Statistical Analysis of Type-1 and Interval Type-2 Fuzzy Logic in dynamic parameter adaptation of the BCO

Leticia Amador-Angulo[1], Oscar Castillo[2]

Division of Graduate Studies
Tijuana Institute of Technology
Tijuana, México
[1]leticia.amadorangulo@yahoo.com.mx, [2]ocastillo@tectijuana.mx

Abstract. A statistical analysis of Type-1 and Interval Type-2 Fuzzy Logic in dynamic parameter adaptation in the Bee Colony Optimization algorithm (BCO) is presented in this paper. The Bee Colony Optimization meta-heuristic belongs to the class of Nature-Inspired Algorithms. The objective of the work is based on the main reasons for the analysis of the approach with Interval Type-2 Fuzzy Logic to find the best parameters of the Beta and Alpha in BCO. We implemented the BCO specifically for tuning membership functions of the fuzzy controller for the benchmark problem, known as the temperature controller.

Keywords: Adjust Dynamic, Bee Colony Optimization, Fuzzy Logic, Uncertainty, Fuzzy Controller, Bees.

1. Introduction

In recent years, many works have been conducted on control system stabilization. However, all these control design methods require the exact mathematical model of the physical systems, which may not be available in practice. On the other hand, fuzzy logic control has been successfully applied for solving many nonlinear control problems. Usually benchmark problems are used to test algorithms and observe their performance, i.e. [11, 12].

The nonlinear characteristics of ill-defined and complex modern plants make classical controllers inadequate for such systems. However, using Fuzzy sets and fuzzy logic principles have enabled researchers to understand better, and hence control, complex systems that are difficult to model. These newly developed fuzzy logic controllers have given control system a certain degree of intelligence [1, 4,11].

Type-2 fuzzy models have emerged as an interesting generalization of fuzzy models based upon fuzzy sets. In Type-2 fuzzy systems, the membership functions can now return a range of values, which vary depending on the uncertainty involved in not only the inputs, but also in the same membership function [25].

BCO has recently received many improvements and applications. The BCO algorithm mimics the food foraging behavior of swarms of honey bees [22]. Honey bees use several mechanisms like waggle dance to optimally locate food sources and search new ones. It is a very simple, robust and population based stochastic optimization algorithm [8].

This paper starts with related work in Section 2. Section 3 describes the Fuzzy Logic System. Section 4 the problem statement. Section 5 describes the Bee Colony Optimization algorithm. Section 6 describes the proposed method. Section 7 describes the simulation results. Section 8 describes the statistical analysis, and finally Section 9 describes the conclusion of this research.

2. Related Works

Many intelligent optimization techniques, such as Ant Colony Optimization [12] and [29], and Particle Swarm Optimization [21], Differential Evolution [24] have been proposed to tune Fuzzy Controllers, however; the BCO algorithm is a new technique to solve complex problems that can be used in this case. In addition, most related works are realized with the classical algorithm, in [18, 19, 20, 1, 27] the dynamic adaptation of the parameters in bio-inspired algorithms has been implemented finding good results. Therefore, we proposed to use the dynamic parameter adaptation of the BCO with type-1 fuzzy logic and interval type-2 fuzzy logic system to find the best Alpha and Beta parameters in the BCO algorithm.

In [2] fixed values of the recommended parameters of Alpha and Beta in the BCO algorithm are both 1.0, and we need to realize several experiments to find the optimum values of these parameters. To overcome this problem, this paper proposes a new method with dynamic adjustment of the parameters of the BCO algorithm applied to benchmark problem in control.
BCO has been implemented in various applications within the control area. As an example, in [26], the fuzzy decentralized state feedback and observer-based decentralized output feedback controllers for a class of continuous nonlinear interconnected systems with

time-delay and the modeling error have been applied. In [28], the sliding mode controller with fuzzy logic has been used for the heading control of the submerged vehicle. In [18], the method of fuzzy control systems for trailers driven by multiple motors inside slipways to haul out ships has been presented.

3. Fuzzy Logic Controller

The main idea of a fuzzy logic system was introduced by Zadeh in 1965 [13, 14, 15], and was first applied to control theory in 1974 by Mamdani [4]. In [16, 25, 26, 27] various applications have been successfully achieved of Fuzzy Controllers.

