International Journal of Computational Intelligence Systems, Vol. 4, No. 5 (September, 2011), 855-862

$\mathscr{H}_\infty$ State Estimation for Takagi-Sugeno Fuzzy Delayed Hopfield Neural Networks

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Received 19 April 2010
Accepted 17 June 2011

Abstract

This paper presents a new $\mathscr{H}_\infty$ state estimator for Takagi-Sugeno fuzzy delayed Hopfield neural networks. Based on Lyapunov-Krasovskii stability approach, a delay-dependent criterion is proposed to ensure that the resulting estimation error system is asymptotically stable with a guaranteed $\mathscr{H}_\infty$ performance. The proposed $\mathscr{H}_\infty$ state estimator can be realized by solving a linear matrix inequality (LMI) problem. An illustrative numerical example is given to verify the effectiveness of the proposed $\mathscr{H}_\infty$ state estimator.

Keywords: $\mathscr{H}_\infty$ state estimation, Takagi-Sugeno fuzzy Hopfield neural networks, linear matrix inequality (LMI), Lyapunov-Krasovskii stability theory

1. Introduction

Neural networks have been intensively studied in the past decade and have found several application in areas such as signal processing, static image processing, and pattern recognition. It has been shown that these applications depend on the stability and performance of neural networks. Among various neural networks, Hopfield neural networks [1] have been extensively studied and successfully applied in combinatorial optimization, signal processing, and pattern recognition [2].

Fuzzy logic method has proven to be an efficient approach to treat the analysis and synthesis problems for complex nonlinear systems. Among several fuzzy methods, Takagi-Sugeno (T-S) fuzzy models provide a useful method to describe complex nonlinear systems using local linear subsystems [3,4]. These local linear subsystems are smoothly blended through fuzzy membership functions. Recently, the T-S fuzzy models are used to express some complex nonlinear systems by having a set of delayed Hopfield neural networks as its consequent parts. Since the T-S fuzzy models have the outstanding approximation ability, T-S fuzzy delayed Hopfield neural networks [6,7,8,9,10] are recently recognized as an appealing tool in approximating nonlinear systems. Stability problems for T-S fuzzy delayed Hopfield neural networks have been investigated in [6,7,8,9,10].

On the other hand, the states of neural networks are not often completely available in the outputs of neural networks in many applications. Thus, it is important to estimate the states of neural networks through measurements in order to make use of neural networks in practical applications. Wang and his colleagues obtained a delay-independent criterion for state estimator design of delayed neural networks in [11]. The authors in [12] investigated a state estimator of delayed neural networks and proposed a delay-dependent condition such that the estimation error system is asymptotically stable. Recently, in [13], the state estimation problem for a class of neural networks with discrete and distributed delays was further considered.

In physical systems, there exist model uncertainties and system noises. It is known that $\mathscr{H}_\infty$ approach is robust to disturbance variances and model uncertainties [14]. Analysis and synthesis in an $\mathscr{H}_\infty$ framework have some good advantages such as effective disturbance attenuation and less sensitivity to uncertainties. Huang and Feng proposed an $\mathscr{H}_\infty$ state estimation method for delayed neural networks [15]. Can we obtain an $\mathscr{H}_\infty$ state estimator for T-S fuzzy delayed Hopfield neural networks? This paper answers this interesting question. To the best of our knowledge, for the $\mathscr{H}_\infty$ state estimation of T-S fuzzy delayed Hopfield neural networks, there is no result in the literature so far.

In this paper, we propose a new $\mathscr{H}_\infty$ state estimator for T-S fuzzy delayed Hopfield neural networks. This state estimation method is a new contribution to the topic of recurrent neural networks. The proposed scheme ensures that the estimation error system is asymptotically stable and the $\mathscr{H}_\infty$ norm from the external disturbance to the estimation error is reduced to a predefined level of disturbance attenuation. Based on Lyapunov-Krasovskii stability approach, a sufficient existence condition for the proposed $\mathscr{H}_\infty$ state estimator is represented in terms of linear matrix inequality (LMI). The LMI problem can be easily solved by using standard convex optimization algorithms [16].

