An Elegant Deadline Calculation for SCED

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Abstract

The future multimedia Internet needs mechanisms to provide QoS (Quality of Service) for users. Service curve (SC) is an effective description of QoS and SCED (service curve based earliest deadline first policy) is an efficient scheduling algorithm to guarantee SCs specified by users. Deadline calculation is the core of SCED. The unique treatable SC as we know is the concave piecewise linear SC (CPLSC). This paper re-derives out a more compact and clearer recursive relation of deadline calculation than that in the original SCED and also modifies some defects of the SCED.

Keywords: Concave piecewise linear service curve, deadline calculation, QoS, SCED.

1. Introduction

The Internet will be full of multimedia flows in the future. Owing to the various diversities of multimedia traffic, different flow/class should be treated differentially to provide different Quality of Service (QoS) required by users. An effective way to describe the requirement of QoS is by way of Service Curve (SC) [1], [2]. The Concave Piecewise Linear SC (CPLSC) [2] is the most useful SC, because CPLSC can provide time-variant service rates for users and it possesses treatable deadline calculation for the scheduler SCED (Service Curve based Earliest Deadline first policy) [2].

The definition of SC is originated by Cruz [1], and then extended by Boudec [3], [4]. Before these definitions, Parekh had proposed the concept of SC [5]. SC is closely linked to network calculus[6]. The above studies focused on the network calculus if a system/node can provide the SC. However, no idea is given about how to provide the SC. Sariowan first developed a scheduler called SCED to guarantee the SC [2]. In SCED, the key factor is the deadline of a packet, which should be found out first to be the base of scheduling. Although [2] has derived the recursive relation to find the deadline, it is still a mission impossible to find the deadline for general SCs. As far as we know, CPLSC is the unique treatable SC proposed by [2].

This paper puts focus on the deadline calculation of SCED. Two works are done. One is to re-derive out the deadline of a packet for CPLSC and to get a clearer and more compact recursive relation. The other is to modify some defects of SCED in [2]. After the enhancement, a precise SCED and an easy approach for the deadline are obtained. In this way, the Internet can put QoS on multimedia flows to carry out the multimedia Internet.

The organization of this paper is as follows. Section 2 reviews and refines the SCED. Section 3 re-derives the recursive relation of deadline calculation for CPLSC and modifies some defects of SCED. Finally, concluding remarks are given in Section 4.

2. Reviewing and Refining SCED

Sariowan [2] defined $Y(q \mid u)$, $q \geq 1$, $u \geq 1$, which we call Key Factor of Deadline (KFD), as

$$Y(q \mid u) = \min\{t: Z(t \mid u-1) \geq A[1, u-1] + q\}, \quad (1)$$

where $Z(t \mid u-1) = \min\{A[1, s] + S(t-s): \tau(u-1) \leq s \leq u-1\}$ [2, (10)], $A[1, s]$ is the Arrival during $[1, s]$, $S(t)$ is the SC, and $\tau(t)$ is the last system reset time before time t. Then, the deadline of the nth arrival denoted by d_n can be expressed as [2, (19)]

$$d_n = \max\{u, Y(q \mid u)\}, \quad (2)$$

where $n = A[1, u-1] + q$ means that the nth packet is also the qth packet arriving during time slot u. If we define $A_0 = 0$ and $Y(q \mid u_0) = 0$, the recursive relation of $Y(q \mid u)$, [2, (22), (23)], is rewritten as

$$Y(q \mid u_m) = \max\{Y(q + A_{m-1} \mid u_{m-1}), u_m - 1 + D(q)\}, \quad (3)$$

where u_m is the mth time slot that has arrivals, A_m is the number of packets arriving during time u_m, and $D(q)$, we call Delay Curve (DC), is defined by [2, (20)]

$$D(q) = \min\{m: m \geq 1 \text{ and } S(m) \geq q\}. \quad (4)$$

Besides, another form of KFD is given to more clearly discern how the KFD is really found out.

Lemma 1: Another form of KFD is given by

$$Y(q \mid u_m) = \max\{u_k - 1 + D(q + \sum_{i=k}^{m-1} A_i): k = 1, \ldots, m\}. \quad (5)$$

Proof: By induction. □

The following theorem is a refinement to illustrate that to take the maximum value of KFD and u in the deadline equation of [2], i.e., (2) is redundant.

