Analysis and optimal design of film profile for power-function-shaped slider bearings lubricated with couple stress fluids

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Abstract. The effects of film profile on steady-state performance of power-function-shaped slider bearings with couple stress fluids have been investigated in this report. Taking account for the couple stress caused by additives blended into Newtonian fluid, the modified one dimensional non-Newtonian Reynolds-type equation is derived from the Stokes motion equations combined with the continuity equation. The steady-state performance is obviously enhanced by the effects of couple stress compared with the traditional Newtonian lubricant case. And the steady-state performance of power-function-shaped slider bearing with convex profile(function-power-larger than 1) is much better than concave profile(function-power smaller than 1). To get the optimal steady-state performance of the power-function-shaped slider bearing, the power of the profile could be chosen between 1.3~1.4.

Introduction

Slider bearing is a frequently-used part in mechanical engineering field. It is often utilized in the cases demanding large load capacity, high stability and long lifespan for its remarkable advantages. The bearing's film profile is the dominant factor impacting working performance. Hence, to improve the working capacity of slider bearing by optimizing the film profile has been an attractive topic for many researchers in recent years.

El-Gamal and Awadconducted an attempt to investigate the obvious influence of geometry parameters of the oscillating units on the characteristics of slider bearing. For arbitrary shapes of oscillating sliders, the steady streaming pressure was acquired with the objective of maximizing load-carrying capacity [1]. On the basis of Brinkman model, Lin JR theoretically optimized friction coefficient and load capacity for one-dimensional porous slider bearing [2]. Cheng and Chang developed a direct problem solver incorporated with an inverse method for determination of slider surface to meet the pressure distribution and load demands under certain working conditions [3]. More recently, Chu HM developed an algorithm depend on inverse method to design the film shape for optimum pressure distribution of slider bearing [4]. Based on the work of Cheng and Chang,Alyaqout et al. took the influence of couple stress fluids into consideration during the optimization of film shape of slider bearing [3,5].

With the improvement of modern design level and strict standards of mechanical products, non-Newtonian lubricants have drawn much attention. Generally, high molecular-weight polymers are added into oils to improve the viscosity, which can obviously enhance the characteristics of lubricants. The micro-continuum theory proposed by Stokesextended the traditional continuum theory by considering the presence of body couples and couple stress [6]. Ramanaiah and Dubeyoptimized the slider profile and load capacity of a slider bearing lubricated by couple stress fluids[7]. Lin et al.derived a general dynamic Reynolds equation for slider-squeezing surfaces with non-Newtonian fluids, they analyzed the dynamic behavior of a wide exponential-shaped slider bearing and the effects of couple stress upon the steady-state performance of a wide parabolic-shaped slider bearing [8].

Nevertheless, among all the investigations above, no literature concerned about the effects of film profiles on steady-state performance of power- function-shaped slider bearings. The bearings shape varies with the function-power, in order to find the optimum film profile in different couple stress fluids, an analysis with respect to the steady-state performance of power-function-shaped slider bearings has been done. The average film thicknesses of different profiles are fixed as constant in the following study.

The analytical approach

An investigation about the steady-state performance of power-function-shaped slider bearings as shown in Fig.°1 with equal average film thickness is conducted based on the micro- continuum theory of Stokes [6]. As shown in the schematic diagram, L denotes the slider bearing length which is fixed, and the upper surface of the slider bearing is at rest while the lower slider surface moves at a constant speed U. To keep the average film thicknesses equal to each other, the profile of power-function-shaped bearing is defined as follow:

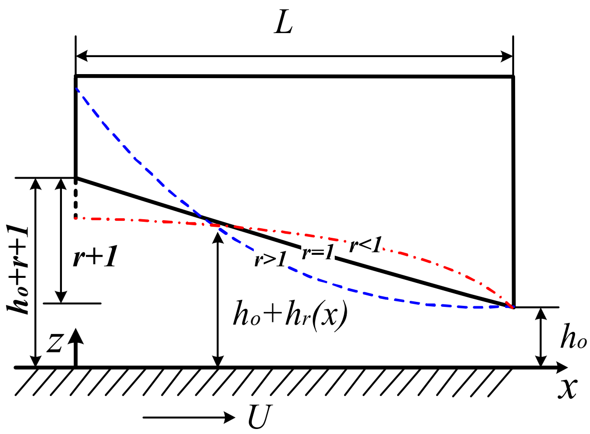

Fig.°1 Profile of the power-function-shaped slider bearing

$$h(x) = h_0 + h_r(x) = h_0 + (r+1)\left(1 - \frac{x}{L}\right)^r \qquad (1)$$

Where h_0 represents the outlet height of the bearing, $h_r(x)$ is the thickness caused by the upper surface, r is the function-power. When r>1, the profile of the upper bearing surface is a convex arc, while r<1, the profile becomes concave. To obtain the average film thickness, we can integrate $h(x)$ with respect to x from inlet to outlet. Different r forms different corresponding profile. For each profile, the average film thickness is h_0+1. The Stokes couple stress fluid was assumed to be used as lubricant between the sliders. In the thin-film assumptions, the body couples and body forces are quite small, and the fluid inertia can also be neglected. Therefore, in the x and z directions, the continuity equation and fluid motion equations can be obtained as:

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{2}$$

$$\frac{\partial^2 u}{\partial z^2}-l^2\frac{\partial^4 u}{\partial z^4}=\frac{1}{\mu}\frac{\partial p}{\partial x} \tag{3}$$

$$\frac{\partial p}{\partial z}=0 \tag{4}$$

In the above equations, p is the film pressure, x represents the direction parallel to the velocity U while z represents the orthogonal direction, u and w represent the velocity components along x and z directions, respectively. According to Stokes, the length parameter l can be defined as $l=(\eta/\mu)^{1/2}$ [6]. Where μ denotes the shear viscosity and η represents a new constant of the material correlates with the couple stress property. And Stokes explained that the value of η is easy to be determined through some experiment. The length l could be considered as the characteristic length or the molecular length of polar addition in the Newtonian lubricant. The boundary conditions are as follows.

At the lower bearing surface (z=0)

$$u=U,\frac{\partial^2 u}{\partial z^2}=0 \tag{5}$$

$$w=0 \tag{6}$$

While at the upper surface (z=h)

$$u=0,\frac{\partial^2 u}{\partial z^2}=0 \tag{7}$$

$$w=0 \tag{8}$$

According to Eq.°4, the value of p is fixed in the orthogonal direction. Then integrate Eq.°3 with respect to z with the boundary conditions of u, the expression of u could be obtained as

$$u=U\left(1-\frac{z}{h}\right)+\frac{1}{2\mu}\frac{dp}{dx}\left\{z\left(z-h\right)+2l^2\left[1-\frac{\cosh\left(\left(2z-h\right)/2l\right)}{\cosh\left(h/2l\right)}\right]\right\} \tag{9}$$

Substitute the expression of u in Eq.°9 into Eq.°2, then integrate Eq.°2 with respect to z with the boundary conditions Eq.°6 and Eq.°8, the one-dimensional modified non-Newtonian Reynolds-type equation governing the hydrodynamic film pressure for the slider bearing with power-function-shaped film can be derived as

$$\frac{d}{dx}\left(s\left(h,l\right)\frac{dp}{dx}\right)=6\mu U\frac{dh}{dx} \tag{10}$$

where the function $s(h, l)$ is defined as

$$s(h,l) = h^3 - 12l^2\left[h - 2l\tanh\left(\frac{h}{2l}\right)\right] \tag{11}$$

In Eq.°11, when couple stress parameter l approaches zero, $s(h,l)$ yields to h^3, then Eq.°10 would reduce to the expression of classical Newtonian lubricant case. To simplify the computational process and have general results, non-dimensional variables are introduced into the following part. The non-dimensional quantities are defined as follow

$$x^* = \frac{x}{L}, z^* = \frac{z}{h_0}, u^* = \frac{u}{U}, w^* = \frac{Lw}{Uh_0},\ p^* = \frac{ph_0^2}{\mu UL}, l^* = \frac{l}{h_0}, h^* = \frac{h}{h_0}, h_r^* = \frac{h_r}{h_0} \tag{12}$$