This paper is organized as follows. In Section 2, we formulate the problem. In Section 3, an LMI problem for the $\mathscr{H}_\infty$ state estimation of T-S fuzzy delayed Hopfield neural networks is proposed. In Section 4, a numerical example is given, and finally, conclusions are presented in Section 5.

2. Problem Formulation

Consider the following delayed Hopfield neural network:

$$\dot{x}(t) = Ax(t) + W\phi(x(t-\tau)) + J(t) + Gd(t), \quad (1)$$

$$y(t) = Cx(t) + Dx(t-\tau) + Hd(t), \quad (2)$$

where $x(t) = [x_1(t) \ \dots \ x_n(t)]^T \in R^n$ is the state vector, $y(t) = [y_1(t) \ \dots \ y_m(t)]^T \in R^m$ is the output vector, $d(t) = [d_1(t) \ \dots \ d_k(t)]^T \in R^k$ is the disturbance vector, $\tau \geqslant 0$ is the time-delay, $A = diag\{-a_1,\dots,-a_n\} \in R^{n\times n}$ $(a_k > 0, k = 1,\dots,n)$ is the self-feedback matrix, $W \in R^{n\times n}$ is the delayed connection weight matrix, $\phi(x(t)) = [\phi_1(x(t)) \ \dots \ \phi_n(x(t))]^T : R^n \to R^n$ is the nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant $L_\phi > 0$, $G \in R^{n\times k}$, $C \in R^{m\times n}$, $D \in R^{m\times n}$, and $H \in R^{m\times k}$ are known constant matrices, and $J(t) \in R^n$ is an external input vector.

Based on the T-S fuzzy model concept, a general class of T-S fuzzy delayed Hopfield neural networks [6,7,8,9,10] is considered here. The model of T-S fuzzy delayed Hopfield neural networks is described as follows:

Fuzzy Rule i:

IF ω_1 is μ_{i1} and $\dots$ ω_s is μ_{is} **THEN**

$$\dot{x}(t) = A_i x(t) + W_i\phi(x(t-\tau)) + J_i(t) + G_i d(t), \quad (3)$$

$$y(t) = C_i x(t) + D_i x(t-\tau) + H_i d(t), \quad (4)$$

where ω_j $(j = 1,\dots,s)$ is the premise variable, μ_{ij} $(i = 1,\dots,r, j = 1,\dots,s)$ is the fuzzy set that is characterized by membership function, r is the number of the IF-THEN rules, and s is the number of the premise variables. Using a standard fuzzy inference method, the system (3)-(4) is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\omega)[A_i x(t) + W_i\phi(x(t-\tau)) + J_i(t) + G_i d(t)], \quad (5)$$

$$y(t) = \sum_{i=1}^{r} h_i(\omega)[C_i x(t) + D_i x(t-\tau) + H_i d(t)], \quad (6)$$

where $\omega = [\omega_1,\dots,\omega_s]$, $h_i(\omega) = w_i(\omega)/\sum_{j=1}^{r} w_j(\omega)$, $w_i : R^s \to [0,1]$ $(i = 1,\dots,r)$ is the membership function of the system with respect to the fuzzy rule i. h_i can be regarded as the normalized weight of each IF-THEN rule and it satisfies

$$h_i(\omega) \geqslant 0, \qquad \sum_{i=1}^{r} h_i(\omega) = 1. \quad (7)$$

For the T-S fuzzy delayed Hopfield neural network (3)-(4), we design the following full-order state estimator:

Fuzzy Rule i:

IF ω_1 is μ_{i1} and $\ldots$ ω_s is μ_{is} **THEN**

$$\dot{\hat{x}}(t) = A_i\hat{x}(t) + W_i\phi(\hat{x}(t-\tau)) + J_i(t) + L(y(t) - \hat{y}(t)), \quad (8)$$

$$\hat{y}(t) = C_i\hat{x}(t) + D_i\hat{x}(t-\tau), \quad (9)$$

where $\hat{x}(t) = [\hat{x}_1(t) \ldots \hat{x}_n(t)]^T \in R^n$ is the state vector of the state estimator, $\hat{y}(t) = [\hat{y}_1(t) \ldots \hat{y}_m(t)]^T \in R^m$ is the output vector of the state estimator, and $L \in R^{n\times m}$ is the gain matrix of the state estimator to be designed. Using a standard fuzzy inference method, the state estimator (8)-(9) is inferred as follows:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\omega)[A_i\hat{x}(t) + W_i\phi(\hat{x}(t-\tau)) + J_i(t) + L(y(t) - \hat{y}(t))], \quad (10)$$

$$\hat{y}(t) = \sum_{i=1}^{r} h_i(\omega)[C_i\hat{x}(t) + D_i\hat{x}(t-\tau)]. \quad (11)$$

