollutants into interval data, and makes a pretreatment for them. Secondly, we obtain the interval comprehensive evaluation values by linear weighted formula, and determine the order of the environmental air quality of five cities using closeness theory.

Introduction

With the improvement of living standards, the environment issues which is closely related to people's lives is a growing concern of society. On the one hand, the rapid development of the productive forces improves the material conditions of human, but on the other hand it causes tremendous damage for the natural environment. For environmental protection, our country has been active in efforts. Especially, for those enterprises that cause serious air pollution, the country will continue to intensify rectification. So, the objective and accurate evaluation for environment air quality, and providing scientific basis for decision-making department is very necessary.

On air quality assessment method, based on different research perspective, domestic and foreign scholars put forward many effective methods. For example, index evaluation method, expert evaluation method, ecology evaluation method, economics evaluation method, fuzzy comprehensive evaluation method. Because factors affecting air quality with a high degree of complexity and uncertainty, air quality comprehensive evaluation using fuzzy set theory is more and more attention by the domestic and foreign scholars, and the study is more in-depth [1-5]. Because of complexity, uncertainties of things and the ambiguity of human mind, it's often more reasonable that the decision-making information and comprehensive evaluation information are represented by interval numbers. Therefore, this paper discusses the issue that studies environmental air quality problems using interval number theory, and presents an air quality assessment method based on interval numbers.

The Basic Conception of Interval Numbers

Let R denote the whole real numbers. For arbitrarily $r^-, r^+ \in R$ ($r^- \le r^+$), we call closed interval $r = [r^-, r^+] = \{x | r^- \le x \le r^+, x \in R\}$ is a interval number. The all interval numbers are denoted by $[R]$. Specifically, if $r^- = r^+$, r is reduced to a real number.

For $r_1, r_2 \in [R]$, the calculation laws involved in this article are as follows:

$r_1 + r_2 = [r_1^- + r_2^-, r_1^+ + r_2^+]$, $kr = [kr^-, kr^+] (k \ge 0)$

Theorem 2.1[6] For any $r_1, r_2 \in [R]$, let

$$d(r_1, r_2) = \sqrt{(\frac{r_1^- + r_1^+}{2} - \frac{r_2^- + r_2^+}{2})^2 + \frac{1}{12}[(r_1^+ - r_1^-) - (r_2^+ - r_2^-)]^2},$$

then

$$N(r_1,r_2)=\begin{cases}\dfrac{1}{\sqrt{1+d(r_1,r_2)}}\ , & d(r_1,r_2)\in[0,1)\\ 0\ , & d(r_1,r_2)\geq 1\end{cases} \qquad (2\text{-}1)$$

is called the closeness degree of interval number r_1 a and r_2.

The Air Quality Evaluation Principles Based Intervals

Table 1 raw data of annual average concentration of urban air pollutants (units: mg/m3)

City	Index	2004	2005	2006	2007	2008	2009	2010
Chongqing	SO_2	0.113	0.073	0.074	0.065	0.063	0.053	0.048
	NO_2	0.067	0.048	0.047	0.044	0.043	0.037	0.039
	PM_{10}	0.142	0.120	0.111	0.108	0.106	0.105	0.102
Yichang	SO_2	0.122	0.048	0.054	0.047	0.046	0.049	0.041
	NO_2	0.047	0.039	0.034	0.027	0.038	0.027	0.022
	PM_{10}	0.129	0.081	0.097	0.084	0.085	0.087	0.086
Jingzhou	SO_2	0.054	0.040	0.031	0.036	0.043	0.038	0.035
	NO_2	0.028	0.022	0.021	0.024	0.027	0.022	0.030
	PM_{10}	0.100	0.080	0.077	0.080	0.089	0.085	0.088
Wuhan	SO_2	0.044	0.045	0.046	0.050	0.051	0.044	0.041
	NO_2	0.052	0.045	0.044	0.048	0.054	0.054	0.057
	PM_{10}	0.130	0.111	0.109	0.108	0.113	0.105	0.108
Wuhu	SO_2	0.016	0.020	0.026	0.022	0.019	0.025	0.029
	NO_2	0.027	0.026	0.026	0.023	0.021	0.024	0.031
	PM_{10}	0.089	0.086	0.067	0.072	0.084	0.072	0.074

Table 2 the weight coefficients of urban air pollutants

City	Index	2004	2005	2006	2007	2008	2009	2010
Chongqing	SO_2	0.432	0.383	0.400	0.379	0.376	0.350	0.328
	NO_2	0.231	0.227	0.228	0.231	0.231	0.220	0.240
	PM_{10}	0.337	0.391	0.372	0.391	0.393	0.430	0.432
Yichang	SO_2	0.499	0.360	0.373	0.381	0.346	0.385	0.359
	NO_2	0.173	0.263	0.211	0.197	0.257	0.191	0.173
	PM_{10}	0.328	0.377	0.416	0.422	0.397	0.424	0.468
Jingzhou	SO_2	0.382	0.365	0.317	0.336	0.351	0.344	0.300
	NO_2	0.178	0.181	0.193	0.201	0.198	0.179	0.232
	PM_{10}	0.439	0.454	0.489	0.463	0.451	0.477	0.468
Wuhan	SO_2	0.257	0.291	0.300	0.312	0.300	0.279	0.257
	NO_2	0.273	0.262	0.258	0.270	0.286	0.308	0.322
	PM_{10}	0.471	0.446	0.441	0.418	0.413	0.413	0.421
Wuhu	SO_2	0.167	0.207	0.286	0.252	0.211	0.274	0.282
	NO_2	0.254	0.242	0.257	0.237	0.210	0.237	0.271
	PM_{10}	0.578	0.552	0.457	0.511	0.579	0.490	0.447

Annual detectable concentration range of the main pollutants in ambient air SO_2 、 NO_2 and respirable particulate matter PM_{10} reflect environmental air pollution situation in the region. How to use the information implied by annual detectable concentration range evaluate the environmental air pollution situation objectively and accurately. We make a comprehensive evaluation about air quality from 2004 to 2010 of five cities along the Yangtze River (Chongqing, Yichang, Jingzhou, Wuhan, Wuhu) [5]. We introduce the evaluation principle of air quality based on interval number.

