Random Finite Element Computation Analysis of Deformation of Multi-pivot Foundation Pit

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Abstract. The paper selects shear strength index of soil as the random variable, and applies elastic foundation beam system finite element method and load incremental method to establish random finite element computing model of multi-pivot piling wall supporting foundation pit deformation and deduct the partial derivative of foundation pit deformation for random variable. Combined with the improved the first-order second-moment method, the paper analyzes the reliability of multi-pivot piling wall supporting foundation pit deformation for providing theoretical and research basis for the design method of deep excavation engineering based on reliability theory.

Introduction

With rapid development of the construction of high-rise buildings and municipal engineering in cities in China, there are more and more deep excavation engineering, and the cutting depth is greater and greater. In order to ensure the stability of foundation pit in construction and reduce the influence of foundation pit excavation on the surrounding buildings, roads and underground engineering facilities, the supporting structure system with inner support is generally used in urban deep foundation pit excavation such as campshed+pre-stressed anchor supporting system, campshed+inner supporting system and underground continuous wall+inner supporting system. We call the supporting system multi-pivot piling wall supporting system.

For the multi-pivot piling wall supporting system, the constant value design method based on security coefficient is generally used. Some scholars have made initial exploration for the reliability of single-pivot piling wall supporting system, but there is no research on the analysis of reliability of multi-pivot piling wall supporting system at home and abroad. The computation method has the disadvantages of large computation amount and low computation efficiency, so the practicality in engineering is worse.

Finite Element Method

For analyzing deformation of foundation pit and internal force of supporting structure, finite element method has the incomparable advantages. The method can simulate the actual excavation process of foundation pit, and fully considers the influence of the deformation of inner support and supporting structure on internal force of structure and deformation, so it is widely applied in the design computation of foundation pit engineering.

According to the characteristics of foundation pit engineering, finite element method can be divided into space finite element method, plane finite element method and bar-system finite element method. Space finite element method and plane finite element method uses continuum mechanics as the theoretical basis, and divide soil and supporting structure into computation units. In computation, the soil and supporting structure can flexibly apply different constitutive models, so the method is perfect theoretically. However, in application process, it has the disadvantages of complicated finite element procedure, large computation amount and low efficiency, so the method is not generally applied in engineering practice.

Bar-system finite element method can use foundation pit supporting structure as elastic foundation beam based on elastic resistance method. And the function of the surrounding soil for supporting structure is replaced by soil pressure and soil spring, which avoids the disadvantage of the above finite element method. The model is simple, the computation is easy and the calculation results are ideal. So the method not only is simple and practical, but also can solve the actual engineering problems.

Based on the above analysis, the paper combines bar-system finite element method and elastic foundation beam method, simulates the actual excavation process of multi-pivot piling wall supporting foundation fit, fully considers the common function of soil, supporting structure and support, and establishes the computation model of inner force of supporting structure and deformation under different working conditions in the excavation process of foundation pit.

Random Finite Element Computation Model of Deformation of Multi-pivot Foundation Pit

For multi-pivot filing wall supporting foundation pit, soil excavation is the first, and the erection of inner support (or anchor stock) is the second. That is, before the erection of support (or anchor stock), the supporting structure has initially deformed, and the initial deformation can't be neglected in the design of foundation pit. In finite element analysis of multi-pivot piling wall supporting foundation pit, load full dose method and incremental method has the same calculation results. However, the analysis of load full-dose method needs to consider the inheritance of working conditions, and the initial displacement of the supporting side needs to be corrected, which makes the computation complicated and makes computer calculation program difficult. And applying load incremental method for analysis is easy and simple.

For the selection of random variables, the paper only selects the internal friction angle φ_i and cohesion c_i of soil as random variables, and the other variables are constants. There are m soil layers in the range of foundation pit excavation, there are $2m$ random variables, $\left(c_1,\varphi_1,c_2,\varphi_2,\cdots,c_m,\varphi_m\right)$. And all random variables meet normal distribution. In order to make expression easy, the random variables are expressed by $\left(X_1,X_2,X_3,\cdots,X_{2m}\right)$.

