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Unidirectional Convergence Problem of nonlinear system

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Abstract. The unidirectional convergence condition and unidirectional convergence domain problems for nonlinear system are studied. Firstly, linear dynamic closed-loop equation is obtained by designing linear switching function and corresponding control law. Secondly, the reaching position of the system is calculated based on the analyzed solution. Thirdly, unidirectional convergence condition and unidirectional convergence domain are achieved by analyzing the sign relationship between the initial state and the reaching state. If the sign of the initial state and the reaching state are same, the system state is unidirectional convergence. Otherwise, the system state is not unidirectional convergence. If the initial state is positioned in unidirectional convergence domain, the movement of the state is unidirectional convergence. Lastly, the simulation results proved the above conclusions.

Introduction

Sliding mode control(SMC) method is widely used in nonlinear control system.The procedure of SMC includes reaching stage and sliding stage. Almost SMC study has payed attention on the sliding stage because its good performance. But there are many projects have requirements on the reaching time and reaching point,and it is necessary to study the reaching stage of SMC. Reaching law is a famous method which used in the reaching stage [1].There are so many study to improve the reaching law to obtain good performance[2,3].Not only this, the motion in phase plane is discussed in [4], the reaching time of the system is estimated, and the relationship of the reaching time and initial state of the system is obtained. The above study are connected with the dynamic performance of the reaching stage and the reaching time, but the topic of reaching position is not mentioned.

With bounded control,[5,6] studied the reaching position problem for linear system.The unidirectional convergence condition is proposed in detail.But the unidirectional convergence problem for nonlinear system is not mentioned. The paper is aimed to study the following problems by the second order nonlinear system as the control plant and the linear switching function as the sliding mode.

The description of the problem

The second order nonlinear system is as

$$\begin{cases} \dot{x}_1 = x_2, x_1(0) \\ \dot{x}_2 = f(x) + b(x)u, x_2(0) \end{cases}. \tag{1}$$

where $x=[x_1,x_2]^T$ is the system state, $x_1(0), x_2(0)$ are the initial state of the system, $f(x), b(x)$ are nonlinear term and $b(x)\neq 0$.Now the linear switching function is designed as

$$s = cx_1 + x_2 \quad . \tag{2}$$

where c is constant and $c>0$. Using the equal control +switching control to design the control law as

$$u = -b^{-1}(x)[cx_2 + f(x) + \eta\,\mathrm{sgn}(s)] \quad . \tag{3}$$

where $\eta>0$.There are three basic problems in the SMC including the existence problem, the reaching problem and the stability problem. If the sliding mode is existence and reachable, the stability is easily to prove. So the existence and reaching will be discussed in detail. According to the linear switching function(2), $\dot{s} = cx_2 + \dot{x}_2$ .Put $\dot{x}_2$ in (1) and (3) into $\dot{s} = cx_2 + \dot{x}_2$, $\dot{s} = -\eta sgn(s)$.Because of $\eta>0$, if $s>0$ then $\dot{s}<0$.Else if $s<0$,then $\dot{s}>0$, so the sliding mode is existence and reachable.

The unidirectional convergence condition

The unidirectional problem is analyzed by the analytical solution of the closed-loop dynamic equation. Put (3) into (1), the closed-loop equation is $\ddot{x}_1 = -cx_2 - \eta\,\mathrm{sgn}(s)$.By using the above designing method, the closed-loop dynamic equation is linear to obtain the analytical solution whether the control plant is linear or not actually. That's mean the analytical solution $x_1(t), x_2(t)$ could be obtained by resolving the closed-loop dynamic equation.

The reaching time t_r is obtained by $\dot{s} = -\eta sgn(s)$,the solution is $t_r = \frac{|s(0)|}{\eta}$,where $s(0)$ is the initial value of switching function. Put t_r into the expression of $x_1(t), x_2(t)$,the reaching position $x_1(t_r), x_2(t_r)$ could be calculated. If $s>0$, $x_1(t) = -\frac{A}{c}e^{-ct} - \frac{\eta}{c}t + B$, $x_2(t) = Ae^{-ct} - \frac{\eta}{c}$,where $A = x_2(0) + \frac{\eta}{c}$, $B = x_1(0) + \frac{A}{c}$ .If $s<0$, $x_1(t) = -\frac{D}{c}e^{-ct} + \frac{\eta}{c}t + E$, $x_2(t) = De^{-ct} + \frac{\eta}{c}$,where $D = x_2(0) - \frac{\eta}{c}$, $E = x_1(0) + \frac{D}{c}$. According to the definition of unidirectional convergence in[6], the conclusion is as following, if $x_1(t_r)\ x_1(0) > 0$, the system state has the same sign in the procedure of reaching stage, that's mean unidirectional convergence.If $x_1(t_r)\ x_1(0) < 0$,the sign of the system state has changed in the reaching stage, that's mean not unidirectional convergence.

