Theorem 13. *Let $C \in \mathcal{C}$. Then $C = C_{I_C}$, that is*

$$C(x,y) = \min\{t \in [0,1] : I_C(x,t) \geqslant y\},\ x,y \in [0,1]. \quad (18)$$

Proof. Let $x, y \in [0,1]$. From definition of I_C and from monotonicity of the conjunction C we obtain

$$\begin{aligned} &I_C(x, C(x,y)) = \\ &\quad = \max\{t \in [0,1] : C(x,t) \leqslant C(x,y)\} \geqslant y. \end{aligned}$$

Thus we have $C(x,y) \in \{t \in [0,1] : I_C(x,t) \geqslant y\}$ and

$$C(x,y) \geqslant C_{I_C}(x,y). \quad (19)$$

From $I_C(x, C_{I_C}(x,y)) \geqslant I_C(x, C_{I_C}(x,y))$ by (RP) we obtain

$$C(x, I_C(x, C_{I_C}(x,y))) \leqslant C_{I_C}(x,y). \quad (20)$$

Because C_I is an implication which is right-continuous with respect to the second variable (from Theorem 10), I_C and C_{I_C} fulfil the residuation principle (from the Theorem 2). Thus, by the trivial inequality $I_{C_I}(x,y) \geqslant I_{C_I}(x,y)$ we obtain $I_C(x, C_{I_C}(x,y)) \geqslant y$. Additionally from (20) and the monotonicity of the conjunction it follows $C(x,y) \leqslant C_{I_C}(x,y)$. Hence and by (19) we obtain the equality $C = C_{I_C}$. □

5. Conclusion

In this contribution the residuation concept that connects the fuzzy implication together with the fuzzy conjunction is examined. The method of regaining the connectives that play the roles of generators are shown in Theorems 12 and 13. Further examination can concern the characterizations of induced implications fulfilling other properties.

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