Substituting Eq.°12 into Eq.° 10 and Eq.°11, the expressions can be transformed into

$$\frac{d}{dx^*}\left(s^*\left(h^*,l^*\right)\frac{dp^*}{dx^*}\right) = 6\frac{dh^*}{dx^*} \tag{13}$$

$$s^*\left(h^*,l^*\right) = h^{*3} - 12l^{*2}\left[h^* - 2l^*\tanh\left(\frac{h^*}{2l^*}\right)\right] \tag{14}$$

Integrating Eq.°13 with respect to x^*, could obtain the following equation

$$\frac{dp^*}{dx^*} = \frac{6h^*}{s^*\left(h^*,l^*\right)} + \frac{e}{s^*\left(h^*,l^*\right)} \tag{15}$$

where the parameter e represents integration constant. The pressure at the inlet is zero, with this boundary condition, integrating Eq.°15 with respect to x^*, could acquire the pressure distribution as follow:

$$p^*\left(x^*\right) = 6s_M^*\left(x^*,l^*\right) + es_N^*\left(x^*,l^*\right) \tag{16}$$

where

$$s_M^*\left(x^*,l^*\right) = \int_0^{x^*}\frac{h^*}{s^*\left(x^*,l^*\right)}dx^* \tag{17}$$

$$s_N^*\left(x^*,l^*\right) = \int_0^{x^*}\frac{1}{s^*\left(x^*,l^*\right)}dx^* \tag{18}$$

At the outlet where the pressure is zero, the integration constant e could be represented as the following expression

$$e = \frac{-6s_M^*\left(x^* = 1,l^*\right)}{s_N^*\left(x^* = 1,l^*\right)} \tag{19}$$

In Eq.°15, when the pressure gradient equals to zero, using the expression of non-dimensional lubricant film thickness, the maximum pressure location could be obtained as follow:

$$x_M^* = 1 - \left(\frac{-2-e}{12} \right)^{1/s} \tag{20}$$

In Eq.°16, let the $x^*=x^*_M$, then the maximum pressure p^*_M along the x direction can be obtained. Integration Eq.°16 over the film region, the dimensionless load carrying capacity could be acquired:

$$W^* = \frac{Wh_0^2}{\mu UL^2 B} = \int_0^1 p^* dx^* = 6S_M^*\left(x^*, l^*\right) + eS_N^*\left(x^*, l^*\right) \tag{21}$$

The parameter B in Eq.°21 represents the bearing's width, and

$$S_M^*\left(x^*, l^*\right) = \int_0^1 s_M^*\left(x^*, l^*\right) dx^* = \int_0^1 \int_0^{x^*} \frac{h^*}{s^*\left(x^*, l^*\right)} dx^* \tag{22}$$

$$S_N^*\left(x^*, l^*\right) = \int_0^1 s_N^*\left(x^*, l^*\right) dx^* = \int_0^1 \int_0^{x^*} \frac{1}{s^*\left(x^*, l^*\right)} dx^* \tag{23}$$

The friction coefficient is defined as:

$$\mu_s = -\frac{F_L L}{Wh_0} = -\frac{F_L^*}{W^*} \tag{24}$$

The non-dimensional volume flow rates are the same at any location in the x direction, so in this report, considering the location of $x^*= x^*_M$, the non-dimensional volume flow rate could be expressed as follow:

$$Q^* = \frac{q_x}{Uh_0 B} = \int_0^{h^*} u^* dx^* = \frac{h^*}{2}\Big|_{x^*=x_M^*} \tag{25}$$

When non-Newtonian lubricant is used in the bearing, with the rise of viscosity, the rise of temperature would be much severer and should be taken into consideration. Following the discussion of Hamrock [27], the non-dimensional temperature rise could be derived as:

$$\Delta t^* = \frac{\rho g J C_p h_0^2}{2\mu UL} \Delta t = -\frac{F_L^*}{h^*} \tag{26}$$

where ρ is lubricant density, g represents the gravitational acceleration, J denotes the Joule's mechanical equivalent of heat, and C_p is the specific heat of the lubricant at constant pressure.