Theorem 1: KFD is deadline, i.e., (2) is actually

$$d_n = Y(q \mid u). \quad (6)$$

Proof: $D(q) \geq 1$ is given in (4) for any positive integer q, then, from (5), $Y(q \mid u_m) = \max\{u_k - 1 + D(q + \sum_{i=k}^{m-1} R_i): k = 1, \ldots, m\} \geq u_m - 1 + D(q) \geq u_m$, so (2) becomes $d_n = \max\{u, Y(q \mid u)\} = Y(q \mid u)$. □

A simple linear SC can provide single service rate. If users require time-variant service rates, the SC can consist of several linear line segments, e.g., CPLSC. We denote a line segment starting from (τ, σ) with slope ρ by ρ: (τ, σ) and a curve $S(t)$ consisting of L line segments by $\sum_{l=1}^{L} \rho_l : (\tau_l, \sigma_l)$, where $\tau_i < \tau_j$ for $i < j$, then $S(t)$ is expressed as $S(t) = \rho_l(t - \tau_l) + \sigma_l$ for $\tau_l \leq t < \tau_{l+1}$. As far as we know, the unique SC that provides time-variant service rates and has low complexity is CPLSC. Its complexity of deadline calculation is $O(L)$, where L is the number of the line segments constructing CPLSC [2].

Definition 1: CPLSC is expressed as [2]

$$\begin{aligned} S(t) &= \max\{0, \min\{\rho_l t + \theta_l: l = 1,\ldots, L\}\} \\ &= \max\{0, \min\{\rho_l t + (\sigma_l - \rho_l \tau_l): l = 1,\ldots, L\}\}, \end{aligned} \quad (7)$$

where slope of EF-l ρ_l is positive, $\rho_i > \rho_j$ for $i < j$, and θ_l is vertical coordinate of the cross point of EF-l and vertical axis. The turning corners, defined as the start points of line segments, are (τ_l, σ_l), $l = 2,\ldots, L$, where τ_l is not less than 1 [2, (26)] and $\tau_i < \tau_j$ for $i < j$. □

The original definition did not specify the value of θ_l. However, because of concavity, it must be $\theta_i < \theta_j$ for $i < j$. τ_1 is not specified, either. Of course, it is $\tau_1 = 0$ for the $S(t)$ defined on $t \geq 0$. In the original definition, σ_1 (= θ_1) is not specified, but σ_l, $l = 2, \ldots, L$, are assumed positive. This definition has ambiguity at $t = 0$. If $\theta_1 < 0$, there is a horizontal line between 0 and $-\theta_1/\rho_1$ and the SC is not concave. If $\theta_1 > 0$, $S(0) = \theta_1$ conflicts with the definition of SC with $S(0) = 0$. The only way is to have $\theta_1 = 0$, which also makes $\min\{\rho_l t + \theta_l: l = 1,\ldots, L\} \geq 0$ for all nonnegative t. Consequently, the max operator in (7) becomes redundant, i.e., it is enough to express $S(t)$ as

$$S(t) = \min\{\rho_l t + \theta_l: l = 1,\ldots, L\}. \quad (8)$$

3. Deadline calculation of CPLSC

We focus on the CPLSC $S(t)$, which is continuous both in its domain and range. Imitating the expression of DC, (4), the expression of continuous DC (CDC) corresponding to $S(t)$ is expressed by

$$C(q) = \inf\{t: t \geq 0, t \in R, \text{ and } S(t) \geq q\}, q \geq 0, q \in R, \quad (9)$$

Because $S(t)$ is a one-to-one and onto function, it yields $C(q) = S^{-1}(q)$. This property makes deadline calculation smoother, because the inverse function of SC can be directly used in deadline calculation. Note that, DC does not own this inverse property. It is easy to show that the relation between DC and CDC is

$$D(q) = \lceil C(q) \rceil, \text{ for integer } q. \quad (10)$$

Similar to (3), if $X(q \mid u_0) = 0$ is defined, the Key Factor of Continuous Deadline (KFCD) denoted by $X(q \mid u_m)$ can be construct recursively from CDC $C(q)$ by

$$X(q \mid u_m) = \max\{X(q + A_{m-1} \mid u_{m-1}), C(q) + u_m - 1\}. \quad (11)$$