3.1. Type-1 Fuzzy Logic System

A fuzzy logic system (FLS) that is defined entirely in terms of Type-1fuzzy sets, is known as Type-1 Fuzzy Logic System (Type-1 FLS), its elements are defined in the following Fig. 1 [7].
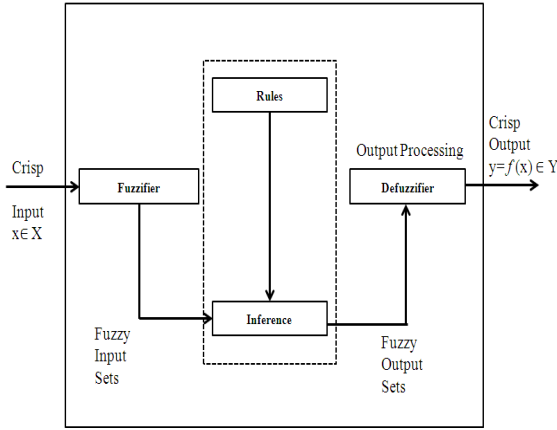

Fig. 1. Architecture of a Type-1 fuzzy logic system.

A fuzzy set in the universe U is characterized by a membership function $u_A(x)$ taking values on the interval [0,1] and can be represented as a set of ordered pairs of an element and the membership value of the set:

$$A = \{(x, u_A(x)) \mid x \in U\} \quad (1)$$

3.2. Interval Type-2 Fuzzy Logic System

AType-2 fuzzy set, $\tilde{A}$,is characterized by.

$$\tilde{A} = \{(x,u), u_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \quad (2)$$

Where $0 \leq u_{\tilde{A}}(x,u) \leq 1$

The Uncertainty affects decisions in a number of different ways. The concept of information is fully connected to the concept of uncertainty. The most fundamental part of this connection is that the uncertainty involved in any solution of a problem is the result of poor information, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some way or another [9].Fig. 2 shows the architecture of a Type-2 fuzzy logic system.
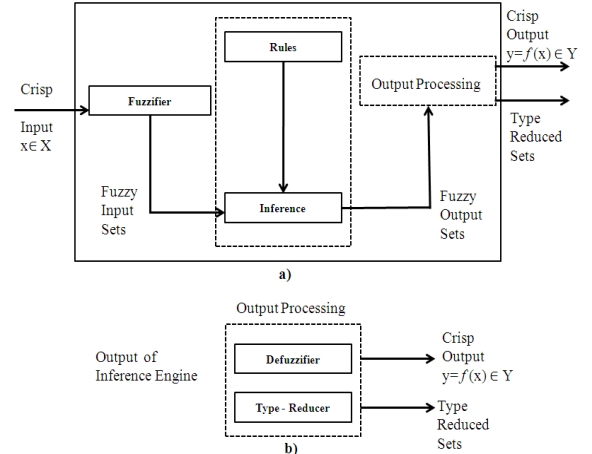

Fig.2. Architecture of a Type-2 fuzzy logic system.

The output processor includes a type-reducer and defuzzifier; it generates a Type-1 fuzzy set output (from the type-reducer) or a crisp number (from the defuzzifier) [10, 19].

A Type-2 FLS is also characterized by IF-THEN rules, but their fuzzy sets are now of Type-2. The FLS can be used when circumstances are too uncertain to determine exact membership degrees, as is the case when the membership functions in a fuzzy controller can take different values and we want to find the distribution of membership functions to show better results in the stability of fuzzy control.

In this paper the optimization of the parameter values of the membership functions with BCO is applied to fuzzy controllers, and we implemented the robustness the interval type-2 fuzzy logic to find the optimal values of the *Beta* and *Alpha* the BCO algorithm, which is detailed in the following sections.

4. Problem Statement

The main problem to study is to regulate the temperature of water flow. The Fuzzy Controller has two inputs and two outputs, called "*temp*", "*flow*" for the inputs and "*cold*" and "*hot*" for the outputs. It has 9 rules, and its membership functions are Trapezoidals and Triangulars. The Structure of the Fuzzy Controller is shown in Fig.3.
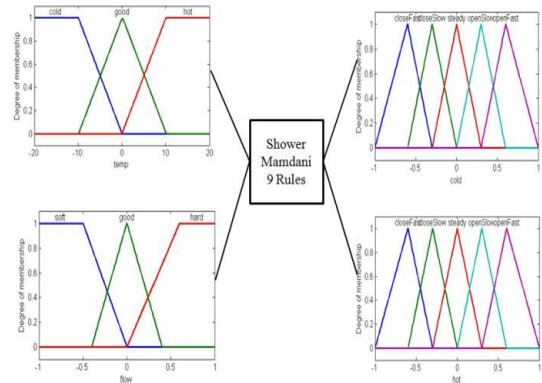

Fig. 3. Structure of the Fuzzy Controller for the temperature controller.

The inference is determined by the following rules shown in Fig. 4.

Fig.4. Rules of the Fuzzy Logic System.