In the computation, the unit wide wall is selected as mechanical analysis object, and it is divided into n-1 computation elements. The length of each computation elements is the same, and is expressed by l_e. The first inner support is located in the second node, the second inner support is placed in the sixth node, and the depth of foundation pit excavation is H.

The first working condition (the first excavation). For the first excavation, the working face of the first inner support is excavated. The first inner support is located in the second 2, so the depth of the first excavation is $H_1=2l_e$, which means to excavate out the third node. And two soil springs are dig out, and the computation sketch is shown in Fig. 1.

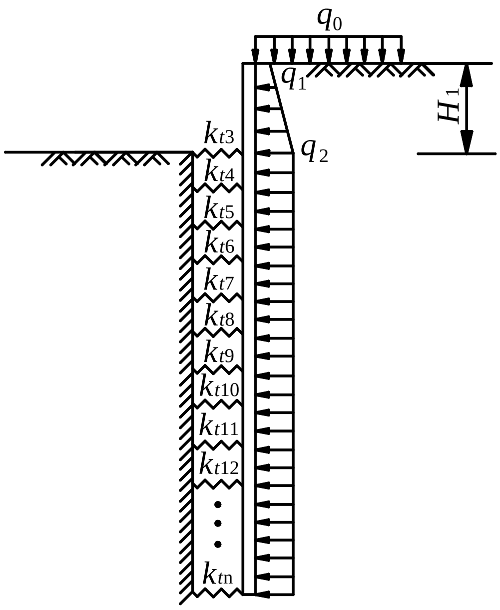

Fig. 1 Calculation diagram of the first working condition

Computation model of incremental method. The stiffness matrix $\boldsymbol{K_1}$ of the supporting structure under the first working condition achieved by the correction of initial stiffness matrix $\boldsymbol{K_0}$ is

$$\boldsymbol{K_1}=\begin{bmatrix} k_{11} & k_{12} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ k_{21} & k_{22} & k_{23} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & k_{32} & k'_{33} & k_{34} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & k_{43} & k'_{44} & k_{45} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & k_{54} & k'_{55} & k_{56} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{65} & k'_{66} & k_{67} & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{76} & k'_{77} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & k'_{(n-1),(n-1)} & k_{(n-1),n} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & k_{n,(n-1)} & k'_{nn} \end{bmatrix} \quad (1)$$

In the formula, $k'_{ii}=\begin{bmatrix} \dfrac{24EI}{l_e^3}+k_{ti} & 0 \\ 0 & \dfrac{8EI}{l_e} \end{bmatrix}$ $(i=3,4,\cdots,n\text{-}1)$;

$$k'_{nn}=\begin{bmatrix} \dfrac{12EI}{l_e^3}+k_{tn} & -\dfrac{6EI}{l_e^2} \\ -\dfrac{6EI}{l_e^2} & \dfrac{4EI}{l_e} \end{bmatrix};$$

And $k_{ti}\left(i=3,4,5,\cdots,n\right)$ means the elastic stiffness coefficient of the i soil spring.

The integral stiffness equation of the supporting structure under the first working condition is

$$\boldsymbol{K_1}\boldsymbol{\varDelta_1}=\boldsymbol{P_1} \quad (2)$$

The column vector $\boldsymbol{\varDelta_1}$ of the displacement increment is

$$\boldsymbol{\varDelta_1}=\left[\Delta y_1^1 \quad \Delta\theta_1^1 \quad \Delta y_2^1 \quad \Delta\theta_2^1 \quad \Delta y_3^1 \quad \Delta\theta_3^1 \ \cdots\ \Delta y_n^1 \quad \Delta\theta_n^1\right]^T \quad (3)$$

The column vector $\boldsymbol{P_1}$ of the equivalent load of the node is

$$\boldsymbol{P}_1 = \begin{bmatrix} f_1^1 & m_1^1 & f_2^1 & m_2^1 & f_3^1 & m_3^1 & \cdots & f_n^1 & m_n^1 \end{bmatrix}^T \tag{4}$$

Partial derivative of displacement increment for random variables

From formula (2), we can get

$$\boldsymbol{\Delta}_1 = \boldsymbol{K}_1^{-1}\boldsymbol{P}_1 \tag{5}$$

In the formula, $\boldsymbol{K}_1^{-1}$ is the inverse matrix of the integral stiffness matrix $\boldsymbol{K}_1$.