Define the estimation error $e(t) = x(t) - \hat{x}(t)$. Then, the estimation error system can be represented as follows:

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(\omega)\Big\{(A_i - LC_i)e(t) - LD_ie(t-\tau) + W_i\phi(x(t-\tau)) - W_i\phi(\hat{x}(t-\tau)) + (G_i - LH_i)d(t)\Big\}. \quad (12)$$

Before stating the main objectives of this paper, the following definitions are introduced.

Definition 1. (Asymptotical state estimation) The state estimator (10)-(11) is an asymptotical state estimator if the estimation error $e(t)$ satisfies

$$\lim_{t\to\infty} e(t) = 0. \quad (13)$$

Definition 2. ($\mathscr{H}_\infty$ state estimation) The state estimator (10)-(11) is an $\mathscr{H}_\infty$ state estimator if the estimation error $e(t)$ satisfies

$$\int_0^\infty e^T(t)Se(t)dt < \gamma^2 \int_0^\infty d^T(t)d(t)dt, \quad (14)$$

for a given level $\gamma > 0$ under zero initial conditions, where S is a positive symmetric matrix. The parameter γ is called the $\mathscr{H}_\infty$ norm bound or the disturbance attenuation level.

Remark 1 *The $\mathscr{H}_\infty$ norm [14] is defined as*

$$\|T_{ed}\|_\infty = \frac{\sqrt{\int_0^\infty e^T(t)Se(t)dt}}{\sqrt{\int_0^\infty d^T(t)d(t)dt}}$$

where T_{ed} is a transfer function matrix from $d(t)$ to $e(t)$. For a given level $\gamma > 0$, $\|T_{ed}\|_\infty < \gamma$ can be restated in the equivalent form (14). If we define

$$H(t) = \frac{\int_0^t e^T(\sigma)Se(\sigma)d\sigma}{\int_0^t d^T(\sigma)d(\sigma)d\sigma}, \quad (15)$$

the relation (14) can be represented by

$$H(\infty) < \gamma^2. \quad (16)$$

In Section 4, through the plot of $H(t)$ versus time, the relation (16) is verified.

This paper designs a state estimator of the form (10)-(11) for T-S fuzzy delayed Hopfield neural networks with a guaranteed performance in the $\mathscr{H}_\infty$ sense if there exists the disturbance $d(t)$. We also will show that this state estimator ensures the asymptotical state estimation when the disturbance $d(t)$ disappears.

3. $\mathscr{H}_\infty$ State Estimator Design

In this section, we design an $\mathscr{H}_\infty$ state estimator for T-S fuzzy delayed Hopfield neural networks. The following theorem presents an LMI-based criterion to obtain the $\mathscr{H}_\infty$ state estimator.

Theorem 1 *For given $\gamma > 0$ and $S = S^T > 0$, assume that there exist common matrices $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $U = U^T > 0$, and M such that*

$$\begin{bmatrix} [1,1] & -MD_i & U & PG_i - MH_i \\ -D_i^TM^T & -R & -U & 0 \\ U & -U & -\frac{1}{\tau}Q & 0 \\ (PG_i - MH_i)^T & 0 & 0 & -\gamma^2 I \\ 0 & I & 0 & 0 \\ I & 0 & 0 & 0 \\ W_i^TP & 0 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{array}{ccc} 0 & I & PW_i \\ I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{1}{L_\phi^2}I & 0 & 0 \\ 0 & -S^{-1} & 0 \\ 0 & 0 & -I \end{array}\right] < 0, \qquad (17)$$