4.1. Proposed Control Diagram

Fig. 5shows the block diagram used for the FLC that obtained the best results for the temperature controller benchmark problem. Generally the fuzzy controller is a closed-loop control, the aim is to make the plant output to follow the input r, and the adder is applied to the system, it is used as a controller in the first the output is connected directly to one of the two inputs of the adder. In the second situation, the output and the model is perturbed by noise in order to introduce uncertainty in the data feedback [17]. The noise is a disturbance applied to the model with the objective that the BCO algorithm further explores its search space and show better results.
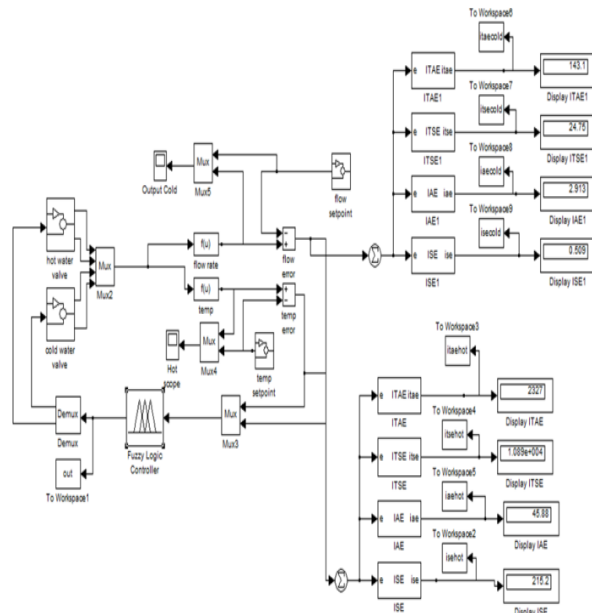

Fig.5. Evaluation of control diagram with Type–1 Fuzzy Inference System.

Fig. 6 shows the output variable called "*Cold*" in the model, the yellow line represents the reference value and the control behavior in the pink line which is optimized fuzzy controller, and the simulation was performed in 50 iterations. The best experiment found by BCO is shown.
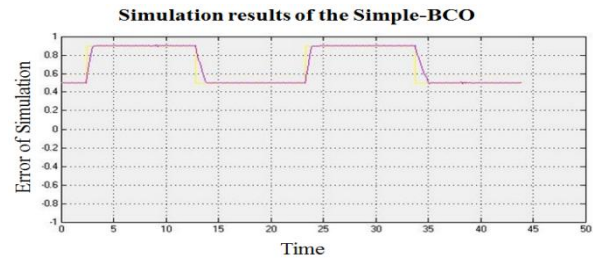

Fig.6. Simulation model using traditional-BCO with Fuzzy Logic Controller.

5. Bee Colony Optimization Algorithm

The BCO is inspired by the bees´ behavior in nature. The basic idea behind the BCO is to create the multi agent system (colony of artificial bees) capabilities to successfully solve difficult combinatorial optimization problems [6, 22].

The population of agents (artificial bees) consisting of Bees collaboratively searches for the optimal solution. Every artificial bee generates one solution to the problem. The algorithm is divided into the forward pass and backward pass. The existence of a large number of different social insect species, and variation in their behavioral patterns, it is possible to describe individual insects as capable of performing a variety of complex tasks [3] [6]. Each bee decides to reach for the nectar source by following a nest mate who has already discovered a path of nectar source dance, in that way trying to convince their nest mate to follow them. If a bee decides to leave the hive to get nectar, she follows the bee dancers to one of the nectar areas. Graphically shown in Fig. 7 is the representation of the smart mechanism that bees use to forage for food in the example of third forward pass. Each circle indicates a path to find a possible solution to the problem.
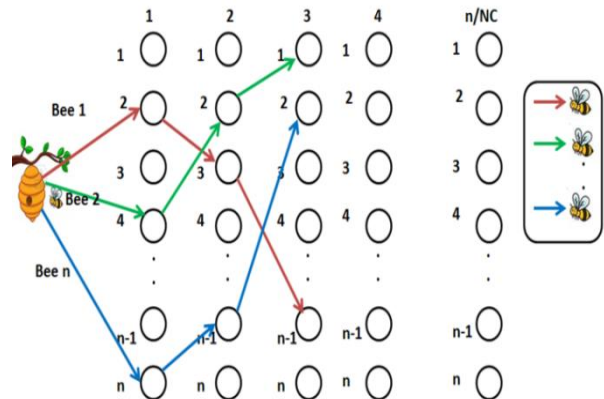

Fig. 7.BCO Representation

The Bee Colony Optimization (BCO) metaheuristic [5, 6, 22] has been introduced fairly recently by *Lučić* and *Teodorović* as a new direction in the field of Swarm Intelligence, and has not been previously applied in Type-2 Fuzzy Controller design. The BCO is an algorithm implementation that adapts the behavior of real bees to solutions of minimum cost path problems on graphs. The general steps of BCO are indicated in Table 1.