Both sides of formula (5) solves partial derivative of random variable X_i at the same time, which can get

$$\frac{\partial \boldsymbol{\Delta}_1}{\partial \boldsymbol{X}_i} = \boldsymbol{K}_1^{-1}\frac{\partial \boldsymbol{P}_1}{\partial \boldsymbol{X}_i} \tag{6}$$

And the expansion is

$$\begin{bmatrix} \frac{\partial \Delta y_1^1}{\partial X_i} \\ \frac{\partial \Delta \theta_1^1}{\partial X_i} \\ \frac{\partial \Delta y_2^1}{\partial X_i} \\ \frac{\partial \Delta \theta_2^1}{\partial X_i} \\ \vdots \\ \frac{\partial \Delta y_n^1}{\partial X_i} \\ \frac{\partial \Delta \theta_n^1}{\partial X_i} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ k_{21} & k_{22} & k_{23} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & k_{32} & k'_{33} & k_{34} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & k_{43} & k'_{44} & k_{45} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & k_{54} & k'_{55} & k_{56} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{65} & k'_{66} & k_{67} & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{76} & k'_{77} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & k'_{(n-1),(n-1)} & k_{(n-1),n} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & k_{n,(n-1)} & k'_{nn} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial f_1^1}{\partial X_i} \\ \frac{\partial m_1^1}{\partial X_i} \\ \frac{\partial f_2^1}{\partial X_i} \\ \frac{\partial m_2^1}{\partial X_i} \\ \vdots \\ \frac{\partial f_n^1}{\partial X_i} \\ \frac{\partial m_n^1}{\partial X_i} \end{bmatrix} \tag{7}$$

Partial derivative of counter force of soil spring for random variables

Under the first working condition, the counter force increment ΔN_k^1 of soil spring is

$$\Delta N_k^1 = k_{tk}\Delta y_k^1 \quad (k = 3,4,5,\cdots,n) \tag{8}$$

In the formula, ΔN_k^1— the counter force of the k soil spring under the first working condition, kN;

Δy_k^1—the deformation increment of the k soil spring under the first working condition, m;

k_{tk}—the stiffness coefficient of the k soil spring, kN/m.

And the total counter force N_k^1 of soil spring is

$$N_k^1 = \Delta N_k^1 = k_{tk}\Delta y_k^1 \quad (k = 3,4,5,\cdots,n) \tag{9}$$

The partial derivative of the total counter force N_k^1 of soil spring for the random variable X_i is

$$\frac{\partial N_k^1}{\partial X_i} = k_{tk}\frac{\partial \Delta y_k^1}{\partial X_i} \quad (k = 3,4,5,\cdots,n) \tag{10}$$

Partial derivative of equivalent load of nodes for random variables

(1) The partial derivative of the equivalent load of the node that the first soil pressure is greater than zero unit for the random variable X_i is

$$\begin{cases} \dfrac{\partial f_k^1}{\partial X_i} = \dfrac{l_e}{20}\left(3\dfrac{\partial q_{k+1}}{\partial X_i} + 7\dfrac{\partial q_k}{\partial X_i}\right) \\ \dfrac{\partial m_k^1}{\partial X_i} = \dfrac{l_e^2}{60}\left(2\dfrac{\partial q_{k+1}}{\partial X_i} + 3\dfrac{\partial q_k}{\partial X_i}\right) \end{cases} \tag{11}$$

