

# Portfolio analysis based on multi-objective optimization algorithm

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**Abstract:** Evolutionary algorithm in risk minimization and the expected return maximization of bi-objective for portfolio optimization applications has received widespread attention. Although the problem is a quadratic programming (QP) problem, the practical investment problems tend to make variables discontinuous and introduce other complexities. Under this circumstance, a normal QP solution is not always capable of finding a feasible solution. In this paper the NSGA-II algorithm is used to deal with the situation for classical QP unconventional methods. Results demonstrate that the evolutionary algorithm NSGA-II can find out the front of the conflict optimization problem which is difficult to achieve through other methods.

## Introduction

Portfolio optimization itself has conflicting standard. Among potential objectives, two objectives which are minimizing risk and maximizing expected return, i.e., the mean-variance model of Markowitz [1] has received the most attention. In these decision variables of the problem is that the initial funding will be allocated to the different proportion of available securities. Such dual objective problems that bring about trade-off solutions must be found in order to study the relationship between risk and return in a problem.

The whole problem is a kind of dual objective quadratic programming (QP) problem, the problem can be resolved via applying the QP solvers under the condition when all the constraints are linear and all of the variables are continuous[2]. However, there can be conditions that make it hard for QP solvers to apply in practice. This gives rise to a demand for decision variables that take on zero value or a non-zero value that correspondence with at least an amount of a minimum investment. In addition, users may only focus on the number of securities portfolio with limited numbers. Furthermore, there can be restrictions which imposed by government augment the complexity of the process, therefore only increase the difficulty of dealing with portfolio via classic way.

Evolutionary multi-objective optimization (EMO) has been conducted experiments in portfolio optimization problems [3]. For instance, Chang et al[4] employed genetic algorithms (GAs), tabu search as well as simulated annealing on portfolio problem with given cardinalities on the number of assets, however, it turns out that the problems on which the methods were tested were small. And other approaches were trying which include simulated annealing, the differential evolution, and local search based on the model for algorithm, etc. In this paper, conflicting objectives are solved via applying the elitist non-dominated sorting NSGA - II[5].

## Markowitz Mean-Variance Model

In this paper the Markowitz mean-variance model is employed which is a portfolio problem that has a restriction on the number of investment. It refers to how investors decide the proportion of investment on each stock after limited the upper and lower bounds of each stock investment in order to obtain a balance on minimum risk or maximum expected return in the future.

Let  $\mathbf{X} = (x_1, x_2, \dots, x_n)^T$ ,  $\mathbf{r} = (r_1, r_2, \dots, r_n)^T$ . The portfolio expected return is defined as  $\mathbf{r}^T \mathbf{x}$ , The variance of portfolio return is defined as  $\mathbf{x}^T \mathbf{\Sigma} \mathbf{x}$ .  $\mathbf{r}^T \mathbf{x}$  is the expected return of portfolio  $\mathbf{x}$ ,  $\mathbf{x}^T \mathbf{\Sigma} \mathbf{x}$  is risk of portfolio  $\mathbf{x}$ .

According to the agreed conditions above, the portfolio model can be expressed as:

$$\begin{cases} \min x^T \Sigma x \\ \max r^T x \\ \sum_{i=1}^n x_i = 1, x \geq 0 \end{cases} \quad (1)$$

Where,  $r_i$  means the  $i^{\text{th}}$  security yield,  $i=1,2,\dots,n$ .  $x_i$  means the proportion of investment on  $i^{\text{th}}$  security in total investment,  $\sum_{i=1}^n x_i = 1$ .  $\Sigma = (\sigma_{ij})_{n \times n}$  means covariance matrix of  $n$  kinds of security yields,  $\sigma_{ij} = \text{cov}(u_i, u_j)$ .

### NSGA-II Algorithm

In this paper, NSGA-II algorithm which has been employed efficiently to the portfolio optimization problem using Markowitz mean variance model. The process of NSGA-II is given as following:

Step1: Initialize parameters: individual number of population  $NN$ ,  $x(0)$  is initial state obtained from simulation,  $gen$  is evolutionary generation of the optimization computation, create initial population  $IniChrom$ , set  $gen = 0$ .

Step2: Optimize objective, obtain individual rank and crowding distance.

Step3: Set  $ParChrom = IniChrom$ , make tournament selection operator in  $ParChrom$ , namely two individuals are selected randomly and the better is saved in  $ParChrom$  according to crowded tournament selection operator;

Step4: Make crossover and mutation operator in  $IniChrom$ , obtain offspring population  $SonChrom$ .

Step5: Combine parent with offspring population, obtain  $MidChrom = ParChrom + SonChrom$ , perform non-dominated sorting and compute crowding distance, obtain  $NN$  optimal individual from  $MidChrom$  and composes  $IniChrom$ .

Step6:  $gen = gen + 1$ , if  $gen \leq MAXGEN$ , turn to step3.