for $i = 1, \ldots, r$, where

$$[1,1] = (PA_i - MC_i)^T + (PA_i - MC_i) + \tau Q + R.$$

Then the $\mathscr{H}_\infty$ state estimation for T-S fuzzy delayed Hopfield neural networks with the disturbance attenuation level γ is achieved. Moreover, the gain matrix of the state estimator (10)-(11) is given by

$$L = P^{-1}M. \qquad (18)$$

Proof. Consider the following Lyapunov-Krasovskii functional

$$\begin{aligned} V(t) &= e^T(t)Pe(t) + \int_{-\tau}^{0}\int_{t+\beta}^{t} e^T(\alpha)Qe(\alpha)d\alpha d\beta \\ &\quad + \int_{-\tau}^{0} e^T(t+\sigma)Re(t+\sigma)d\sigma \\ &\quad + \left[\int_{-\tau}^{0} e(t+\sigma)d\sigma\right]^T U \left[\int_{-\tau}^{0} e(t+\sigma)d\sigma\right]. \end{aligned} \qquad (19)$$

Its time derivative along the trajectory of (12) is

$$\begin{aligned} \dot{V}(t) &= \dot{e}(t)^T Pe(t) + e^T(t)P\dot{e}(t) + \tau e^T(t)Qe(t) \\ &\quad - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)d\sigma + e(t)^T Re(t) \\ &\quad - e^T(t-\tau)Re(t-\tau) + [e(t) - e(t-\tau)]^T U \\ &\quad \times \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T U \\ &\quad \times [e(t) - e(t-\tau)] \\ &= \sum_{i=1}^{r} h_i(\omega)\Big\{ e^T(t)[(A_i - LC_i)^T P + P(A_i - LC_i)] \\ &\quad \times e(t) - e^T(t)PLD_i e(t-\tau) - e^T(t-\tau)D_i^T L^T \\ &\quad \times Pe(t) + e^T(t)PW_i(\phi(x(t-\tau)) - \phi(\widehat{x}(t-\tau))) \\ &\quad + (\phi(x(t-\tau)) - \phi(\widehat{x}(t-\tau)))^T W_i^T Pe(t) \\ &\quad + e(t)^T P(G_i - LH_i)d(t) + d^T(t)(G_i - LH_i)^T \\ &\quad \times Pe(t)\Big\} + \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)d\sigma \\ &\quad + e(t)^T Re(t) - e^T(t-\tau)Re(t-\tau) + [e(t) \\ &\quad - e(t-\tau)]^T U \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T \\ &\quad \times U[e(t) - e(t-\tau)]. \end{aligned} \qquad (20)$$

If we use the inequality $X^TY + Y^TX \leqslant X^T\Lambda X + Y^T\Lambda^{-1}Y$, which is valid for any matrices $X \in R^{n\times m}$, $Y \in R^{n\times m}$, $\Lambda = \Lambda^T > 0$, $\Lambda \in R^{n\times n}$, we have

$$\begin{aligned} & e^T(t)PW_i(\phi(x(t-\tau)) - \phi(\widehat{x}(t-\tau))) \\ & + (\phi(x(t-\tau)) - \phi(\widehat{x}(t-\tau)))^T W_i^T Pe(t) \\ & \leqslant (\phi(x(t-\tau)) - \phi(\widehat{x}(t-\tau)))^T(\phi(x(t-\tau)) \\ & - \phi(\widehat{x}(t-\tau))) + e^T(t)PW_iW_i^T Pe(t) \\ & \leqslant L_\phi^2(x(t-\tau) - \widehat{x}(t-\tau))^T(x(t-\tau) - \widehat{x}(t-\tau)) \\ & + e^T(t)PW_iW_i^T Pe(t) \\ & = L_\phi^2 e^T(t-\tau)e(t-\tau) + e^T(t)PW_iW_i^T Pe(t). \end{aligned} \qquad (21)$$