(2)The partial derivative of the equivalent load of any middle node for the random variable X_i is

$$\begin{cases} \dfrac{\partial f_k^1}{\partial X_i} = \dfrac{l_e}{20}\left(3\dfrac{\partial q_{k-1}}{\partial X_i} + 7\dfrac{\partial q_k}{\partial X_i} + 7\dfrac{\partial q_k'}{\partial X_i} + 3\dfrac{\partial q_{k+1}}{\partial X_i}\right) \\ \dfrac{\partial m_k^1}{\partial X_i} = \dfrac{l_e^2}{60}\left(2\dfrac{\partial q_{k-1}}{\partial X_i} + 3\dfrac{\partial q_k}{\partial X_i} + 3\dfrac{\partial q_k'}{\partial X_i} + 2\dfrac{\partial q_{k+1}}{\partial X_i}\right) \end{cases} \tag{12}$$

(3)The partial derivative of the equivalent load of the n node for random variable X_i is

$$\begin{cases} \dfrac{\partial f_n^1}{\partial X_i} = \dfrac{l_e}{20}\left(3\dfrac{\partial q_{n-1}}{\partial X_i} + 7\dfrac{\partial q_n}{\partial X_i}\right) \\ \dfrac{\partial m_n^1}{\partial X_i} = -\dfrac{l_e^2}{60}\left(2\dfrac{\partial q_{n-1}}{\partial X_i} + 3\dfrac{\partial q_n}{\partial X_i}\right) \end{cases} \tag{13}$$

The partial derivative of the active soil pressure strength q of each node for the random variable X_i is

$$\begin{cases} \dfrac{\partial q}{\partial c_i} = -2\tan\left(45° - \dfrac{\varphi_j}{2}\right) & (i = j) \\ \dfrac{\partial q}{\partial c_i} = 0 & (i \neq j) \\ \dfrac{\partial q}{\partial \varphi_i} = \sec^2\left(45° - \dfrac{\varphi_j}{2}\right)\left[2\sum \gamma h \tan\left(45° - \dfrac{\varphi_j}{2}\right) - 2c_j\right] & (i = j) \\ \dfrac{\partial q}{\partial \varphi_i} = 0 & (i \neq j) \end{cases} \tag{14}$$

The second working condition (erecting the first support and pre-add strutting axial force F_{N1}). Based on the first working condition, the first inner support (the elastic stiffness coefficient is k_{z1}) is erected, and the axial force F_{N1} (a constant) is pre-added. The computation sketch of the load increment of the working condition is shown in Figure 2.

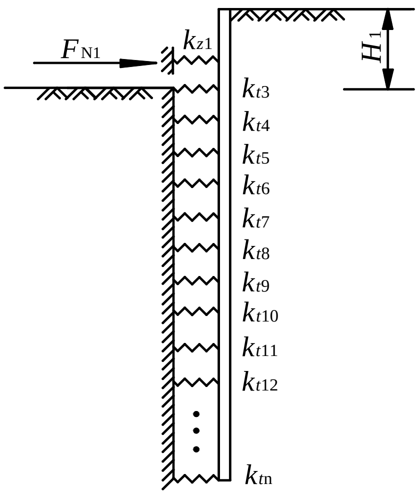

Fig. 2 Calculation diagram of the second working condition

Computation model of increment method

The integral stiffness matrix $\boldsymbol{K}_2$ of the supporting structure under the second working condition which is achieved by the correcting of the integral stiffness matrix $\boldsymbol{K}_1$ under the first working condition is

$$\boldsymbol{K}_2=\begin{bmatrix} k_{11} & k_{12} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ k_{21} & k_{22}^{z1} & k_{23} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & k_{32} & k'_{33} & k_{34} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & k_{43} & k'_{44} & k_{45} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & k_{54} & k'_{55} & k_{56} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{65} & k'_{66} & k_{67} & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{76} & k'_{77} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & k'_{(n-1),(n-1)} & k_{(n-1),n} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & k_{n,(n-1)} & k'_{nn} \end{bmatrix} \quad (15)$$

In the formula, $k_{22}^{z1}=\begin{bmatrix} \dfrac{24EI}{l_e^3}+k_{z1} & 0 \\ 0 & \dfrac{8EI}{l_e} \end{bmatrix}$;