Pseudocode of BCO
1. Initialization: an empty solution is assigned to every bee;
2. For every bee: //the forward pass
a) Set k=1; //counter for constructive moves in the forward pass;
b) Evaluate all possible constructive moves;
c) According to the evaluation, choose on move using the roulette wheel;
d) k=k+1; if k ≤ NC goto step b.
3. All bees are back to the hive; //backward pass stars.
4. Evaluate (partial) objective function value for each bee;
5. Every bee decides randomly whether to continue its own exploration and become a recruiter, or to become a follower;
6. For every follower, choose a new solution from recruiters by the roulette wheel;
7. If solutions have not completed goto step 2;
8. Evaluate all solutions and find the best one;
9. If stopping condition is not met goto step 2;
10. Output the best solution found.
B: Represents the number of bees in the hive.
NC: Represents the number of constructive moves during one forward pass.

Table 1: Basic steps of the BCO algorithm

Each bee represents a possible solution to the problem and the possible parameter values. Fig. 8 indicates the representation of the BCO in the design of the Fuzzy Logic System.
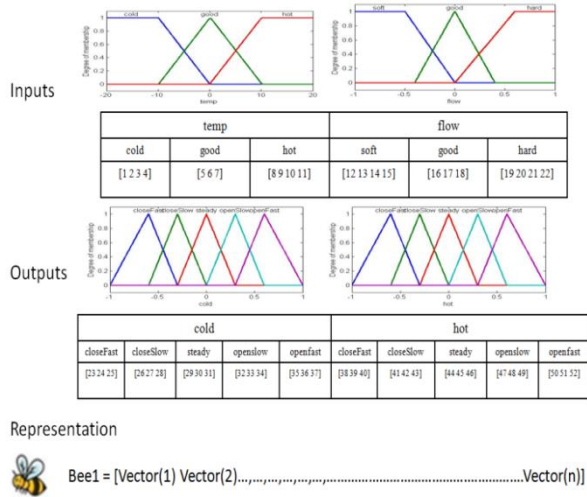

Fig. 8.BCO Architecture.

In BCO algorithm, a bee represents the values of the distribution of the each membership function. The triangular membership function has three values and the trapezoidal membership function has four values, obtaining a total of 52 values for two inputs and two outputs that design the Fuzzy Logic System.

6. Proposed Method

The dynamics of BCO are defined in Equations (3)–(6); the Equation 6 shows the probability of a bee located one best solution, and *alpha* and *beta* are variables that determine the heuristics of the algorithm:

$$P_{ij,n} = \frac{[\rho_{ij,n}]^{\alpha}.[\frac{1}{d_{ij}}]^{\beta}}{\sum_{j \in A_{i,n}} [\rho_{ij,n}]^{\alpha}.[\frac{1}{d_{ij}}]^{\beta}} \quad (3)$$

$$D_i = K.\frac{Pf_i}{Pf_{colony}} \quad (4)$$

$$Pf_i = \frac{1}{L_I}, L_i = \text{Tour Length} \quad (5)$$

$$Pf_{colony} = \frac{1}{N_{Bee}} \sum_{i=1}^{N_{Bee}} Pf_i \quad (6)$$

Equation (3) represents the probability of a bee k located on a node i selects the next node denoted by j, where, N^k_i is the set of feasible nodes (in a neighborhood) connected to node i with respect to bee k, β is the probability to visit the following node. Note that the ρ_{ij} is inversely proportional to the city distance. d_{ij} represents the distance of node i until node j, for this algorithm indicates the total the dance that a bee have in this moment. Finally $\propto$ is a binary variable that is used for to find better solutions in the algorithm.

Equation (4) represents the fact that a waggle dance will last for certain duration, determined by a linear function, where K denotes the waggle dance scaling factor, Pf_i denotes the profitability scores of bee i as defined in Equation (5) and Pf_{colony} denotes the bee colony's average profitability as in Equation (6) and is updated after each bee completes its tour. For this research the waggle dance is represented for the mean square error that all models to find once that is done the simulation in the iteration of the algorithm [2,8].

