A Calibration Method for A Linear Structured Light System with Three Collinear Points

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Abstract

A calibration method to calibrate both the camera intrinsic parameters and the structured light projector based on three collinear points with any known distance is presented. First, fix one point of the target and move it in the camera field of view with unknown viewpoint at least six times. Next, use camera to obtain the image of the points so as to identify the camera intrinsic parameters. Then, move the target in the structure light plane at least twice. Using the calibrated camera obtains the images of the three points and adopts the principle of triangulation to work out the coordinates of the calibration points in the camera coordinate system. Finally, employ the random sample consensus algorithm to eliminate the outlier and adopt the genetic algorithm for system identification to find the coefficients of the structured light plane. The average absolute error and the average relative error of the segment measurement with this structured light system are 0.13 mm and 0.04%, respectively. Those show that the measuring accuracy of this system achieves the requirements of the basic error limit. Comparison results show that the measurement accuracy of our method is superior to the other method for the same targets.

Keywords: Structured light plane calibration; Camera calibration; Three collinear points; RANSAC; Genetic algorithm

1. Introduction

Structured light is the projection of a light pattern (like spot, strip, or more complex shape) at a certain angle onto an object. This technology has the advantages of low cost, small size, non-contact, fast speed, and etc., so it is very helpful in imaging and acquiring dimensional information, and is widely used in industrial measurement, three-dimensional (3-D) objects measurement and objects' 3-D reconstruction, automobile vehicle guiding system and other fields.

Structured light combining with cameras can be consisted of the active vision sensing system. We can use this system to complete the intelligent tracking of the automobile vehicles and their positioning with respect to some landmarks rather than using magnetic ruler technique.

According to the light projection pattern of the laser projector, the sensing method of the structured light can be classified as point, line, grid, and point lattice structure. Compared with the point mode, the line mode can gain more information in the case of without increasing the complexity of architecture of the

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structured light system and compared with other modes, the line mode does not need to match the light stripe and its corresponding image. Besides, it can provide enough information but it is less complicated than other modes in the process of its calibration. For these reasons, the utilization of the line structured light in the active vision sensing system in industrial field is getting more and more widespread. One of the important applications of the line structured light projector is to replace one camera of the passive binocular stereo vision system to form the active binocular stereo vision system. So, in this paper, we will use the linear structured light projector to replace one of two cameras to compose our sensing system. This replacement can avoid the limitation of features matching in the two camera images.

As a measuring tool, the calibration of the linear structured light active vision system is critical and methods for its calibration can be classified into two categories: one is two-step calibration, i.e. camera calibration and structured light projector calibration respectively; the other is one-step calibration with some scene points and their correspondence points in images. In this paper, we take the former method. For the camera calibration, according to the dimension of the calibration objects, its calibration techniques can be divided into four types: 3-D reference object-based calibration, two-dimensional (2-D) plane-based calibration, one-dimensional (1-D) object-based calibration and zero-dimensional (0-D) approach (self-calibration). For linear structured light projector, its calibration methods are mainly classified as follows: self-generated target method, jugged target method, cross-ratio invariance of three-dimensional target. These methods need precision calibration auxiliary facilities, the calibration process is complicated, and they cannot be performed in the working field. In order to solve these problems, some methods with the planar objects as the calibration targets are proposed. As for these methods, the calibration targets’ production is simple and they are suitable for field calibration, but these methods need to establish the local coordinate and to carry out the coordinate transformation many times.

In this paper, we present a new calibration method for a linear structured light active vision system with the three collinear points. First, use these three points and adopt the method in Ref. 15 to calibrate the camera’s intrinsic parameters. Then, we use these three points, their perspective view and the principle of triangulation technique to accomplish the simple and fast calibration of the structured light projector. Compared with the linear structured light projector’s calibration method mentioned in Ref. 21, this method does not need to detect the light stripe. That reduces one factor of influencing the system accuracy. Our method just needs to project the laser to the three points and detect the images of them. Our method does not need to establish the light plane coordinate system separately because of its calibration is in the camera coordinate system. So, we also do not need to calibrate the light plane’s position with respect to the camera.

