Research and Simulation of the Damage from Missile Attack

Xiao Hu\textsuperscript{1, a}, Zhigang Zhang\textsuperscript{2, b}, MengQi Zhang\textsuperscript{3, c}

\textsuperscript{1,2,3}School of Mathematics and Physics, USTB, Beijing 100083, China
\textsuperscript{a}445342098@qq.com, \textsuperscript{b}zzgcyf@263.net, \textsuperscript{c}zhangmengqigood@163.com

Keywords: Non-explosive missile; Damage degree; Monte Carlo simulation

Abstract. In the career of missile, the extent of the damage caused by the missile can effectively determine the resources demand of the battlefield repair, which is necessary to develop repair plan, improve equipment battlefield repair ability, maintain and restore equipment operational capability [1]. In this paper, we mainly study the non-explosive missile attack on the tall building and simulate the flying process as well as damage degree combined with mathematical modeling, probability analysis and Monte Carlo simulation method. The effective simulation by virtual experiment reasonably evaluates the ground-weapon damage caused by missiles and has significant guidance to the battlefield repair.

Introduction

Nowadays, the damage of equipment caused by various types of missile can be broadly divided into two types: one is to be a direct hit; another is not a direct hit [2], and we study the first case. After the missile was fired to high altitude, there will be a certain flight trajectory [3]. This paper simulated the trajectory of the missile. The missile position is set to the origin, hypothesis of this missile from the ground height $h_1$, the distance of target horizontal distance for $S$, Speed $V_x$, $V_y$. And we know that the effect of missile flight process will have environmental factors. In order to consider the role of environmental factors around, we use the random factors $a_x$, $a_y$, the process is shown in Fig. 1. The simple kinematic equations can be established in (1) (2), if there are no random factors, the kinematic equations are as follows in (3) (4):

\[
\begin{align*}
V_x t + \frac{a_x t^2}{2} &= s \\
V_y t + \frac{(a_y + g)t^2}{2} &= h
\end{align*}
\] (1)

\[
\begin{align*}
V_x' &= V_x + a_x t \\
V_y' &= V_y + (a_y + g)t
\end{align*}
\] (2)

\[
\begin{align*}
V_x' t &= S \\
V_y' t + \frac{gt^2}{2} &= h
\end{align*}
\] (3)

\[
\begin{align*}
V_x' &= V_x \\
V_y' &= V_y + gt
\end{align*}
\] (4)

Theoretical analysis

Fig. 1 Simulation chart of missile motion in 2D case

Fig. 2 Missile strikes height building simulation map

Fig. 3 Building simplified side view
For a certain height of the building, from the Fig. 2, we give that the building’s height $H$, fixed missile away from the distance of the target building $s$. From (1) (2) (3) (4), we can easily draw the distance of missile:

$$
\begin{cases}
V_s t + \frac{a_s t^2}{2} = x \\
V_i t + \frac{(a_i + g)t^2}{2} = y
\end{cases}
$$

(5)

Table 1 Degree of damage

<table>
<thead>
<tr>
<th>Layer</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>$\frac{H}{4}$</th>
<th>$\frac{H+1}{4}$</th>
<th>$\frac{H+2}{4}$</th>
<th>...</th>
<th>$\frac{H+4}{4}$</th>
<th>$\frac{H}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>2</td>
<td>4</td>
<td>...</td>
<td>$\frac{H}{2}$</td>
<td>1</td>
<td>2</td>
<td>...</td>
<td>$\frac{H-1}{2}$</td>
<td>$\frac{H}{2}$</td>
</tr>
</tbody>
</table>

From the formulas, the horizontal and vertical moving distance is a function of $a_x, a_y$, so we create two random numbers to meet the normal distribution, which can calculate the $h$ [4]. Then, we can get if $H < h$, there is no damage; if $H \geq h$, the missile will hit the building. Therefore, We assume transmitting the $N$ missiles, of which there are $k$ gold meet $H \geq h$ [5], so we can calculate the damage probability for $p = \frac{k}{N}$ (6). We also can calculate the speed of this is $V' = \sqrt{V_x'^2 + V_y'^2}$ and the kinetic energy, tangle is:

$$
E_k = \frac{m}{2} (V_x'^2 + V_y'^2)
$$

(7)

$$
\alpha = \tan^{-1} \left( \frac{V_y'}{V_x'} \right)
$$

(8)

In order to measure the damage caused by the missile strike, internal structure by the data can guess in the Fig. 3. By guessing, We put the extent of damage as a table, which sets the degree of injury as $H_1$, the injury is shown in Table 1.