Results and Discussion

The non-dimensional parameter $l^*=l/h_0=(\eta/\mu)^{1/2}/h_0$ is used as the characteristic parameter to investigate the effects of couple stresses upon the slider bearing. In this research, four values of the non-dimensional parameter l^* (0, 0.1, 0.3, 0.5) were chosen, and the function-power r varies from 0.2 to 4.0. For each certain value of r, the average film thickness is set to be the same. In this condition, with the change of r, the effects on the bearing's steady-state performance could be considered to be caused only by the varying film profiles.

Pressure

Lines in Fig.°2 depicts the distribution of non-dimensional film pressure at $l^*=0.2$, for $x^*=0\sim1$ and r=1, 2, 3, 4, respectively. It clearly shows that with the rising of the value of r, the pressure is decreased within $x^*=0\sim0.33$, while in $x^*>0.33$, the magnitude of film pressure for each r varies diversely, but the variation tendencies are similar. With the rising of the value of r, the location of the maximum pressure is shifting toward the inlet gradually, and the value of the maximum pressure becomes larger at first and then reduces. Fig.°3 presents the dimensionless maximum pressure p_M^* for r=0.2~4.0 at l^*=0, 0.1, 0.3, 0.5, respectively. It is apparent that with different values of l^*, the variation tendency of the dimensionless maximum pressure p_M^* is similar. When r>1.8, the curves of different l^* are almost parallel to each other which indicates that variation tendencies are nearly the same. It also shows that with larger value of the couple stress parameter l^*, the effects on the non-dimensional maximum pressure are much more obvious. And for each value of l^*, the p_M^* reaches the maximum value at r=2. This result would be quite useful for determining the optimal profile during the design of slider bearings.

Load capacity

Fig.°4 describes the variation of dimensionless load capacity for r=0.2~4.0 at l^*=0, 0.1, 0.3, 0.5. It clearly reveals that effects of couple stresses upon load carrying capacity is not obvious when l^* is small (l^*=0.1). With rising of the value of l^* (l^*=0.3, 0.5), the discrepancies between couple stress lubricants and Newtonian case are more and more apparent. When r<1, i.e. the profile of the bearing is concave, increasing of the load carrying capacity are sharp. While r>1, i.e. the profile of the bearing is convex, the variations are relatively gentle. For all the values of l^*, with equal average film thickness, the non-dimensional load carrying capacity W^* reaches the maximum value at r=1.5.

Fig.°5 shows the changes of friction parameters at l^*=0, 0.1, 0.3, 0.5 for r=0.2~4.0. For all the values of l^*, the values of the friction parameter reach minimum values at r=1.0. When r<0.6, the reducing tendency of the friction parameter is quite sharp, while r>0.6, the variation becomes relatively gentle gradually.

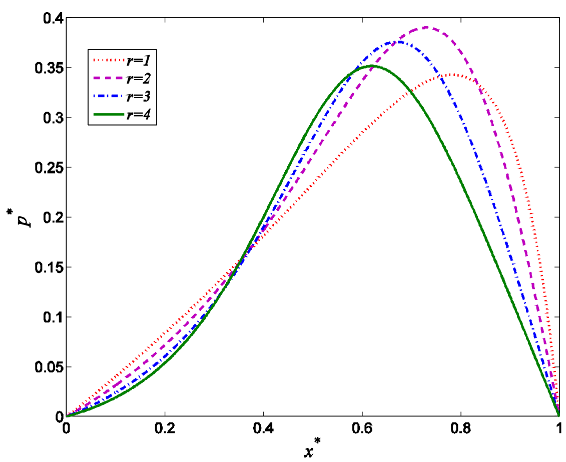

Fig.°2 Dimensionless film pressure distribution at l^*=0.2

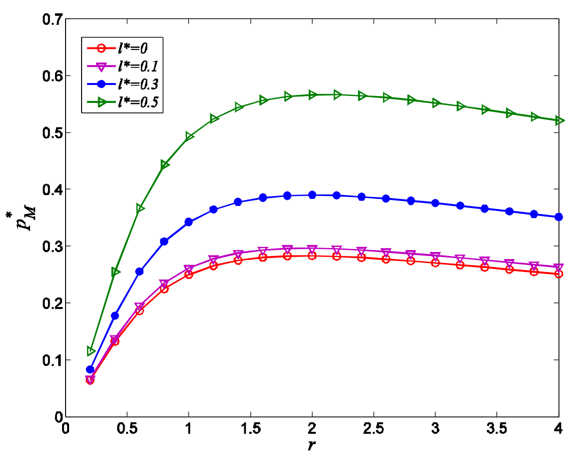

Fig.°3 Change of maximum pressure P^*_Mat l^*=0, 0.1, 0.3, 0.5 for r=0.2~4.0.