The remainder of this article is structured as follows: Section 2 describes the geometric model of the linear structured light active vision sensing system, containing the camera model and the structured light projector model. Section 3 analyzes the calibration principle of the sensing system, especially analyzes the new proposed structured light projector calibration method in detail. Section 4 provides segment measurement and reconstruction test and the comparison with another calibration method with the same target. Section 5 concludes the paper and gives the future work.

2. Geometric model

A structured light active vision sensing system consists of a camera and a linear structured light projector. This system can realize the depth information (which losing during the camera imaging process) when the camera gains a laser light stripe on an object illuminated by the projector. The geometric model of the linear structured light active vision sensing system is presented in Fig. 1. Here camera is modelled by the usual pinhole and the linear structured light projector is modelled by the space plane.
2.1. Camera model

Camera is described by the pinhole model on condition that its lens’ aberration is insignificant. Camera calibration is to determine the camera model’s parameters, i.e. the pinhole model’s parameters, which contain intrinsic parameters and extrinsic parameters. The intrinsic parameters represent the optical and geometric characteristics of the camera. It is given by

\[
K = \begin{bmatrix}
\alpha & \gamma & u_0 \\
0 & \beta & v_0 \\
0 & 0 & 1
\end{bmatrix}
\] (1)

\(K\) is the transformation matrix between pixel coordinates \(m\) and the normalized coordinates \(M\) of a point. where \(\alpha\) and \(\beta\) are the scale factors in pixel image \(u\) and \(v\) axes, and \(\gamma\) is the parameter describing the skew of the two image axes, and \(u_0, v_0\) are pixels coordinates of the principal point.

The extrinsic parameters represent the position and orientations of the camera relative to a given reference coordinate system. The given reference coordinate system is usually set as world coordinate system. It is given by

\[
T = \begin{bmatrix}
R & t \\
0_{3 \times 3} & 1
\end{bmatrix}
\] (2)

\(R\) and \(t\) describe the orientation and the translation of the camera relative to the world coordinate system, respectively. In this paper, we set \(R = I\) and \(t = 0\), that is to say, we set the camera coordinate system as the reference coordinate. Therefore, the projection from a 3-D point in the camera coordinate system to a 2-D image point \(m = [u,v]\) in the image plane can be given as followings

\[
s\tilde{m} = K\tilde{M}
\] (3)

Where \(s\) is an arbitrary scale factor, \(\tilde{m}\) and \(\tilde{M}\) indicates homogeneous coordinates of the image point \(m\) and the space 3-D point \(M\). According to pinhole imaging principle and the formula (1), for one point, we have

\[
\begin{bmatrix}
u \\
v \\
1
\end{bmatrix} = \begin{bmatrix}
\alpha & \gamma & u_0 \\
0 & \beta & v_0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
X_C \\
Y_C \\
Z_C
\end{bmatrix}
\] (4)

2.2. Linear structured light projector model

The laser projector, unlike the camera, can be described by the pinhole model of perspective projection. It can be seen from Fig. 1 that the light is not exactly illuminated from the optical centre of the projector; it projects a laser beam, which is contained in a plane, so the light beam can be expressed by a space plane equation. In the camera coordinate system, the laser light beam plane is

\[aX_c + bY_c + cZ_c + d = 0\] (5)

Where \(a,b,c,d\) are coefficients of the plane equation. \(a,b,c\) are any real number. It is known that if the sensing system does work the laser light beam cannot pass through the origin of the camera coordinate system, so \(d\) can be chosen as any real number except zero. The projector calibration is to get the plane equation coefficients.

3. Vision System Calibration

In this paper, we consider the calibration of the whole system. We present a calibration method for the structured light sensor system by observing three collinear points. Fig. 2 is the calibration diagram of the structured light sensing system. Our method to calibrate the sensing system is divided into two parts: camera
intrinsic parameters calibration and the structured light projector calibration.

3.1. Camera calibration

In the stage of camera calibration, camera parameters are computed by the method in Ref.15. The camera is fix point A and move the target in the camera field of the view around A. Use the camera to be calibrated to obtain the target’s image at least six times in any orientation.