The damage degree of buildings caused by missile is not only related to the height but also related to the incident angle and kinetic energy. For kinetic energy, the more velocity of missile is, the more certainly damage degree of building it will be. For the incident angle, by Fig. 4, Fig. 5 shows, the smaller the angle is, the greater injury it will be. Therefore, we set damage degree to $\frac{1}{\alpha}$.

Fig. 4 The range of $\alpha$-angle      Fig. 5 The angle when the missile enters a vertical building

Fig. 6 Flow chart of the simulation experiment under the condition of the height of the building

To sum up, the damage degree of influence factors are defined to three aspects ($H_1, E_k, \frac{1}{\alpha}$), the multiplication can be obtained finally the injured degree measure, defined as $Q$, namely

$$
Q = H_1 \cdot E_k \cdot \frac{1}{\alpha}
$$

(9)

Simulation

In process simulation, we need to confirm statistical placement, placement of the cumulative average and the average damage degree. In the experiment, random factors are considered for mean $0$ variance $\sigma^2$ of the normal distribution, for the general Monte Carlo simulation[6]:

1. Experiment flow

We put the process of the experiment in the Fig. 6.
2. Result and Analysis

(1) Study the relationship between $\varepsilon$ and placement:

Due to random number generation is satisfy the normal distribution $N(0, \varepsilon^2)$, we study different effects of different placement. This paper produce different $\varepsilon$ value to observe the corresponding placement, the final results is shown in Fig. 7 and Fig. 8.

![Fig. 7 $\varepsilon=0.1$ placement map](image)

![Fig. 8 $\varepsilon=0.2$ placement map](image)

From the figure, we can see that different placement plan result in different values of $\varepsilon$. From the graph, the bigger $\varepsilon$ is, the higher curves of the dispersion degree are. The smaller $\varepsilon$ is, the smaller fluctuations are and it has better fitting degree.

(2) Schematic diagram of missile and placement theory

Next, the paper simulated the missile flight of 1000 times ($\varepsilon = 0.1$), we marked building location (yellow box) in Figure. Red line is the theoretical value with simulating 1000 times. The results are shown in Fig. 9.

![Fig. 9 1000 simulation of missile](image)

![Fig. 10 Missile impact point theory](image)

Meanwhile, the simulation respectively hits buildings on two sections (green line) and center point of the theoretical line (red line). At this time, the blue line is very close to the red line. It is proved that the placement of the ideal placement is at the center of the building, as shown in Fig. 10.

(3) Placement distribution under different parameters:

In order to observe the placement objectively and compare actual value with the theoretical value, in this paper, image distribution form of the results is given. Finally, results are shown below: the red points and blue line is value of placement using the value of simulation in the specified interval frequency. The pink line is the mean and the red line is the theoretical value, we can see the result in Fig. 11 and Fig. 12.

![Fig.11 $\varepsilon=0.1$ placement interval frequency diagram](image)

![Fig.12 $\varepsilon=0.2$ placement interval frequency diagram](image)
From the figure, we can see that different parameters are different in the interval of the frequency distribution, the smaller $\varepsilon$ is, and the closer cumulative mean and the theory of value are. It is proved that it is the selection for the final impact point distribution that has a crucial role.

(4) Cumulative mean, theoretical value fluctuations curve chart:

To see the changes in the placement values, we use contrasting method to simulate (1000 times). Final results are shown in the figure below. Pink line is cumulative mean and red line is the theoretical value, as well as the green line is the point average fluctuation curve each time ($\varepsilon=0.1$), we can see the result in Fig. 13.

![Fig. 13 Cumulative mean theoretical value volatility diagram](image)

The picture shows that, in the previous stage, the pink segment is fluctuated, and the former is volatile. The posterior segment tends to be stable, and the final is basic flat.

3. Conclusions

This paper uses Monte Carlo method to simulate and analyze ground weapon damage caused by the missile directly. We obtained the corresponding damage probability and damage degree, and in the course of the study, results of different parameters are analyzed and discussed. Now, in order to have more precise guidance results on the battlefield, the actual analog input is too high. It is necessary to use computer simulation to develop repair plan, improve equipment battlefield repair plan and maintain or restore equipment operational capability.

References