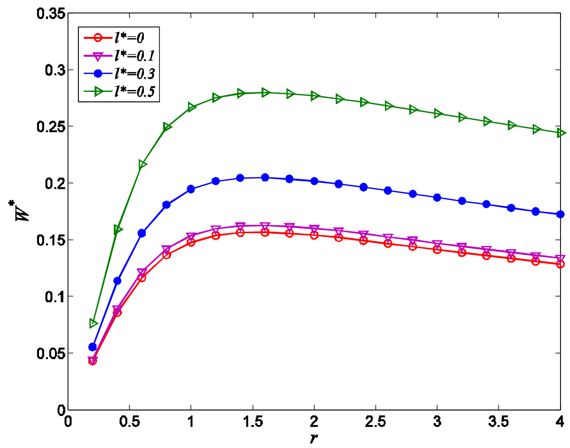

Fig.°4 Load carrying capacity at l^*=0, 0.1, 0.3, 0.5 for r=0.2~4.0.

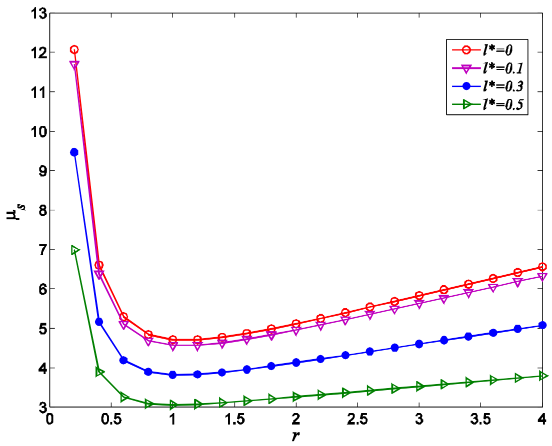

Fig.°5 Friction parameters at l^*=0, 0.1, 0.3, 0.5 for r=0.2~4.0.

Volume flow rate

Fig.°6 exhibits the changes of volume flow rate for r=0.2~4.0 with different l^*. As the rise of the value of r, the values of the volume flow rate reduce for all of the couple stress parameter l^*. When r<2, the decrease of the volume flow rate is quite fast, while r>2, the variation is relatively slight. For all the values of the couple stress parameter l^*, the differences of volume flow rates corresponding to each l^* are distinguishing, but not as obvious as the parameters analyzed previously.

Temperature rise

Fig.°7 manifests the temperature rise under different values of couple stress parameters l^* respect to the variations of r. With the increasing of the value of r, the temperature rise becomes larger. The variations of the temperature rise during r>2 are much smaller than r<2. And as the value of l^*becomeslarger, the differences among the values of the temperature rise are more obvious.

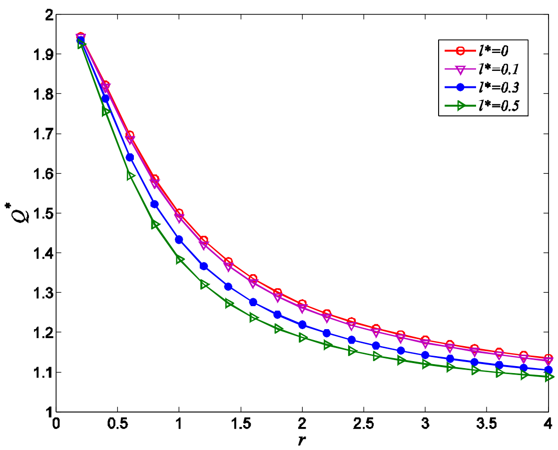

Fig.°6 Volume flow rate at l^*=0, 0.1, 0.3, 0.5 for r=0.2~4.0.