The relation between KFD and KFCD is

$$Y(q \mid u_m) = \lceil X(q \mid u_m) \rceil, \quad (12)$$

which can be inductively proven by (3), (10) and (11). Similar to Lemma 1, another form of KFCD is

$$X(q \mid u_m) = \max\{C(q + \sum_{i=k}^{m-1} A_i) + u_k - 1: k = 1,\ldots, m\}. \quad (13)$$

In general, $C(q)$ is not necessarily linear, so the complexity to find KFCD $X(q \mid u_m)$ is $O(m)$. On the other hand, if $C(q)$ is linear, the complexity is $O(1)$, see next lemma. A CPLSC and its inverse (i.e., CDC) consist of linear functions, so the linear CDC is investigated first.

Lemma 2: If $C(q) = \eta q + d$, then

$$X(q \mid u_m) = C(q) + K_m \quad (14)$$

where $K_0 = 0$ and

$$K_m = \max\{K_{m-1} + \eta A_{m-1}, u_m - 1\}. \quad (15)$$

Proof: By induction. □

Lemma 2 reveals if $C(q)$ is linear, the unknown in deadline calculation is K_m only, which can be recursively obtained by K_{m-1} and some other known parameters, so the complexity of deadline calculation is $O(1)$.

3.1. Combinatorial Function

If a function is synthesized by some functions, e.g., $g(x) = F(f_1(x),\ldots, f_k(x))$, where $g(\cdot)$, $F(\cdot)$, and $f_i(\cdot)$, $i = 1,\ldots, k$, are some kinds of functions, we call $g(x)$ a Combinatorial Function (CF) and call its constituents, $f_i(x)$, $i = 1,\ldots, k$, Element Functions (EFs). If $F(\cdot)$ is the maximum function, i.e., $g(x) = max(f_1(x),\ldots, f_k(x))$, we say the CF possesses Property of Maximum of Combination (PMC). If all EFs of a CF are linear, we call the CF a linear CF (LCF).

Theorem 2: CDC $C_0(q)$ is a CF that consists of EFs $C_l(q)$, $l = 1,\ldots, L$, and possesses PMC, i.e.,

$$C_0(q) = \max\{C_l(q): l = 1, \ldots, L\}, \quad (16)$$

and each $C_l(q)$ can recursively create $X_l(q \mid u_m)$ by

$$X_l(q \mid u_m) = \max\{X_l(q + A_{m-1} \mid u_{m-1}),$$

$C_l(q) + u_m-1\}, l = 0, 1, \ldots, L,$ (17)

where $X_l(q \mid u_0) = 0$ for any l and q, then the corresponding KFCD $X_0(q \mid u_m)$ is also a CF with EFs $X_l(q \mid u_m)$, $l = 1, \ldots, L$ and possesses PMC, i.e.,

$$X_0(q \mid u_m) = \max\{X_l(q \mid u_m): l = 1, \ldots, L\}. \quad (18)$$

Proof: By induction. □

In Theorem 2, $C_l(q)$, $l \geq 1$, is not necessary to be a CDC, so there may exist some q such that $C_l(q)$, even $X_l(q \mid u_m)$ in (17), is negative. However, it does not affect the final correct $X_0(q \mid u_m)$.

Lemma 3: In Theorem 2, even there exists some p at a certain u_m causes $C_j(p) < 0$ and/or $X_j(p \mid u_m) < 0$ for some j. It does not affect the final correct $X_0(\cdot \mid u_m)$.