As shown in Fig. 2, the length of \( AB \) is known to be \( l_1 + l_2 \), we have

\[
AB = l_1 + l_2 \quad (6)
\]

The position of point P is also known with respect to A and B, and, then

\[
P = \frac{l_1}{l_1 + l_2} A + \frac{l_2}{l_1 + l_2} B \quad (7)
\]

According to (4), we have the unknown depths of A, P, and B

\[
\begin{align*}
A &= Z_{ca} K^{-1} a \\
P &= Z_{cp} K^{-1} p \\
B &= Z_{cb} K^{-1} b
\end{align*}
\]

Substituting (8) into (7) yields

\[
Z_{cp} K^{-1} p = \frac{l_1}{l_1 + l_2} Z_{ca} K^{-1} a + \frac{l_1}{l_1 + l_2} Z_{cb} K^{-1} b \quad (9)
\]

Eliminate \( K^{-1} \) from both sides and perform cross product on both side of equation (9) with \( p \), we have

\[
\frac{l_1}{l_1 + l_2} Z_{ca} (a \times p) + \frac{l_1}{l_1 + l_2} Z_{cb} (b \times p) = 0
\]

In turn, we have

\[
Z_{cb} = -Z_{ca} \frac{l_1 (a \times p) - (b \times p)}{l_1 (b \times p) - (b \times p)} \quad (10)
\]

From (6) (8), we have

\[
\left\| K^{-1} (Z_{cb} b - Z_{ca} a) \right\| = l_1 + l_2 \quad (11)
\]

Substituting (10) into (11)

\[
Z_{ca} \left\| K^{-1} (a + Z_{ca} \frac{l_1 (a \times p) - (b \times p)}{l_1 (b \times p) - (b \times p)} b) \right\| = l_1 + l_2 \quad (12)
\]

Then, we have

\[
Z_{ca}^2 h^T K^{-T} K^{-1} h = (l_1 + l_2)^2 \quad (13)
\]

With

\[
h = a + \frac{l_1 (a \times p) - (b \times p)}{l_1 (b \times p) - (b \times p)} b
\]

Expression \( K^{-T} K^{-1} \) contains the intrinsic parameters of the camera. So, figure out the equation (13) and decompose \( K^{-T} K^{-1} \) we can get the intrinsic parameters of the camera. Finally, use the maximum likelihood criterion to refine them. The detailed solution process can refer to the literature 15.

3.2. Linear structured light projector calibration

We use the collinear three points to calibration the laser light beam plane. In this process, the camera and the projector are fixed. Move the target in the light beam plane at least twice. Using the calibrated camera obtain their images. The three image points’ coordinates in the image physical coordinate system \( O - XY \) are \((X_{s}, Y_{s})\), \((X_{p}, Y_{p})\) and \((X_{b}, Y_{b})\). Their coordinate in \( O - XYZ \) are \((X'_{s}, Y'_{s}, f)\), \((X'_{p}, Y'_{p}, f)\) and \((X'_{b}, Y'_{b}, f)\).

Suppose \( \alpha_1 \) and \( \alpha_2 \) are the angles between the vectors \( Oa \) and \( Op \), vectors \( Op \) and \( Ob \), respectively. In the camera coordinate system, from the definition of the vector dot product, we have
A Calibration Method for

\[
\begin{align*}
\alpha_1 &= \arccos\left( \frac{Oa \cdot Op}{\|Oa\| \|Op\|} \right) \\
\alpha_2 &= \arccos\left( \frac{Ob \cdot Op}{\|Ob\| \|Op\|} \right)
\end{align*}
\]  

(14)

\( \alpha_1 \) and \( \alpha_2 \) are also the angles between vectors \( Oa \) and \( Op \), vectors \( Ob \) and \( Ob \).

In order to figure out the three points coordinates in the camera coordinate system, we establish a plane coordinate system \( O-xy \) on the plane \( OAPB \). As Fig. 3 shows, its origin is the origin of the camera coordinate system, and the x axis is coincided with \( Op \), and the y axis is perpendicular to \( Op \).