For measuring the iterations of the algorithm, it was decided to use the percentage of iterations as a variable, i.e. when starting the algorithm the iterations will be considered ''low'', and when the iterations are completed it will be considered ''high'' or close to 100%. To represent this idea we use the variable:

$$\text{Iteration} = \frac{\text{Current Iteration}}{Maximum \text{ of Iterations}} \quad (7)$$

The diversity measure is defined by Equation (8), which measures the degree of dispersion of the bee, i.e. when the bees are closer together there is less diversity as well as when bees are separated then diversity is high. As the reader will realize the equation of diversity can be considered as the average of the Euclidean distances between each bee and the best bee.

$$Diversity(S(t)) = \frac{1}{n_s} \sum_{i=1}^{n_x} \sqrt{X_{ij}(t) - \overline{X}_j(t))^2} \quad (8)$$

The error measure is defined by Equation (9), which measures the difference between the population and the best bee, by averaging the difference between the fitness of each bee and the fitness of the best bee.

$$\varepsilon = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(X_t - \hat{X}_t)^2} \qquad (9)$$

Where x_t is the estimated value of the control - signal reference- $\hat{x}_t$ is the observed value - control-signal, and N represents the total number of observed samples.

Therefore, for designing the fuzzy systems, which dynamically adjust the parameters of *alpha* and *beta*, the two measures described above are considered as inputs. It is obvious that for each fuzzy system the outputs are alpha and beta.

The distribution of the membership functions in inputs and outputs were realized symmetrically. The design of the input and output variables can be appreciated in Figs.9, 10, 11 and 12 for Type-1 Fuzzy Logic.
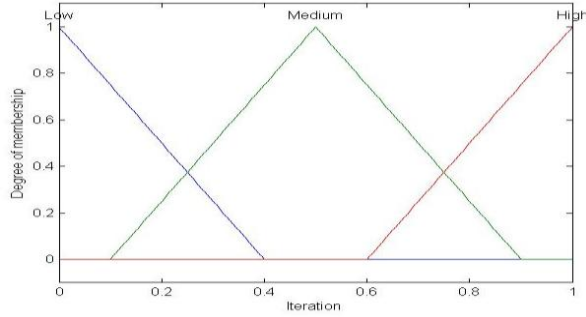
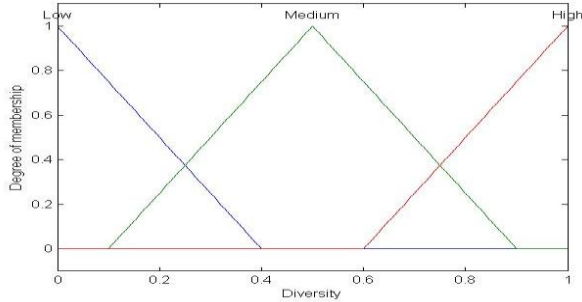
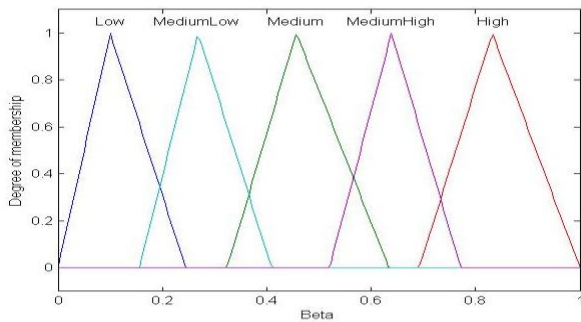Fig. 9. Input 1. Iteration ITFLS

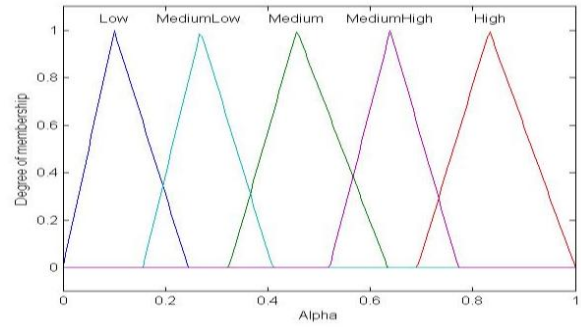Fig. 10. Input 2.Diversity ITFLS

Fig. 11. Output 1.Beta ITFLS

Fig. 12. Output 2.Alpha ITFLS

The design and the rules of the fuzzy system are shown in Figs. 13 and 14, respectively.
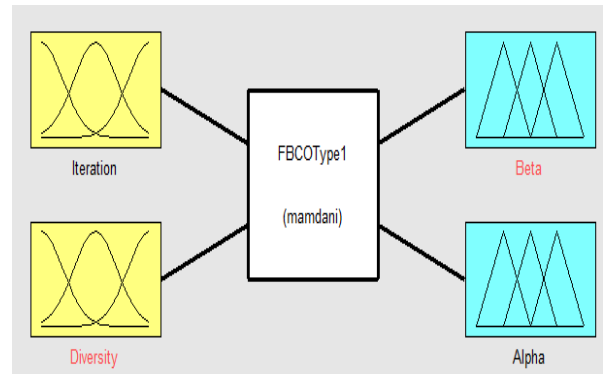

Fig. 13. Type-1 Fuzzy Logic System.