Proof: Because $C_0(q)$ is a CDC, it has $\max\{C_l(p): l = 1,\ldots, L\} = C_0(p) \geq 0$, which means there exists at least a $C_i(p) \geq 0$, $i \neq j$. That is any negative $C_j(p)$ does not affect the value of $C_0(p)$. From (13), we have $X_0(q \mid u_m) = \max\{ C_0(q + \sum_{i=k}^{m-1} A_i) + u_k - 1: k = 1,\ldots, m\}$, which must be nonnegative, because of nonnegative $C_0(\cdot)$. That is, $X_0(q \mid u_m)$ is not influenced by any negative $C_j(\cdot)$. Similarly, from (18), $X_0(p \mid u_m) = \max\{X_l(p \mid u_m): l = 1,\ldots, L\} \geq 0$ results in that there must exist at least an $X_i(p \mid u_m) \geq 0$, $i \neq j$, even if $X_j(p \mid u_m) < 0$. That is, any negative $X_j(p \mid u_m)$ does not influence the final correct $X_0(p \mid u_m)$. □

The above Lemma also roughly illustrates the defect in [2] that will be specified in next subsection. In fact, corresponding (13) with (18), if $C_i(q)$ is not a linear function, KFCD is the maximum element of the $m \times L$ matrix constructed by $C_l(\cdot)$, so the complexity is $O(mL)$. The complexity grows with time u_m. It is unacceptable. One way to eliminate the effect of time is to have SC an LCF.

Theorem 3: If an LCF $C(q) = \sum_{l=1}^{L} \eta_l : (\sigma_l, \tau_l)$ possesses PMC, its KFCD is

$$X(q \mid u_m) = \max\{ C_l(q) + K_{l,m}: l = 1,\ldots, L\}, \quad (19)$$

where $C_l(q) = \eta_l(q - \sigma_l) + \tau_l$, $K_{l,0} = 0$, and

$$K_{l,m} = \max\{K_{l,m-1} + \eta_l A_{m-1}, u_m - 1\}. \quad (20)$$

Proof: From Lemma 2, the $X_l(q \mid u_m)$ corresponding to $C_l(q)$ is $C_l(q) + K_{l,m}$, where $K_{l,m} = \max\{K_{l,m-1} + \eta_l A_{m-1}, u_m-1\}$. Then Theorem 2 gives

$$\begin{aligned} X(q \mid u_m) &= \max\{X_l(q \mid u_m): l = 1, \ldots, L\} \\ &= \max\{ C_l(q) + K_{l,m}: l = 1,\ldots, L\}, \end{aligned} \quad (21)$$

so the proof is finished. Furthermore, the complexity of deadline calculation of an LCF $C(q)$ is $O(L)$, because the complexity of $X_l(q \mid u_m)$ is $O(1)$. □

3.2. CPLSC

This subsection will finish two tasks. One is to do the approach of deadline calculation of CPLSC by our methodology; the other is to modify some defects of [2]. Because CPLSC $S(t) = \sum_{l=1}^{L} \rho_l : (\tau_l, \sigma_l)$ is invertible, we can find $C(q)$ by turning the t-q plant to q-t plant. Then the corresponding CDC $C(q)$ consists of $C_l(q) = S_l^{-1}(q)$, $l = 1, \ldots, L$, i.e., $C(q) = \sum_{i=1}^{L} \eta_i : (\sigma_i, \tau_i)$, where $\eta_l = 1/\rho_l$ and $\eta_i < \eta_j$ for $i < j$, because $\rho_i > \rho_j$, i.e., $C(q)$ is a convex LCF. Next theorem will prove that CPLSC possesses PMC after the following Lemma proven.

Lemma 4: For two points (x_1, y_1) and (x_2, y_2), $x_1 < x_2$, on a convex curve $y = f(x)$, it obtains $f'(x_1) \leq (y_2 - y_1)/(x_2 - x_1) \leq f'(x_2)$, where $y_i = f(x_i)$ for $i = 1$ and 2.

Proof: Assume there are N turning corners at t_i, $i = 1, \ldots, N$ in interval (x_1, x_2). Also, let $t_0 = x_1$ and $t_{N+1} = x_2$, then $y_2 = y_1 + \int_{y_1}^{y_2} dy = y_1 + \sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} f'(x)dx$. Because of convexity, we have $f'(x_1) \leq f'(x) \leq f'(x_2)$ for $x_1 \leq x \leq x_2$ no matter from the right or the left hand side, such that

$$\sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} f'(x_1)dx \leq \sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} f'(x)dx \leq \sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} f'(x_2)dx,$$

i.e., $f'(x_1)(x_2 - x_1) \leq y_2 - y_1 \leq f'(x_2)(x_2 - x_1)$ or $f'(x_1) \leq (y_2 - y_1)/(x_2 - x_1) \leq f'(x_2)$. □

Theorem 4: For an LCF, it possesses PMC, if and only if it is convex.