In the integral stiffness equation of the supporting structure under the second working condition is

$$\boldsymbol{K}_2\boldsymbol{\Delta}_2=\boldsymbol{P}_2 \quad (16)$$

The column vector $\boldsymbol{\Delta}_2$ of the displacement increment is

$$\boldsymbol{\Delta}_2=\left[\Delta y_1^2 \quad \Delta\theta_1^2 \quad \Delta y_2^2 \quad \Delta\theta_2^2 \quad \Delta y_3^2 \quad \Delta\theta_3^2 \ \cdots \ \Delta y_n^2 \quad \Delta\theta_n^2\right]^T \quad (17)$$

The column vector $\boldsymbol{P}_2$ of the equivalent load increment of the node is

$$\boldsymbol{P}_2=\left[f_1^2 \quad m_1^2 \quad f_2^2 \quad m_2^2 \quad f_3^2 \quad m_3^2 \ \cdots \ f_n^2 \quad m_n^2\right]^T \quad (18)$$

In the formula, $f_2^2=-F_{N1}$, and the others are zero.

Mean and variance of displacement increment

From formula (16), we can get

$$\boldsymbol{\Delta}_2=\boldsymbol{K}_2^{-1}\boldsymbol{P}_2 \quad (19)$$

In the formula, $\boldsymbol{K}_2^{-1}$—the inverse matrix of the integral stiffness matrix $\boldsymbol{K}_2$.

Both sides of formula (19) solve derivative of random variable X_i at the same time, which can get

$$\frac{\partial \Delta_2}{\partial X_i} = K_2^{-1} \frac{\partial P_2}{\partial X_i} \tag{20}$$

$\partial P_2 / \partial X_i = 0$, so

$$\frac{\partial \Delta y_k^2}{\partial X_i} = \frac{\partial \Delta \theta_k^2}{\partial X_i} = 0 \tag{21}$$

From formula (19), we can get the horizontal displacement value Δy_k^2 (constant) of each node, and we can get the mean $\mu_{\Delta y_k^2}$ and the variance $\sigma_{\Delta y_k^2}^2$ of the horizontal displacement value Δy_k^2 of each node.

$$\begin{cases} \mu_{\Delta y_k^2} = \Delta y_k^2 \\ \sigma_{\Delta y_k^2}^2 = 0 \end{cases} \tag{22}$$

Partial derivative of counter force of soil spring for random variables

Under the second working condition, the increment ΔN_k^2 of the counter force of soil spring is

$$\Delta N_k^2 = k_{tk} \cdot \Delta y_k^2 \qquad (k = 3, 4, 5, \cdots, n) \tag{23}$$

In the formula: ΔN_k^2—the increment of the counter force of the k soil spring under the second working condition, kN;

Δy_k^2—the displacement increment of the k node under the second working condition, m;

k_{tk}—the stiffness coefficient of the k soil spring, $\mathrm{kN/m}$.

And the total counter force of soil spring is

$$N_k^2 = N_k^1 + \Delta N_k^2 = k_{tk} \left(\Delta y_k^1 + \Delta y_k^2 \right) \qquad (k = 3, 4, 5, \cdots, n) \tag{24}$$

The partial derivative of the total counter force N_k^2 of soil spring for random variable X_i is

$$\frac{\partial N_k^2}{\partial X_i} = k_{tk} \frac{\partial \Delta y_k^1}{\partial X_i} = \frac{\partial N_k^1}{\partial X_i} \qquad (k = 3, 4, 5, \cdots, n) \tag{25}$$

Computation of Reliability of Deformation of Foundation Pit

The final horizontal deformation displacement of the k node of the supporting wall is

$$y_k = \Delta y_k^1 + \Delta y_k^2 + \Delta y_k^3 + \Delta y_k^4 + \Delta y_k^5 \tag{26}$$

The failure function of the horizontal deformation displacement of the k node of the supporting wall is

$$g_k = [y] - y_k = [y] - \left(\Delta y_k^1 + \Delta y_k^2 + \Delta y_k^3 + \Delta y_k^4 + \Delta y_k^5 \right) \tag{27}$$

In the formula, $[y]$—the limiting value of the horizontal deformation of the supporting structure of the foundation pit, and can take 0.3%～0.5% of the depth of foundation pit excavation.