![Diagram of O-xy coordinate system](image)

Fig. 3. The O-xy coordinate system

The points A, P and B in the coordinate \( O-xy \) are written as \( (x_A, y_A) \), \( (x_p, y_p) \) and \( (x_B, y_B) \). In the similar triangles \( APA \) and \( BPB \), we have

\[
\begin{align*}
\|y_A\| &= \frac{l_1}{l_2} \\
\|y_B\| &= \frac{l_1}{l_2} \\
\|x_p - x_A\| &= \frac{l_1}{l_2} \\
\|x_B - x_p\| &= \frac{l_1}{l_2} \\
(y_B - y_p)^2 + (x_B - x_p)^2 &= l_2^2
\end{align*}
\]  

(15)

(16)

Use the slopes of line \( OA \) and line \( OB \), we have

\[
\begin{align*}
y_A &= x_A \tan \alpha_1 \\
y_B &= x_B \tan \alpha_2
\end{align*}
\]  

(17)

From (15) (16) (17) we can uniquely extract each point coordinate as

\[
\begin{align*}
x_B &= \sqrt{\frac{(l_1^2(1 + l_1) \tan \gamma_2)^2}{(l_1^2 + l_2) \tan \gamma_2 + (l_2 \tan \gamma_2 - l_1 \tan \gamma_2)^2}} \\
y_B &= -x_B \tan \gamma_2 \\
x_A &= \frac{l_1 \tan \gamma_2}{l_2} x_B \\
y_A &= \frac{l_1 \tan \gamma_2}{l_2} y_B \\
x_p &= \frac{l_1 (\tan \gamma_2 + \tan \gamma_2) - x_B}{(l_1 + l_2) \tan \gamma_2} \\
y_p &= 0
\end{align*}
\]

We can compute the distance from the origin of \( O-xy \) coordinate to the three points

\[
\begin{align*}
\|OA\| &= \sqrt{x_A^2 + y_A^2} \\
\|OP\| &= \sqrt{x_p^2 + y_p^2} \\
\|OB\| &= \sqrt{x_B^2 + y_B^2}
\end{align*}
\]

These are also the distances from the origin of the camera coordinate system to the three points. So,

\[
\begin{align*}
D_A &= \|OA\| \\
D_p &= \|OP\| \\
D_B &= \|OB\|
\end{align*}
\]

Therefore, the three points’ coordinates in the camera coordinate system are

\[
\begin{align*}
X_A &= \frac{D_A}{d_a} \bar{X}_a \\
Y_A &= \frac{D_A}{d_a} \bar{Y}_a \\
Z_A &= \frac{D_A}{d_a} \bar{Z}_a \\
X_p &= \frac{D_p}{d_p} \bar{X}_p \\
Y_p &= \frac{D_p}{d_p} \bar{Y}_p \\
Z_p &= \frac{D_p}{d_p} \bar{Z}_p
\end{align*}
\]  

(18a)

(18b)
\[
\begin{align*}
X_b &= \frac{D_a}{d_a} X_a \\
Y_b &= \frac{D_a}{d_a} Y_a \\
Z_b &= \frac{D_a}{d_a} Z_a
\end{align*}
\]
(18c)

Where
\[
\begin{align*}
d_a &= \sqrt{X_a^2 + Y_a^2 + f^2} \\
d_p &= \sqrt{X_p^2 + Y_p^2 + f^2} \\
d_b &= \sqrt{X_b^2 + Y_b^2 + f^2}
\end{align*}
\]

Move the target in the light plane at least twice and use the calibrated camera obtains the image of the target in different locations we can get different points in the light plane, \((X_{am}, Y_{am}, Z_{am})\), \((X_{pm}, Y_{pm}, Z_{pm})\), \((X_{bm}, Y_{bm}, Z_{bm})\), \(m = 1, \ldots, n (n \geq 2)\) is the number of the image.