Proof: Denote the LCF by $C(q) = \sum_{l=1}^{L} \eta_l : (\sigma_l, \tau_l)$ with $\sigma_i < \sigma_j$ for $i < j$. This theorem is to prove $\eta_i < \eta_j$ for $i < j$ if and only if $C(q) = \max\{C_l(q): l = 1,\ldots, L\}$, where $C_l(q) = \eta_l(q - \sigma_l) + \tau_l$ is the EF-l. First we prove that if $\eta_i < \eta_j$ for $i < j$, then $C(q) = \max\{C_l(q): l = 1,\ldots, L\}$. For $q \in [\sigma_n, \sigma_{n+1})$, $n = 1, \ldots, L$, where $\sigma_{L+1} = \infty$ is defined, one has $C(q) = C_n(q) = \eta_n(q - \sigma_n) + \tau_n = \tau_{n+1} - \eta_n(\sigma_{n+1} - q)$. For $l < n$ and from Lemma 4, the right-hand side derivative of $C(q)$ at σ_l is $C'(\sigma_l^+) = \eta_l \leq (\tau_n - \tau_l)/(\sigma_n - \sigma_l)$, i.e., $\eta_l(\sigma_n - \sigma_l) + \tau_l \leq \tau_n$, so $C_l(q) = \eta_l(q - \sigma_l) + \tau_l = \eta_l(q - \sigma_n) + \eta_l(\sigma_n - \sigma_l) + \tau_l \leq \eta_l(q - \sigma_n) + \tau_n \leq C_n(q)$. For $l > n$, the left-hand side derivative of $C(q)$ at σ_{l+1} is $C'(\sigma_{l+1}^-) = \eta_l \geq (\tau_{l+1} - \tau_{n+1})/(\sigma_{l+1} - \sigma_{n+1})$, i.e., $\tau_{l+1} - \eta_l(\sigma_{l+1} - \sigma_{n+1}) \leq \tau_{n+1}$, so $C_l(q) = \tau_{l+1} - \eta_l(\sigma_{l+1} - q) = \tau_{l+1} - \eta_l(\sigma_{l+1} - \sigma_{n+1}) - \eta_l(\sigma_{n+1} - q) \leq \tau_{n+1} - \eta_l(\sigma_{n+1} - q) < C_n(q)$. Finally, $C(q) = \max\{C_l(q): l = 1,\ldots, L\} = C_n(q)$.

We prove the reverse direction by contradiction. Assume there exists an EF-l such that $\eta_l < \eta_{l-1}$. For $\sigma_l < q < \sigma_{l+1}$, LCF makes $C(q) = C_l(q)$. But, $\eta_l < \eta_{l-1}$ results in $C_l(q) = \eta_l(q - \sigma_l) + \tau_l < \eta_{l-1}(q - \sigma_l) + \tau_l = \eta_{l-1}(q - \sigma_l) + \eta_{l-1}(\sigma_l - \sigma_{l-1}) + \tau_{l-1} = \eta_{l-1}(q - \sigma_{l-1}) + \tau_{l-1} = C_{l-1}(q)$. It means $C(q) = C_l(q) < C_{l-1}(q) \leq \max\{C_l(q): l = 1,\ldots, L\}$, which conflicts with premise $C(q) = \max\{C_l(q): l = 1,\ldots, L\}$, so no such EF-$l$ exists. □

Now, the deadline calculation of CPLSC becomes an easy task by our approach. The CDC of CPLSC is a convex LCF, so it has PMC, and then the deadline can be found easily by Theorem 3 and (12). The recursion and the expression are clearer than those of [2].