### Simulation Study

**Data Preparation** In this paper four securities are selected as the research object for portfolio optimization analysis which are respectively 600569, (Anyang iron and steel), 000878 (Yunnan copper), 600229 (Qingdao Soda Ash), 000585 (Northeast electric).

The historical data of stock price and the future trend of the data are the basis of the portfolio investment analysis. These historical data mainly include the daily opening price, the highest and the lowest price, closing price, volume, etc. This article will exploit monthly closing price in 2007 which has a health development in the market as the data analysis of stock analysis optimization object.

In this paper, the maximum generation is 200, the crossover probability is 0.7, the mutation probability is 0.03. Suppose the upper and lower limits of investment percentage are  $L$  and  $U$  respectively. In this paper, set  $L=[0.055,0.025,0.015,0.015]$ ,  $U=[0.255,0.325,0.435,0.25]$ . Monthly return of four kinds of securities is shown in table 1. The aim of this article is to solve a dual objective optimization problem via finding a relatively optimal investment between the expected return and risk balance.

Table 1 Monthly Stock

Stock Ticker <sup>a</sup>		600569 <sup>a</sup>	000878 <sup>a</sup>	600229 <sup>a</sup>	000585 <sup>a</sup>
Monthly Stock Returns	1 <sup>a</sup>	27.60	23.15	26.68	22.61
	2 <sup>a</sup>	25.56	34.92	27.02	60.94
	3 <sup>a</sup>	16.33	2.97	14.91	-2.33
	4 <sup>a</sup>	57.31	25.67	46.36	49.70
	5 <sup>a</sup>	19.80	17.15	4.88	8.23
	6 <sup>a</sup>	-13.88	25.32	-23.93	-36.81
	7 <sup>a</sup>	16.00	41.80	4.87	30.49
	8 <sup>a</sup>	13.28	42.68	15.36	13.24
	9 <sup>a</sup>	16.41	32.92	-7.02	-5.65
	10 <sup>a</sup>	-13.01	-14.30	-15.43	-24.79
	11 <sup>a</sup>	-10.33	-36.55	0.52	1.11
	12 <sup>a</sup>	19.27	8.64	25.85	12.09

The expected return is:

$$R = (0.1453, 0.1703, 0.1001, 0.1074)^T$$

The covariance is:

$$\Sigma = \begin{bmatrix} 0.03797 & 0.02240 & 0.03367 & 0.04269 \\ 0.02240 & 0.05344 & 0.01114 & 0.02615 \\ 0.03367 & 0.01114 & 0.03847 & 0.04555 \\ 0.04269 & 0.02615 & 0.04555 & 0.07512 \end{bmatrix}$$

**Optimization Results** Figure 1 gives the Pareto front of expected return and variance. The x axis is the expected return and y axis is the variance. It can be seen from figure 1 that with the increase of the expected rate of return of portfolio, the risk of portfolio value gradually reduced. When the expected return is close to 0.16, reached the lowest value of the variance, the portfolio risk value is approximately 0.02 which is the optimal solution in this paper. At this point, the expected return is 0.15839, variance is 0.0195.

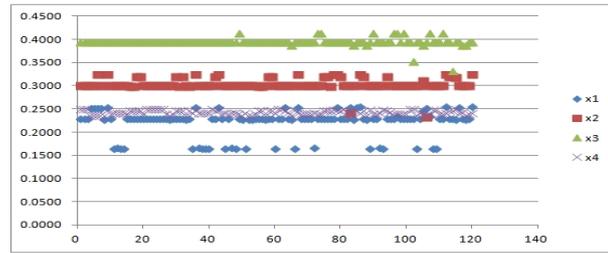
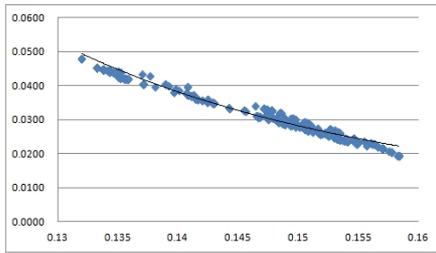


Fig.1 Pareto curve of expected return and variance

Fig.2 Investment percentage of each stock

Figure 2 describes the investment percentage of each stock, the distribution curve of x3, x2, x4 and x1 are listed from top to bottom respectively. It can be seen from figure 2 that with the increase of population evolution generation, four types of investment percentage tends to be stable in gradual. While the x3 in this area mainly distributed from 0.35 to 0.40, the x2 mainly fluctuate up and below 0.30, the x4 mainly distributed from 0.20 to 0.25 and the x1 is mainly distributed within the range of 0.15 to 0.25. From the case in the graph, the final optimization results of four investment ratio values will appear in the four main distribution ranges as shown in the figure below.

Figure 3 gives a portfolio of all the expected return from 1 to 450 generations. As it can be seen from the figure 3, the expected return of the portfolio in the first 30 generations has an obvious upward trend, increased from 0.04 to 0.15. After 30th generation, the expected return of the portfolio gradually remains stable at 0.14. Overall, the maximum expectation will appear after 30 generations, and its value remains around 0.15.