1. If (Iteration is Low) and (Diversity is Low) then (Beta is High)(Alpha is Low) (1)
2. If (Iteration is Low) and (Diversity is Medium) then (Beta is MediumHigh)(Alpha is Medium) (1)
3. If (Iteration is Low) and (Diversity is High) then (Beta is MediumHigh)(Alpha is MediumLow) (1)
4. If (Iteration is Medium) and (Diversity is Low) then (Beta is MediumHigh)(Alpha is MediumLow) (1)
5. If (Iteration is Medium) and (Diversity is Medium) then (Beta is Medium)(Alpha is Medium) (1)
6. If (Iteration is Medium) and (Diversity is High) then (Beta is MediumLow)(Alpha is MediumHigh) (1)
7. If (Iteration is High) and (Diversity is Low) then (Beta is Medium)(Alpha is High) (1)
8. If (Iteration is High) and (Diversity is Medium) then (Beta is MediumLow)(Alpha is MediumHigh) (1)
9. If (Iteration is High) and (Diversity is High) then (Beta is Low)(Alpha is High) (1)

Fig. 14. Rules for the Type-1 Fuzzy Logic System.

With the same methodology, we can design the Interval Type-2 Fuzzy Logic System. In Figs. 15, 16, 17 and 18 we show the inputs, outputs and the fuzzy logic system, respectively.
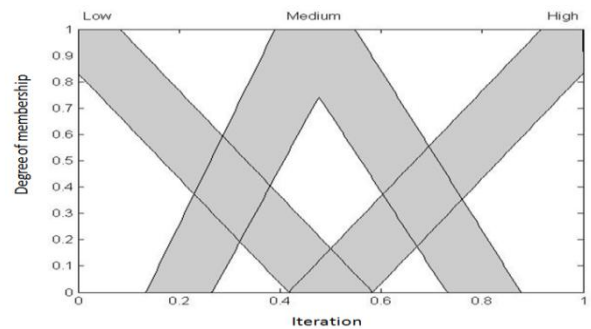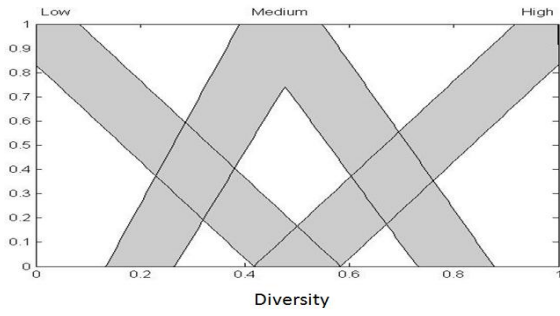

Fig. 15. Input 1. Iteration IT2FLS.
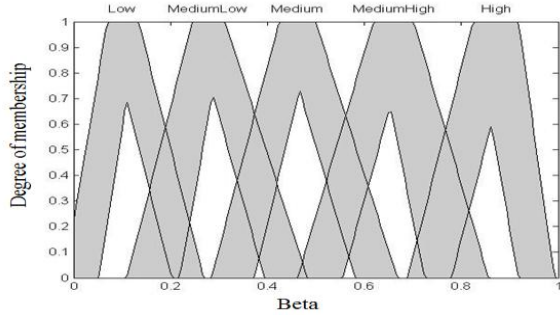
Fig. 16. Input 2. Diversity IT2FLS.
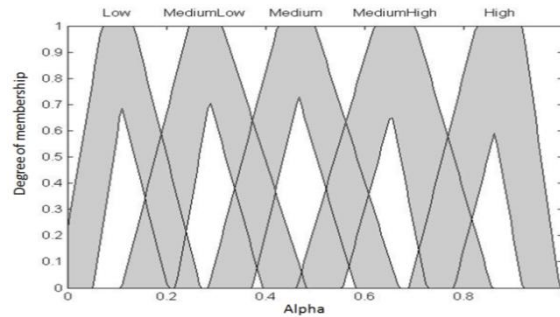

Fig. 17. Output 1. Beta IT2FLS.

Fig. 18. Output 2. Alpha IT2FLS.