Furthermore, the approach in [2] has a defect, although the final results are the same as ours. While [2] derived KFD of CPLSC [2, the first paragraph of the right column on p.676], the $S(t)$ is expressed as $S(t) = \min\{S_l(t): l = 1,\ldots, L\}$, where

$$S_l(t) = \max\{0, \rho_l t + \theta_l\}. \tag{22}$$

Sariowan temporarily took $S_l(t)$ as an $S(t)$ to find KFD, so we also temporarily consider $S_l(t)$ only. Because $\rho_l t + \theta_l$ is nonnegative for all nonnegative t, the max operator in (22) is redundant (see the derivation of (8)). Thus, we modify the expression of SC in (22) and also the last equation of the left column on p.676 in [2] by removing the max operator. One more modification is aimed at $D_l(q) = \lceil \eta_l(q - \theta_l) \rceil$ [2, the first paragraph of the right column on p.676]. Because there may exist some $\theta_l > 0$ to cause an unreasonable negative $D_l(q)$ for small q, the expression of $D_l(q) = \lceil \eta_l(q - \theta_l) \rceil$ is not correct. From definition of $D(q)$, (4), $D_l(q)$ must be not less than 1, so the correct expression is

$$D_l(q) = \max\{1, \lceil \eta_l(q - \theta_l) \rceil\}. \tag{23}$$

The cause of this mistake comes from inverting cause and effect. The derivation of KFD in [2, (21), (22), (23)] first used (4) [2, (20)], where $D_l(q)$ is not necessarily linear and $D_l(q) \geq 1$. Then [2] gave a linear $D_l(q)$ [2, last paragraph on p.675] to find out the specific KFD, in which $D_l(q)$ plays the role of cause, rather than the role of effect. However, in deriving the KFD corresponding to a line segment of CPLSC, the given premise is the line segment $S_l(t)$ [2, the first paragraph of the right column on p.676], rather than $D_l(q)$ given directly. In this way, $D_l(q)$ needs to be found out from $S_l(t)$. At this moment, the cause is $S_l(t)$ and the effect is $D_l(q)$, which is not guaranteed to conform with the condition of DC [2, (20)] to be larger than or equal to 1. To meet the requirement, the actual result must be (23), which also influences the KFD. The CDC corresponding to (23) is $C_l(q) = \max\{1, \eta_l(q - \theta_l)\}$. It is a LCF with PMC, so, by (19), (14) and (15), one has

$$X_l(q \mid u_m) = \max\{u_m, \eta_l(q - \theta_l) + K_{l,m}\}, \tag{24}$$

where $K_{l,m}$ is as (20). Then $Y_l(q \mid u_m) = \max\{u_m, \lceil \eta_l(q - \theta_l) + K_{l,m} \rceil\}$ that is different from $Y_l(q \mid u_m) = \lceil K_{l,m} + \eta_l(q - \theta_l) \rceil$ of [2] (in the compact expression of this paper). The result of [2] may lead to $Y_l(q \mid u_m) < u_m$ that means deadline is before the arrival time. Although [2, (19)] may modify the problem, however, Theorem 1 has proven that the deadline is KFD and to take the maximum value with u_m is not necessary. In a word, the deadline finally achieved in [2] is the same as that derived in this paper, but there are some defects in the procedure of [2].

The last phrase is aimed on a single EF (a line segment). Its object is to highlight the defect in [2]. Actually, to find the deadline corresponding to the whole curve, it is unnecessary to do the maximum operation of (24) for every EF. Lemma 3 has illustrated the value of KFCD corresponding to each EF can be directly used while utilizing the PMC, no matter whether the value is negative or not. In a word, our approach is an efficient and precise method to find the deadline of CPLSC. Furthermore, the final recursive relation is more elegant than that in [2].

4. Concluding Remarks

Although [2] has proposed the scheduling algorithm SCED and derived the recursive relation of deadline, some defects appeared in [2] and the recursive relation is almost untreatable for general SCs. Even for CPLSC, the recursive relation in [2] can be enhanced to have a more compact and clearer expression. This study finishes some modifications of SCED and derives out a better recursive relation for CPLSC. Consequently, our recursive relation makes the deadline calculation clearer and easier.

About SCED, do other kinds of SCs except the CPLSC possess an easy recursive relation of deadline calculation similar to that derived here? It is a further study topic.

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