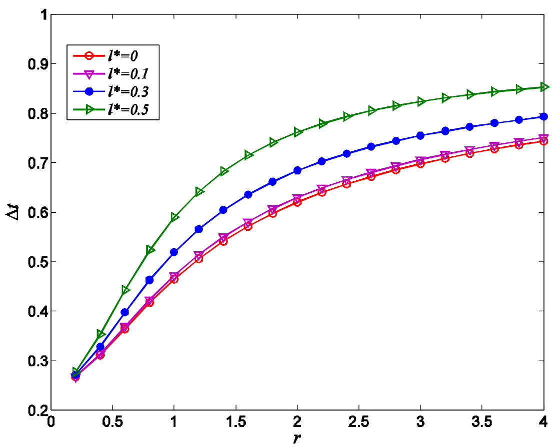

Fig.°7 Temperature rise at l^*=0, 0.1, 0.3, 0.5 for r=0.2~0.4.

Conclusions

With the discussions above, the following conclusions could be obtained

(a) As the value of the non-dimensional couple stress parameter l^* becomes larger, the effects of the couple stress on the steady-state performances become more obvious correspondently. When the couple stress parameter l^*=0.1, the difference with the Newtonian lubricant case is quite small. When the temperature rise is tolerable, to improve the steady-state performance of the bearings, the couple stress parameter of the lubricant should be chosen larger enough.

(b) When the profile of bearing is concave, i.e. r<1, the variations of the dimensionless parameters of steady-state performances compared to the convex profile, i.e. r>1, are quite sharp. And the steady-state performances with convex profile are much better than concave ones. So during the designing procedure, the convex profile should be considered preferentially.

(c) The non-dimensional maximum pressure reaches maximum value at r=1.5. When r=1.4, the load carrying capacity reaches the peak. The friction parameter reaches minimum value at r=1.0. For all the different values of l^*(l^*=0, 0.1, 0.3, 0.5), each of these conclusions is validated. So when the mean film thickness is a constant, to get the optimal steady-state performance, the power of the profile could be chosen as r=1.0~1.5.

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References

[1] El-Gamal, H. A. and T. H. Awad., "Optimum model shape of sliding bearings for oscillating motion." *Tribology International*, Vol. 27 No.3(2005), pp. 189-196.

[2] Lin, J. , "Optimal design of one-dimensional porous slider bearings using the Brinkman model." *Tribology International*, Vol. 34 No.1 (2001), pp. 57-64.

[3] Chang, M. and C. Cheng. , "Optimal design of slider shape for satisfying load demands under specified operation conditions." *Tribology International*, Vol. 38 No. 8 (2005), pp. 757-768 .

[4] Chu, H., "Shape optimum design of slider bearings using inverse method." *Tribology International*, Vol. 40 No. 5 (2007), pp. 906-914.

[5] Alyaqout, S. F. and A. A. Elsharkawy. , "Optimal film shape for two-dimensional slider bearings lubricated with couple stress fluids." *Tribology International,* Vol. 44 No. 3 (2011), pp. 336-342.

[6] Stokes, V. K. , "Couple Stresses in Fluids". *Physics Fluids,* Vol. 9 No. 9 (1966), pp. 1709-1715.

[7] Ramanaiah, G. and J. N. Dubey., "Optimum slider profile of a slider bearing lubricated with a micropolar fluid." *Wear,* Vol. 42 No. 1(1977), pp. 1-7.

[8] Lin JR and Hung CR., "Analysis of dynamic characteristics for wide slider bearings with an exponential film profile." *Journal of Marine Science and Technology*, Vol. 12 No. 3 (2004), pp. 217-221.

[9]Lin JR, Lu RF and Chang TB. "Derivation of dynamic couple-stress Reynolds equation of sliding-squeezing surfaces and numerical solution of plane inclined slider bearings." *Tribology International,* Vol. 36 No. 9 (2003)pp. 679-685.

[10] Lin JR and Lu Y M. "Steady-state performance of wide parabolic-shaped slider bearings with a couple stress fluid." *Journal of Marine Science and Technology*, Vol. 12 No. 4(2004), pp. 239-246.