The partial derivative of the failure function g_k for the random variable X_i is

$$\frac{\partial g_k}{\partial X_i} = -\left(\frac{\partial \Delta y_k^1}{\partial X_i} + \frac{\partial \Delta y_k^3}{\partial X_i} + \frac{\Delta y_k^5}{\partial X_i} \right) \tag{28}$$

g_k obeys normal distribution

If Δy_k^1, Δy_k^2, Δy_k^3, Δy_k^4 and Δy_k^5 obeys normal distribution, from formula (27), we can know that the failure function g_k obeys normal distribution. And the man of the failure function g_k is

$$\mu_{g_k} = [y] - \sum_{i=1}^{5} \mu_{\Delta y_k^i} \qquad (29)$$

The variance of g_k is

$$\sigma_{g_k}^2 = \sum_{i=1}^{5} \sigma_{\Delta y_k^i}^2 + \sum_{i=1}^{5} \sum_{j=1\, j \neq i}^{5} \mathrm{cov}\left(\Delta y_k^i, \Delta y_k^j\right) \qquad (30)$$

In the formula, $\mathrm{cov}\left(\Delta y_k^i, \Delta y_k^j\right)$—the covairance of random variable Δy_k^i and Δy_k^j.

The reliability indicator β_k of the horizontal deformation displacement of the k node of the supporting wall is

$$\beta_k = \frac{\mu_{g_k}}{\sigma_{g_k}} = \frac{[y] - \sum_{i=1}^{5} \mu_{\Delta y_k^i}}{\sqrt{\sum_{i=1}^{5} \sigma_{\Delta y_k^i}^2 + \sum_{i=1}^{5} \sum_{j=1\, j \neq i}^{5} \mathrm{cov}\left(\Delta y_k^i, \Delta y_k^j\right)}} \qquad (31)$$

g_k is abnormal distribution

When g_k is abnormal distribution, the value of reliability indicator β_k can't be achieved simply from the formula (31). And achieving the value of reliability indicator β_k needs iterative solution.

$X^* = \left(X_1^*, X_2^*, X_3^*, \cdots, X_{2m}^*\right) = \left(c_1^*, \varphi_1^*, c_2^*, \varphi_2^*, \cdots, c_m^*, \varphi_m^*\right)$ is design checking point. The failure function g_k receives Taylor linear expansion at the design checking point X^*,

$$Z_k \approx g_k\left(X_1^*, X_2^*, ..., X_{2m}^*\right) + \sum_{i=1}^{2m} \left(X_i - X_i^*\right) \left(\frac{\partial g_k}{\partial X_i}\right)\Bigg|_{X^*} \qquad (32)$$

The calculation process of the reliability indicator β_k of the horizontal deformation displacement of the k node of the supporting wall is as follows.

(1) The initial value of X^* and β_k is $X^* = \mu_{X_i}$ and $\beta_k = 3$.

(2) Based on Ranking active spoil pressure theory, the load increment q_k^i of each node under working conditions is calculated, and formula (14) is used to figure out $\left.\frac{\partial q_k^i}{\partial X_j}\right|_{X=X^*}$.

(3) Substituting $\left.\frac{\partial q_k^i}{\partial X_j}\right|_{X=X^*}$ into formula (11)～(13) can get $\left.\frac{\partial f_k^i}{\partial X_j}\right|_{X=X^*}$ and $\left.\frac{\partial m_k^i}{\partial X_j}\right|_{X=X^*}$.

(4) Substituting $\left.\frac{\partial f_k^i}{\partial X_j}\right|_{X=X^*}$ and $\left.\frac{\partial m_k^i}{\partial X_j}\right|_{X=X^*}$ into formula (17) and (24) can get $\left.\frac{\partial \Delta y_k^i}{\partial X_j}\right|_{X=X^*}$.