Use the random sample consensus algorithm (RANSAC) to eliminate the outlier and adopt the least square method to identify the structured light plane equation’s coefficients
\[
\theta = (H^T H)^{-1} H^T D
\]
(19)

Where \(\theta = [a \ b \ c]^T\) is the vector of the coefficients; \(H\) is \(n \times 3\) vector of the points coordinates; \(D\) is \(n \times 1\) vector of the coefficient \(d\), owing to the light plane cannot passing through the origin of the camera coordinate system, so \(d\) can be chosen as any real number except zero.

4. Experimental Results and Precision Analysis

The linear structured light system of our experiment consists of a linear structured light projector and a charge coupled device (CCD) camera DH-HV1300FC whose resolution is 1024*1280. The targets is a homemade three collinear points with the distance \(m_{1} = 300\)mm and \(m_{2} = 150\)mm.

4.1. Experimental results

4.1.1. Camera calibration results

Fix point A, move the target in the camera field of view ten times, using Zhang’s camera calibration method with 1-D objects\(^{15}\), getting camera’s intrinsic parameters’ values. The identified camera intrinsic parameters are shown in Table1.

<table>
<thead>
<tr>
<th>K</th>
<th>2298.711</th>
<th>0.000</th>
<th>689.341</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2288.767</td>
<td>537.903</td>
<td></td>
</tr>
</tbody>
</table>

4.1.2. Projector calibration results

Project the structured light passing through the three points, using the camera to obtain their image. Then move the target nine times, getting ten groups of the points which contain thirty points totally. Then, adopt the Harris corners detection algorithm to detect the image points and get their pixel coordinates and computer their image physical coordinates. The image physical coordinates are shown in Fig. 4.

Fig. 4 The image physical coordinates of calibration points

Then computer their coordinates in the camera coordinate system. Finally, using the RANSAC algorithm to eliminate the outer points, whose loop is 100 000, and the inner point limited in 24, and the threshold of the loop is 0.94 mm which is the distance from the point to plane. Finally, Then use the genetic algorithm to identify the light plane equation coefficients. Set their lower and upper are \([-1.7 -0.16 -0.7]\) and \([-1.6 -0.15 -0.6]\). The population type is double vector and the population size is 3000. The generations are 300 and the stall generations are 300. The light plane equation is
\[-1.6179 X - 0.1551Y - 0.7216 Z + 1000 = 0\]
Its normalization equation is
\[-0.9098X - 0.0864Y - 0.4058Z + 562.3337 = 0\]
The calibration points and the structured light plane in the camera coordinate system are shown in Fig. 5.
4.2. Measurement test

4.2.1. Length measurement

We test the accuracy of this structured light sensing system by measuring the segment EF which is a standard length segment, project the linear structured light on the segment, i.e. the point on the light plane, so

\[-1.6179X_e - 0.1551Y_e - 0.7216Z_e + 1000 = 0 \quad (20)\]
\[-1.6179X_f - 0.1551Y_f - 0.7216Z_f + 1000 = 0 \quad (21)\]

From triangulation technique, we have

\[
\begin{align*}
X_e &= \frac{X}{J}Z_e \\
Y_e &= \frac{Y}{J}Z_e \\
X_f &= \frac{X}{J}Z_f \\
Y_f &= \frac{Y}{J}Z_f
\end{align*}
\]

From the above four equations, we can get the point coordinates \( E(X_e, Y_e, Z_e) \), \( F(X_f, Y_f, Z_f) \) in the camera coordinate system, so the length of EF is

\[
EF = \sqrt{(X_e - X_f)^2 + (Y_e - Y_f)^2 + (Z_e - Z_f)^2} \quad (24)
\]

Take three standard length segments as test objects. Their lengths are 150mm, 300m, and 450mm. Their range of motion is about from 900mm to 1300mm, which is the distance from the camera’s optical centre to the test segments. The distance from the camera’s optical centre to the laser projector is about 620mm. The measurement results are shown in Table 2. The experimental results indicate that the average absolute error and the average relative error of this structured light system are 0.13mm and 0.04%, respectively.