Using (21), we obtain

$$\begin{aligned} \dot{V}(t) &\leqslant \sum_{i=1}^{r} h_i(\omega)\Big\{ e^T(t)[(A_i - LC_i)^T P + P(A_i - LC_i) \\ &\quad + PW_iW_i^T P]e(t) - e^T(t)PLD_i e(t-\tau) - e^T(t-\tau) \\ &\quad \times D_i^T L^T Pe(t) + L_\phi^2 e^T(t-\tau)e(t-\tau) + e(t)^T P \\ &\quad \times (G_i - LH_i)d(t) + d^T(t)(G_i - LH_i)^T Pe(t)\Big\} \\ &\quad + \tau e^T(t)Qe(t) - \int_{t-\tau}^{t} e^T(\sigma)Qe(\sigma)d\sigma + e(t)^T R \\ &\quad \times e(t) - e^T(t-\tau)Re(t-\tau) + [e(t) - e(t-\tau)]^T \\ &\quad \times U\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] + \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T U[e(t) \\ &\quad - e(t-\tau)]. \end{aligned} \qquad (22)$$

Using Jensen's inequality [17]

$$\begin{aligned} & \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T Q \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] \\ & \leqslant \tau \int_{t-\tau}^{t} e(\sigma)^T Qe(\sigma)d\sigma, \end{aligned} \qquad (23)$$

we have

$$
\begin{aligned}
\dot{V}(t) \leqslant \sum_{i=1}^{r} h_i(\omega)\Big\{ & e^T(t)[(A_i-LC_i)^T P + P(A_i-LC_i) \\
& + PW_iW_i^T P]e(t) - e^T(t)PLD_ie(t-\tau) - e^T(t-\tau) \\
& \times D_i^T L^T Pe(t) + L_\phi^2 e^T(t-\tau)e(t-\tau) + e(t)^T P \\
& \times (G_i-LH_i)d(t) + d^T(t)(G_i-LH_i)^T Pe(t)\Big\} \\
& + \tau e^T(t)Qe(t) - \frac{1}{\tau}\left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T Q \\
& \times \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] + e(t)^T Re(t) - e^T(t-\tau)R \\
& \times e(t-\tau) + [e(t)-e(t-\tau)]^T U \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right] \\
& + \left[\int_{t-\tau}^{t} e(\sigma)d\sigma\right]^T U[e(t)-e(t-\tau)] \\
= \sum_{i=1}^{r} h_i(\omega) & \left\{ \begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)d\sigma \\ d(t) \end{bmatrix}^T \right. \\
& \times \begin{bmatrix} (1,1) & -PLD_i & U & P(G_i-LH_i) \\ -D_i^T L^T P & (2,2) & -U & 0 \\ U & -U & -\frac{1}{\tau}Q & 0 \\ (G_i-LH_i)^T P & 0 & 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau) \\ \int_{t-\tau}^{t} e(\sigma)d\sigma \\ d(t) \end{bmatrix} \\
& \left. - e^T(t)Se(t) + \gamma^2 d^T(t)d(t) \right\}, \qquad (24)
\end{aligned}
$$

where

$$
\begin{aligned}
(1,1) &= (A_i-LC_i)^T P + P(A_i-LC_i) + PW_iW_i^T P \\
&\quad + S + \tau Q + R, \\
(2,2) &= L_\phi^2 I - R.
\end{aligned}
$$

If the following matrix inequality is satisfied

$$
\begin{bmatrix} (1,1) & -PLD_i & U & P(G_i-LH_i) \\ -D_i^T L^T P & (2,2) & -U & 0 \\ U & -U & -\frac{1}{\tau}Q & 0 \\ (G_i-LH_i)^T P & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \qquad (25)
$$

for $i = 1, \ldots, r$, we have

$$
\begin{aligned}
\dot{V}(t) &< \sum_{i=1}^{r} h_i(\omega)\{-e^T(t)Se(t) + \gamma^2 d^T(t)d(t)\} \\
&= -e^T(t)Se(t) + \gamma^2 d^T(t)d(t). \qquad (26)
\end{aligned}
$$

Integrating both sides of (26) from 0 to ∞ gives

$$
\begin{aligned}
& V(\infty) - V(0) \\
& < -\int_0^\infty e^T(t)Se(t)dt + \gamma^2 \int_0^\infty d^T(t)d(t)dt.
\end{aligned}
$$