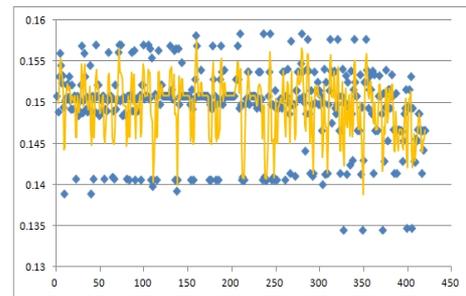
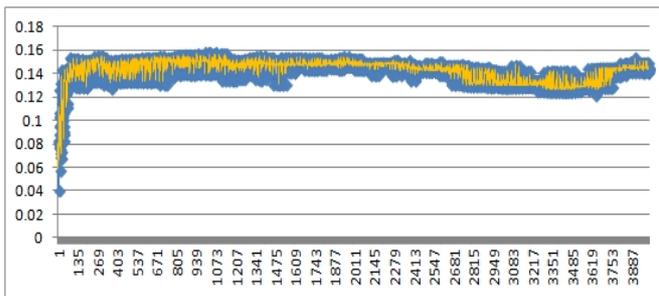


Fig.3 Distribution of portfolio expected return

Fig.4 Part distribution of portfolio expected return

Figure 4 gives the distribution of a part of portfolio expectation return, the x axis represents the number of x that has evolved, and the y axis is the expected return. It is a partial distribution of expected return which contains the optimal solution of portfolio optimization. Along with the increase of population evolution generation, the portfolio expected return in a state of fluctuation as shown in the figure. It can be seen from figure 4 that with the increase of population evolution generation, after many times of initial population evolution, the expected return tends to be stable gradually. As shown in the figure 4, the maximum expected return distributed in the range of 0.155 to 0.16, while the maximum expected return of portfolio optimization is 0.15839 in this paper.

Figure 5 contains a portfolio of all the variance from 1 to 450 generations. In contrast to the trend line of the expected return which can be seen from the figure 6, the variance of portfolio in the first 30 generations decline significantly, from 0.15 to 0.15. After 30th generation population, the expected

return of the portfolio gradually remains stable at 0.02. As can be seen from the overall trend, minimum variance will appear in the range after the 30<sup>th</sup> generation, and its value is in the range of around 0.02.

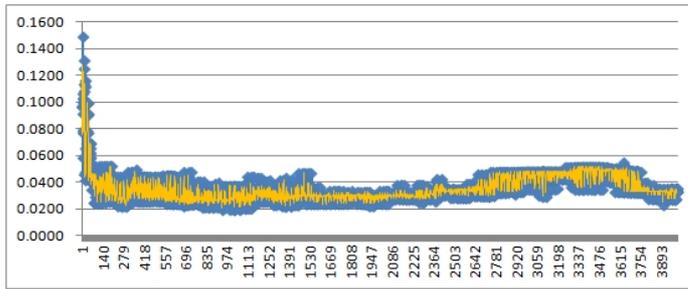


Fig.5 Distribution of portfolio variance

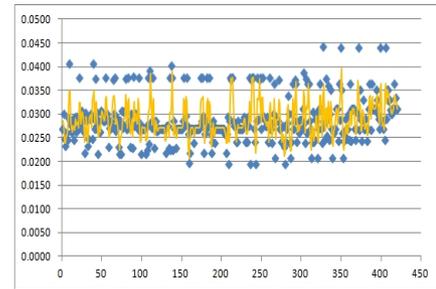


Fig.6 Part distribution of portfolio variance

Figure 6 is the diagram of portfolio variance (risk) which shows the change of variance as the increase of the evolution generation. As can be seen from figure 6, the trend line, along with the increase in population evolution generation, after many times of initial population evolution, the investment value of variance gradually tends to be stable. In contrast to the trend of the expected return, another objective of this paper is to minimize the risk value. Therefore, in the process of evolution, there is a need to find a minimum variance under the premise of the maximum expected return. As can be seen from the diagram, the minor value of risk fluctuates up and down around 0.02, in line with investor's demand; the minimum risk value is 0.0195.

Therefore, after 450 generations of population optimization, consider the target of maximizing the expected return and minimizing the risk, the relative optimal portfolio solution is:

$$X_1 = 0.2528; X_2 = 0.3240; X_3 = 0.3962; X_4 = 0.2497;$$

The proportions of four kinds of stock investment are respectively:

$$X = \{0.2528, 0.3240, 0.3962, 0.2497\};$$

The expected return:  $r^T x = 0.15839$ , Variance:  $x^T \Sigma x = 0.0195$

The results obtained at this time reached a relatively optimal solution, within the scope of this optimization can meet investors' expectations of maximizing and minimizing risk.

## Summary

In this report a multi-objective evolutionary algorithm NSGA-II is used to solve the portfolio optimization problem. From the simulation results, it can be seen that the Markowitz mean variance model can identify good Pareto solutions and maintain sufficient diversity.

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