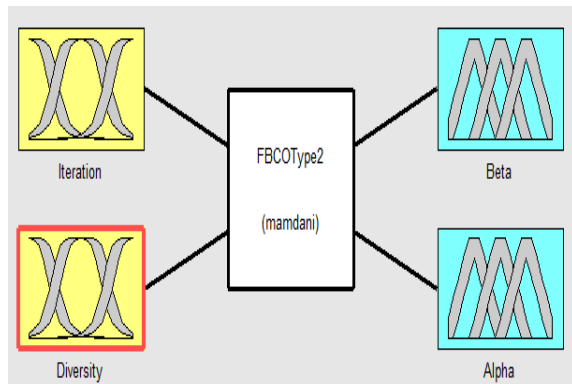The design of the fuzzy logic system for the IT2FBCO is shown in Fig. 19.

Fig. 19. Interval Type-2 Fuzzy Logic System.

In the design for two fuzzy logic system we implemented the same rules and the style is Mamdani.

7. Simulations Results

Experimentation was performed with various scenarios of external perturbations and, specific noise generators were used Band-limited white noise. Where the height of the Power Spectral Density of the band-limited white noise was set to 0.01 and 0.05, respectively.

The test criteria area series of Performance Indices; the indices used for the BCO are RMSE, we used as well ISE, IAE, ITSE and ITAE, respectively shown in Equations. (10-13).

$$ISE = \int_0^\infty e^2(t)dt \ (10)$$

$$IAE = \int_0^\infty |e(t)|dt \ (11)$$

$$ITSE = \int_0^\infty e^2(t)tdt \ (12)$$

$$ITAE = \int_0^\infty |e(t)|tdt \ (13)$$

The configurations of the experiments are: 30 experiments, the initial value for the population, bee follower and iterations are 3, 2, 3, respectively, and we are increasing by 3 units in each experiment; where Simple BCO is the Simple-BCO, IT2BCO represents the dynamic adjustment of the parameters of the BCO with Type-1 Fuzzy Logic system and IT2FBCO represents the dynamic adjustment to Interval Type-2 Fuzzy Logic system.

Simulation results in Table 2shows the best experiment without perturbation in the fuzzy logic controller for each bio-inspired algorithm. These results are of the best result and average of the RMSE of 30 experiments for each method.

Performance Index	Cold			Flow		
	Simple BCO	T1FBCO	IT2FBCO	Simple BCO	T1FBCO	IT2FBCO
ITAE	84.119	152.71	80.503	2040.61	1961.64	2791.65
ITSE	19.112	21.894	20.431	8417.86	8634.08	10966.98
IAE	1.674	3.082	1.637	39.383	38.382	53.680
ISE	0.3777	0.438	0.4070	160.106	169.23	207.5262
AVERAGE of RMSE	0.00218	0.00186	0.001941	Not Applicable		

Table 2: Simulations results without noise

Results shown that the Simple-BCO obtained the best results, however, by adding perturbation of band-limited white noise disturbance to the plant model is when our proposed method with fuzzy logic systems is better because it's observed that IT2FBCO analyzes uncertainty therefore the stability of the model, the results are shown in Table 3with the noise of 0.05.

Performance Index	Cold			Flow		
	Simple BCO	T1FBCO	IT2FBCO	Simple BCO	T1FBCO	IT2FBCO
ITAE	7005	7148	6949.45	224126.85	224287.55	214694.2
ITSE	10604.12	10964.24	10493.87	10471212	10515460	10391625
IAE	138.20	140.5	137.30	4482.87	4490.64	4480.01
ISE	207.28	212.4	204.44	208186.71	209153.83	207450.8
AVERAGE of RMSE	0.0130	0.0113	0.0117	Not Applicable		

Table 3: Performance Index results when inserting band-limited white noise perturbations of value 0.05.

In Tables 2 and 3 we can note that when we increase the level of the noise them our methods obtain better results. Fig. 20 shows the behavior applying the disturbance in the model of 0.05 for Simple-BCO, T1FBCO and IT2FBCO.
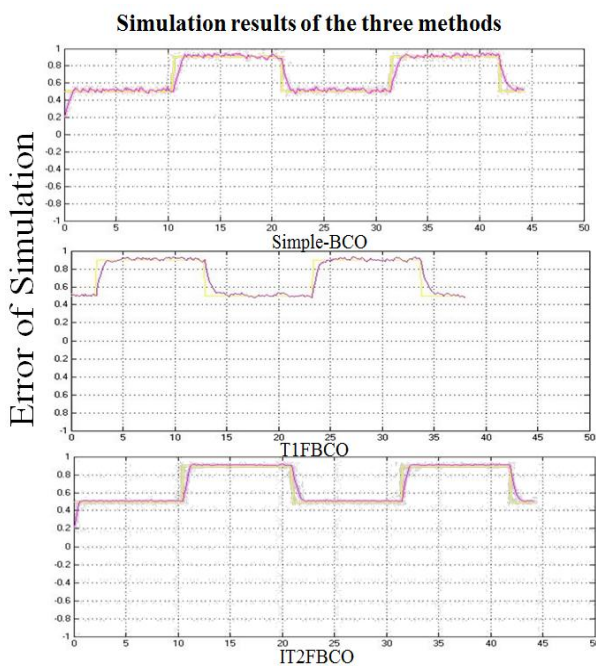

Fig. 20. Behavior of the model with Simple-BCO, T1FBCO and IT2FBCO.