Since $V(\infty) \geqslant 0$ and $V(0) = 0$, we have the relation (14). From Schur complement, the matrix inequality (25) is equivalent to

$$
\begin{bmatrix}
\{1,1\} & -PLD_i & U & P(G_i-LH_i) & 0 & I & PW_i \\
-D_i^T L^T P & -R & -U & 0 & I & 0 & 0 \\
U & -U & -\frac{1}{\tau}Q & 0 & 0 & 0 & 0 \\
(G_i-LH_i)^T P & 0 & 0 & -\gamma^2 I & 0 & 0 & 0 \\
0 & I & 0 & 0 & -\frac{1}{L_\phi^2}I & 0 & 0 \\
I & 0 & 0 & 0 & 0 & -S^{-1} & 0 \\
W_i^T P & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix} < 0, \qquad (27)
$$

where

$$
\{1,1\} = (A_i-LC_i)^T P + P(A_i-LC_i) + \tau Q + R.
$$

If we let $M = PL$, (27) is equivalently changed into the LMI (17). Then the gain matrix of the state estimator is given by $L = P^{-1}M$. This completes the proof. ∎

Corollary 1 *Without the external disturbance, if we use the state estimator (10)-(11) with the gain matrix L (18), the asymptotical state estimation for T-S fuzzy delayed Hopfield neural networks is guaranteed.*

Proof. When $d(t) = 0$, we obtain

$$\dot{V}(t) < -e^T(t)Se(t) \leqslant 0 \tag{28}$$

from (26). This guarantees

$$\lim_{t\to\infty} e(t) = 0 \tag{29}$$

from Lyapunov-Krasovskii stability theory. This completes the proof. ∎

Based on Theorem 1, the optimal $\mathscr{H}_\infty$ norm bound for the proposed state estimator is obtained.

Corollary 2 *For a given $S > 0$, the optimal $\mathscr{H}_\infty$ norm bound γ is obtained by solving the following semi-definite programming problem:*

$$\min_{\gamma>0} \gamma^2 \tag{30}$$

subject to the LMI (17), $P > 0$, $Q > 0$, $R > 0$, and $U > 0$.

Remark 2 *The zero-initial condition is for $e(t)$. Since the transfer function matrix T_{ed} is defined under zero-initial condition, there exists no the initial state error $e(0)$ in the definition of the $\mathscr{H}_\infty$ norm. However, since there exists the initial state error $e(0)$ in real problems, the performance criterion (14) can be slightly modified into*

$$\begin{aligned}
&\int_0^\infty e^T(t)Se(t)dt \\
&< \gamma^2 \int_0^\infty d^T(t)d(t)dt + e^T(0)Pe(0) \\
&+ \int_{-\tau}^0 \int_\beta^0 e^T(\alpha)Qe(\alpha)d\alpha d\beta \\
&+ \int_{-\tau}^0 e^T(\sigma)Re(\sigma)d\sigma \\
&+ \left[\int_{-\tau}^0 e(\sigma)d\sigma\right]^T U \left[\int_{-\tau}^0 e(\sigma)d\sigma\right].
\end{aligned}$$

In this case, we can easily obtain Theorem 1.

Remark 3 *The LMI problem given in Theorem 1 is called the feasibility problem. In addition, the solution of the problem in Corollary 2 can be obtained by solving eigenvalue problem in γ, which is a convex optimization problem. Many LMI problems can be solved efficiently by using standard convex optimization algorithms [16]. In order to solve the LMI problems, this paper utilized MATLAB LMI Control Toolbox [18].*

4. Numerical Example

Consider the following T-S fuzzy delayed Hopfield neural network:

Fuzzy Rule 1:

IF ω_1 is μ_{11} and $\ldots$ ω_s is μ_{1s} **THEN**

$$\begin{aligned}
\dot{x}(t) &= A_1x(t) + W_1\phi(x(t-1)) + J_1(t) \\
&\quad + G_1d(t), &(31)\\
y(t) &= C_1x(t) + D_1x(t-1) + H_1d(t), &(32)
\end{aligned}$$

Fuzzy Rule 2:

IF ω_1 is μ_{21} and $\ldots$ ω_s is μ_{2s} **THEN**

$$\begin{aligned}
\dot{x}(t) &= A_2x(t) + W_2\phi(x(t-1)) + J_2(t) \\
&\quad + G_2d(t), &(33)\\
y(t) &= C_2x(t) + D_2x(t-1) + H_2d(t), &(34)
\end{aligned}$$

where

$$\begin{aligned}
x(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix}, \\
\phi(x(t)) &= \begin{bmatrix} \frac{1}{1+e^{-x_1(t)}} \\ \frac{1}{1+e^{-x_2(t)}} \end{bmatrix}, \\
A_1 &= \begin{bmatrix} -3 & 0 \\ 0 & -2.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -4.2 & 0 \\ 0 & -3.5 \end{bmatrix}, \\
G_1 &= \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 1 & 0 \\ 0.8 & 1 \end{bmatrix}, \\
W_1 &= \begin{bmatrix} -1 & 0.4 \\ 0 & -0.1 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 1 & -0.8 \\ 0.4 & 0.5 \end{bmatrix}, \\
J_1(t) &= \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}, \quad J_2(t) = \begin{bmatrix} -\cos(t) \\ \sin(2t) \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \\
D_1 &= \begin{bmatrix} 0.5 & 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -1 & 0.3 \end{bmatrix}, \\
H_1 &= \begin{bmatrix} -1 & 0.4 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.1 & -0.3 \end{bmatrix}.
\end{aligned}$$

The fuzzy membership functions are taken as $h_1(\omega) = \sin^2(x_1(t))$ and $h_2(\omega) = \cos^2(x_1(t))$. For the design objective (14), let the $\mathscr{H}_\infty$ performance be specified by $\gamma = 0.75$ with $S = I$, where $I \in R^{2\times 2}$ is an identity matrix. If we solve the LMI (17) by the convex optimization technique of MATLAB, we have

$$P = \begin{bmatrix} 1.0591 & -0.4367 \\ -0.4367 & 1.4127 \end{bmatrix},$$
$$M = \begin{bmatrix} -0.0672 \\ 0.1233 \end{bmatrix}.$$

Figure 1 shows the plot of $H(t)$ versus time when $d_1(t) = \cos(20t)$ and $d_2(t) = \sin(10t)$. This figure verifies $H(\infty) < \gamma^2 = 0.5625$, which means that the $\mathscr{H}_\infty$ norm from the external disturbance $d(t)$ to the estimation error $e(t)$ is reduced within the $\mathscr{H}_\infty$ norm bound γ. When the initial conditions are given by

$$x(0) = \begin{bmatrix} 1.5 \\ -1 \end{bmatrix}, \qquad \widehat{x}(0) = \begin{bmatrix} -2 \\ 1.3 \end{bmatrix}, \qquad (35)$$

and the external disturbance $d_i(t)$ $(i = 1,2)$ is given by a Gaussian noise with mean 0 and variance 1, the simulation results for the $\mathscr{H}_\infty$ state estimator design are shown in Figures 2-4. Figures 2 and 3 show the true states $x_1(t)$ and $x_2(t)$ and their estimations $\widehat{x}_1(t)$ and $\widehat{x}_2(t)$, respectively. Figure 4 shows the responses of the estimation error $e(t)$. These simulation results demonstrate that the proposed estimator reduces the effect of the external disturbance $d(t)$ on the estimation error $e(t)$.