<table>
<thead>
<tr>
<th>Standard length</th>
<th>Experimental length</th>
<th>Absolute error</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>150.02</td>
<td>0.02</td>
<td>0.01%</td>
</tr>
<tr>
<td>150</td>
<td>150.04</td>
<td>0.04</td>
<td>0.03%</td>
</tr>
<tr>
<td>300</td>
<td>300.12</td>
<td>0.12</td>
<td>0.04%</td>
</tr>
<tr>
<td>300</td>
<td>299.86</td>
<td>0.14</td>
<td>0.05%</td>
</tr>
<tr>
<td>450</td>
<td>450.40</td>
<td>0.40</td>
<td>0.09%</td>
</tr>
<tr>
<td>450</td>
<td>450.03</td>
<td>0.03</td>
<td>0.01%</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>0.13</td>
<td>0.04%</td>
</tr>
</tbody>
</table>

4.2.2. Segment reconstruction

The segment is to be reconstructed shows in Fig. 6. Project the structured light onto the segment and use the calibrated camera gets one picture of it.

![Segment to be reconstructed](image1)

Then use image binaryization and image thinning image processing algorithms, we get 776 pixel points of the segment and the binary image of the segment. The binary image is shown as Fig. 7.

![The binary image of segment](image2)
Use the triangulation principle to compute the 776 points coordinates in camera coordinate system, and then reconstruct the segment. The reconstructed segment is shown in Fig. 8.

![Reconstructed Segment Image](image)

**Fig. 8. The reconstructed image of segment**

We can see from Fig. 8 that the segment is composed of a series of discrete pixels points, so the reconstructed segment shown in Fig. 8 is formed by many little segments between two adjacent pixels points. Owing to the impact of the two points at segment’s both ends, it’s both ends seem a bit bent.

**4.3. Comparison experiment**

We compare the accuracy of the structured light system with another structured light system in literature 21, which is also use the collinear three points as the calibration target to calibration the structured light plane. Take the same segment length as literature 21’s, the test segment lengths are 20mm, 40mm and 60mm. The measurement results by their calibrated system are shown in Table 3.

<table>
<thead>
<tr>
<th>Standard length</th>
<th>Experimental length</th>
<th>Absolute error</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>20.18</td>
<td>0.18</td>
<td>0.90%</td>
</tr>
<tr>
<td>20</td>
<td>20.15</td>
<td>0.15</td>
<td>0.75%</td>
</tr>
<tr>
<td>20</td>
<td>20.12</td>
<td>0.12</td>
<td>0.60%</td>
</tr>
<tr>
<td>40</td>
<td>40.25</td>
<td>0.25</td>
<td>0.63%</td>
</tr>
<tr>
<td>40</td>
<td>40.31</td>
<td>0.31</td>
<td>0.78%</td>
</tr>
<tr>
<td>60</td>
<td>60.39</td>
<td>0.39</td>
<td>0.65%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td><strong>0.23</strong></td>
<td><strong>0.72%</strong></td>
</tr>
</tbody>
</table>

From the data, we can see the measurement average absolute error and the average relative error by the method proposed in Ref. 21 are 0.23mm and 0.72%, respectively. However, the measurement average absolute error and the average relative error by our method are 0.03mm and 0.08%, respectively. The experimental results indicate that the accuracy of our sensing system is better than that of the linear structured light sensing system referred in Ref. 21.

Analyze measurement process of this linear structured light active vision system, and we can obtain the factors that influence the system’s accuracy as follows:
- the thickness of the structured light beam,
- the coincidence of the structured light and the centroids of the three collinear points,
- extraction accuracy of the three collinear points,
- the calibration accuracy of the camera intrinsic parameters,
- Whether the lens distortion is taken into account (in our method the lens distortion is not being taken into account).

**5. Conclusions**

This paper described a linear structured light active vision sensing system calibration method using the collinear three points. We proposed a new approach based on three collinear points to calibrate the light plane, which is computed in the camera coordinate system. Furthermore, we test and compare the accuracy of our sensing system by measuring standard segments in different location in the field of the camera view. The results show that this system can achieve the requirement of the basic error limit of the measurement tools which smallest measurement unit is millimeter and
its accuracy is better than the similar linear structured light sensing system calibrated by another method. In the future, we will fully give thought to the factors that affect the system accuracy to further improve the accuracy of the system.

Acknowledgements

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