The best convergence of the IT2FBCO with the noise of the 0.01 is shown in Fig. 21; in this figure we are using the RMSE in the cold output, the best value of beta and alpha are 0.294 and 0.47205, respectively.
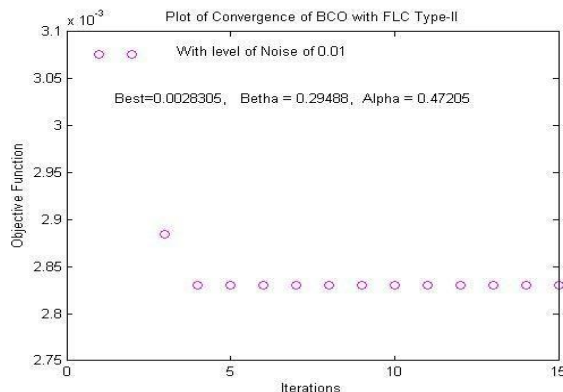

Fig. 21. Best convergence our method.

8. Statistical Analysis

The statistical test used for result comparison is the z-test, whose parameters are defined in Table 4. We realized the statistical test with a sample of 30 experiments randomly for each method, obtaining the results contained in Table 5. In applying the statistic z-test, with a significance level of 0.05, and the alternative hypothesis says that the results found of the proposed method (T1FBCO) is lower than the of simple BCO, and of course the null hypothesis tells us that the results of the proposed method are greater than or equal to the results of simple BCO, with a rejection region for all values that fall below **-1.645**. So the statistical test results are that: for the T1FBCO and IT2FBCO, there is significant evidence to reject the null hypothesis.

Parameter	Value
Level of significance	95 %
Alpha	0.05%
Ha	$\mu_1 < \mu_2$
Ho	$\mu_1 \geq \mu_2$
Critical value	-1.645

Table 4: Parameters for the statistical z-test

Our method	Simple Method	Z value	Evidence
T1FBCO	Simple-BCO	-1.309	Significant
IT2FBCO	Simple-BCO	-1.309	Significant

Table 5:The results of applying statistical z - tests.

Fig. 22 shows a comparison with the results with the three methods considered in the work, i.e. in the experiment number 23 the parameters are a population of 18, bee followers of 13, iterations of 20 and noise of 0.05. The RMSE is shown in Fig. 22.
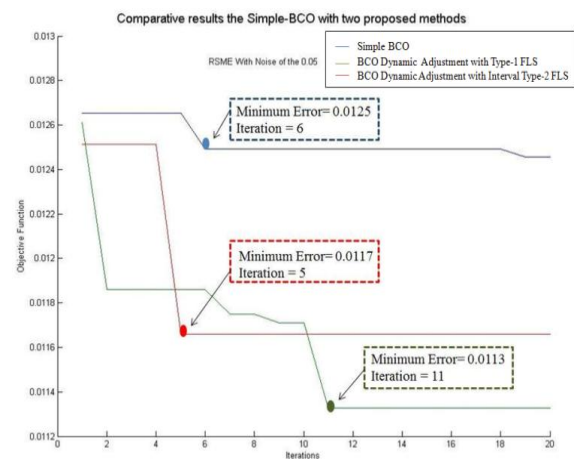

Fig. 22. Comparative results of the RMSE in the three methods.

9. Conclusions

We conclude that dynamically adjusting parameters of an optimization method (in this case the Bee Colony

Optimization BCO), can improve the quality of results and increase the diversity of solutions to a problem.

Two proposed methods were designed for adjusting the parameters for bee colony optimization with the fuzzy logic system. It was obtained in two systems a statistical study of an improvement in the quality of the results of the method of bee colony optimization when applied in the minimization of the benchmark problem in fuzzy controller.

By comparing the proposed methods and the simple method of BCO, in the design of fuzzy logic system applied to fuzzy control it was found that in the method based for the experiments that is proposed this work it was possible to develop a method for adjusting the parameters alpha and beta of the BCO using fuzzy logic. And in this way improving the results compared with the simple method of BCO.

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