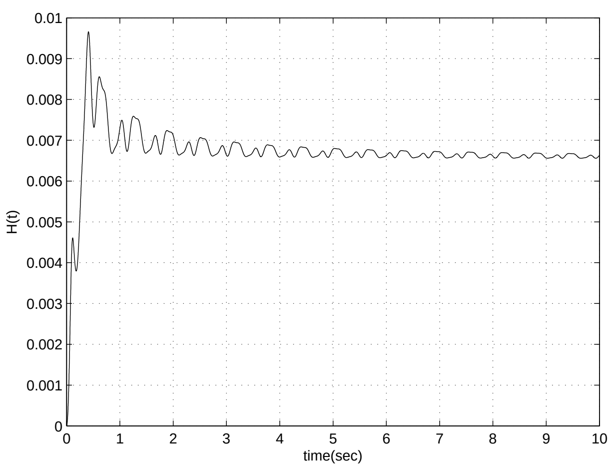

Fig. 1. The plot of $H(t)$ versus time

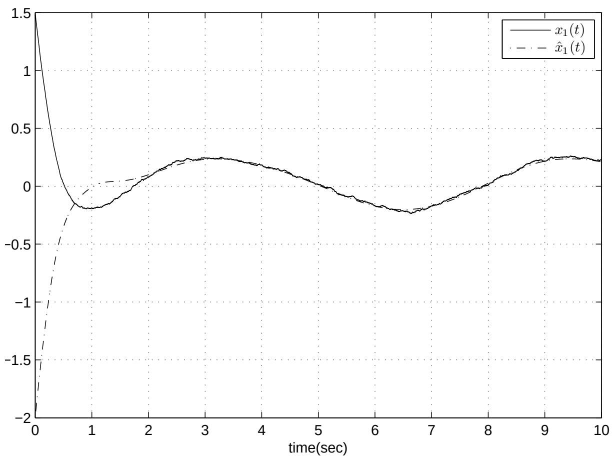

Fig. 2. Responses of the state $x_1(t)$ and its estimation $\widehat{x}_1(t)$

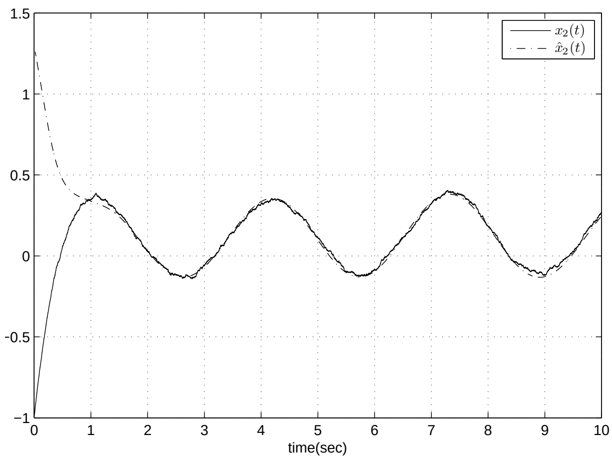

Fig. 3. Responses of the state $x_2(t)$ and its estimation $\widehat{x}_2(t)$

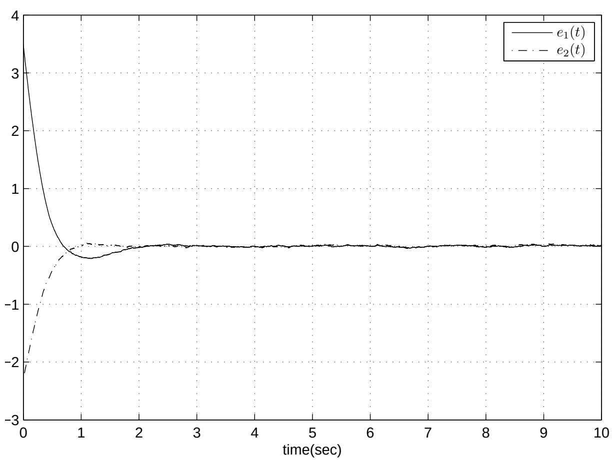

Fig. 4. Responses of the estimation error $e(t)$

5. Conclusion

In this paper, for the first time, an $\mathscr{H}_\infty$ state estimation method for T-S fuzzy delayed Hopfield neural networks is proposed. This estimator ensures the asymptotical state estimation and reduces the $\mathscr{H}_\infty$ norm from the external disturbance to the estimation error within a predefined level of disturbance attenuation. It has been shown that the gain matrix of the proposed $\mathscr{H}_\infty$ estimator can be realized by solving the LMI problem. A numerical simulation example is provided to show the effectiveness of the proposed $\mathscr{H}_\infty$ estimator. The proposed estimation scheme can be used in several applications. A possible scheme is to use T-S fuzzy delayed Hopfield neural network to build a mathematical model from experimental data and then design a nonlinear controller, based on this model. Algorithms already exist which ensure error convergence for identifiers of T-S fuzzy delayed Hopfield neural networks [9,10].

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