(5) Substituting $\left.\frac{\partial \Delta y_k^i}{\partial X_j}\right|_{X=X^*}$ into formula (28) can get $\left.\frac{\partial g_k}{\partial X_i}\right|_{X=X^*}$.

(6) Substituting $\left.\frac{\partial g_k}{\partial X_i}\right|_{X=X^*}$ into formula (32) can figure out the sensitivity coefficient α_i.

$$\alpha_i = \frac{\sum_{i=1}^{2m} \frac{\partial g_k}{\partial X_i}\Big|_{X^*} \rho_{ij}\sigma_{X_i}}{\left[\sum_{i=1}^{2m}\sum_{j=1}^{2m} \frac{\partial g_k}{\partial X_i}\cdot\frac{\partial g_k}{\partial X_j}\Big|_{X^*} \rho_{ij}\sigma_{X_i}\sigma_{X_j}\right]^{\frac{1}{2}}} \tag{33}$$

In the formula, ρ_{ij} —the correlation coefficient of random variable X_i and X_j.

(7) Substituting α_i and β_k into formula (33) can figure out the new X^*.

$$X_i^* = \mu_{X_i} - \beta_k \alpha_i \sigma_{X_i} \tag{34}$$

(8) According to formula (34), the new β_k is figured out.

$$\beta_k = \frac{g_k\left(X_1^*, X_2^*, \cdots, X_{2m}^*\right) + \sum_{i=1}^{2m}\left(\mu_{X_i} - X_i^*\right)\frac{\partial g_k}{\partial X_i}\Big|_{X^*}}{\sum_{i=1}^{2m} \alpha_i \sigma_{X_i} \frac{\partial g_k}{\partial X_i}\Big|_{X^*}} \tag{35}$$

New X_i^* and β_k is the initial value, the step from (2)～（8）is repeated until the difference of β_k is less than the required calculation accuracy, which can get the final reliability indicator β_k and the design checking point $X^* = \left(c_1^*, \varphi_1^*, c_2^*, \varphi_2^*, \cdots, c_m^*, \varphi_m^*\right)$.

The reliability indicator β of the horizontal deformation displacement of supporting structure of foundation pit is

$$\beta = \min\left\{\beta_1, \beta_2, \beta_2, \cdots, \beta_n\right\} \tag{36}$$

The above calculation process is written into the calculation procedure, which can can realize iteration calculation by computers.

Conclusions

Based on finite element method and load increment method of elastic foundation beam system, the paper uses the computation model of elastic soil pressure,and establishes random finite element computation model of supporting structure deformation analysis under working conditions of multi-pivot piling wall supporting foundation pit. The shearing intensity of soil is selected as random variable to deduct the computation formula of partial derivative of horizontal displacement of supporting structure for random variables. And the paper uses the improves the first-order second-moment method to figure out the reliability indicator of horizontal deformation of foundation pit, which forms analysis method of horizontal deformation of multi-pivot piling wall supporting foundation.

Combined with the engineering projects, the paper analyzes the variable coefficient of shearing strength indicator of soil and the influence of position of soil layer on reliability indicator of horizontal deformation of foundation soil, and the research indicates that

(1) The reliability indicator β of horizontal deformation of foundation pit reduces with the increase of the soil cohesion c and the variance coefficient of internal friction angle φ. The variance coefficient of internal friction angle φ has greater influence on reliability indicator β of deformation of foundation pit, and the variance coefficient of cohesion c has little influence on reliability indicator β.

(2) The place of soil layer in the excavation range of foundation pit has great influence on the reliability indicator β of horizontal deformation of foundation pit, and the influence of the variance coefficient of shearing strength indicator on reliability indicator β of horizontal deformation of

foundation fit is greater than that of the soil layer under excavation face. For the soil above excavation face, the shearing strength indicator of thicker soil layer has greater influence on indicator indicator β of horizontal deformation of foundation pit.

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