The air quality pollution from 2004 to 2010 of five cities (Chongqing, Yichang, Jingzhou, Wuhan, Wuhu) are shown in Table 1, the weight coefficients of pollutants are shown in Table 2.

Getting original interval data.

The annual average concentrations between the minimum and maximum of the three air pollutant make up an interval number, get the interval decision matrix of these five urban air quality as shown in Table 3.

Table 3 decision matrix A

pollutants	SO_2	NO_2	PM_{10}
Chongqing a_1	$[0.048, 0.113]$	$[0.037, 0.067]$	$[0.102, 0.142]$
Yichang a_2	$[0.041, 0.122]$	$[0.022, 0.047]$	$[0.081, 0.129]$
Jingzhou a_3	$[0.031, 0.054]$	$[0.021, 0.030]$	$[0.077, 0.100]$
Wuhan a_4	$[0.041, 0.051]$	$[0.044, 0.057]$	$[0.105, 0.130]$
Wuhu a_5	$[0.016, 0.029]$	$[0.021, 0.027]$	$[0.067, 0.089]$

Pretreatment of interval data.

We normalize the decision matrix. According to the proportion of the value of the standardized method[7], we can get standardized matrix in Table 4.

Table 4 standardized matrix R

pollutants	SO_2	NO_2	PM_{10}
Chongqing r_1	$[0.0445, 0.1369]$	$[0.0656, 0.1578]$	$[0.0994, 0.1671]$
Yichang r_2	$[0.0412, 0.1603]$	$[0.0935, 0.2647]$	$[0.1095, 0.2105]$
Jingzhou r_3	$[0.0931, 0.2120]$	$[0.1465, 0.2773]$	$[0.1412, 0.2214]$
Wuhan r_4	$[0.0986, 0.1603]$	$[0.0771, 0.1323]$	$[0.1086, 0.1624]$
Wuhu r_5	$[0.1734, 0.4108]$	$[0.1628, 0.2773]$	$[0.1587, 0.2545]$

Determining the weights.

We determine the final weight of air pollutants in Table 5 using the formula

$$w_i = \frac{1}{7}\sum_{j=2004}^{2010} w_{ij}$$

Table 5 the weight of air pollutants

pollutants	SO_2	NO_2	PM_{10}
Chongqing r_1	0.3783	0.2297	0.3923
Yichang r_2	0.3861	0.2093	0.4046
Jingzhou r_3	0.3421	0.1946	0.4630
Wuhan r_4	0.2851	0.2827	0.4319
Wuhu r_5	0.2399	0.2440	0.5163

Solving comprehensive evaluation interval value.

According to the linear weighted formula $\tilde{r}_i = \sum_{j=1}^{3} w_{ij} r_{ij}$, we calculate comprehensive evaluation interval values as shown in Table 6.

Table 6 comprehensive evaluation values

Chongqing r_1	Yichang r_2	Jingzhou r_3	Wuhan r_4	Wuhu r_5
$[0.0709, 0.1536]$	$[0.0798, 0.2025]$	$[0.1257, 0.2290]$	$[0.0968, 0.1532]$	$[0.1633, 0.2976]$

Determining the merits order of environmental air quality.

①Selecting the superior and inferior interval numbers：let $r_i = \left[r_i^-, r_i^+\right]\left(i = 1,2,3,4,5\right)$,then by table 6 , we get the inferior interval

$r^{+} = [\max\{r_1^-, r_2^- \cdots r_5^-\}, \max\{r_1^+, r_2^+ \cdots r_5^+\}] = [0.1633, 0.2976]$

and superior interval

$r^{-} = [\min\{r_1^-, r_2^- \cdots r_5^-\}, \min\{r_1^+, r_2^+ \cdots r_5^+\}] = [0.0709, 0.1532]$

②Calculating the closeness degree between r_1 and the superior and inferior interval numbers according to formula (2-1)

$N(r_1, r^+) = \dfrac{1}{\sqrt{1 + d(r_1, r^+)}} = 0.8935$, $N(\tilde{r}_1, r^-) = 0.9998$

Then we also calculate the sorting index value of r_1

$T(r_1) = \dfrac{N(r_1, r^+) + (1 - N(r_1, r^-))}{2} = 0.4469$

Similarly, we get

$T(r_2) = 0.4742$; $T(r_3) = 0.5052$; $T(r_4) = 0.4587$; $T(r_5) = 0.5533$

③Sorting them according to the size of sorted index values

According to $T(r_5) \succ T(r_3) \succ T(r_2) \succ T(r_4) \succ T(r_1)$, we get comprehensive evaluation results as follows

$r_5 \succ r_3 \succ r_2 \succ r_4 \succ r_1$

Based on the above method, we can conclude the comprehensive evaluation results of the five urban air quality, ie, the pollution degree of Wuhu is the largest, others were: Jingzhou, Yichang, Wuhan, Chongqing. The result is consistent with literature [5]. Therefore, the evaluation method based on the interval number is valid, and the calculation method is simple, and can be more completely reflect the degree of air pollution. So it's a more reasonable and practical evaluation method.

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