The unidirectional convergence domain

According to the above analysis, if the initial state $x(0)$ and the control are given, the reaching position is different. If the control is limited, there must be a convergent domain which could guarantee $x(0)$ is unidirectional convergent.

Besides considering the sign of $x_1(t_r)$ $x_1(0)$, the sign of initial state $s(0)$ could decide the expression of $x_1(t_r)$. So the unidirectional convergence condition is descripted as following,if $x_1(0)<0$ and $s>0$,then unidirectional convergence condition is $x_1(t_r)<0$.Put the expression of A, B and t_r into $x_1(t)=-\frac{A}{c}e^{-ct}-\frac{\eta}{c}t+B$,the unidirectional convergence domain is $\frac{\eta}{c}<[x_2(0)+\frac{\eta}{c}]e^{-c\frac{s(0)}{\eta}}$.If $x_1(0)>0$ and $s(0)>0$, then unidirectional convergence condition is $x_1(t_r)>0$ in.Put the expression of A, B and t_r into $x_1(t)=-\frac{A}{c}e^{-ct}-\frac{\eta}{c}t+B$,the domain is $\frac{\eta}{c}>[x_2(0)+\frac{\eta}{c}]e^{-c\frac{s(0)}{\eta}}$. If $x_1(0)>0$ and $s(0)<0$, then unidirectional convergence condition is $x_1(t_r)>0$ .Put the expression of D, E and t_r into $x_1(t)=-\frac{D}{c}e^{-ct}+\frac{\eta}{c}t+E$,the domain is $-\frac{\eta}{c}>[x_2(0)-\frac{\eta}{c}]e^{c\frac{s(0)}{\eta}}$ .IF $x_1(0)<0$ and $s(0)<0$, then unidirectional convergence condition is $x_1(t_r)<0$ .Put the expression of D, E and t_r into $x_1(t)=-\frac{D}{c}e^{-ct}+\frac{\eta}{c}t+E$,the domain is $-\frac{\eta}{c}<[x_2(0)-\frac{\eta}{c}]e^{c\frac{s(0)}{\eta}}$.

In conclusion, if c,η is given,the unidirectional convergence domain could be obtained. If $c=0.5,\eta=0.5$,there are two system states named as P1,P2, the related value are shown in Tab.1,the position is shown in Fig.1.

Tab.1 the coordinate of P1~P2

point	coordinate	Wether in convergent domain	$x_1(t_r)$	unidirectional convergence?
P1	(-2,2)	yes	-0.2073	yes
P2	(1,-2)	no	-4.2313	no

Simulation Results

To verify the above analysis, (1) is the control plant and $f(x)=x_1x_2$, $b(x)=1$, (2) and (3) are respectively the switching function and control law, where $c=0.5,\eta=0.5$.Using P1~P2 as the initial position, the simulation results are shown in Fig.2~Fig.4.

In Fig.2,P1,P2 are both convergent from initial state to the sliding mode, and they are agree with the requirements of the SMC.Fig.3 is shown the sign of $x_1(0)$ are same as the sign of $x_1(t_r)$ for P1,it is unidirectional convergence. Fig.4 is shown the convergent procedure of P2, $x_1(0) > 0$, $x_1(t_r) < 0$,the sign of the state is changed, it is not unidirectional convergence. So the simulation results could verify the correctness of theory analysis.

Conclusion

（1）The unidirectional convergence problem must be solved in sliding mode control. Though it is not included in the requirement of the sliding mode, it is an urgent problem in the engineering.（2）If the switching function is linear, the closed-loop dynamic equation is easy to obtain the analyzed solution whether the control plant is linear or nonlinear. This is convenient to study the unidirectional convergent problem.（3）To solve the above unidirectional convergent problem, the key stage is to obtain the linear closed-loop dynamic equation. If the switching function is changed, the analysis solution is not obtained directly. This is the following study target.

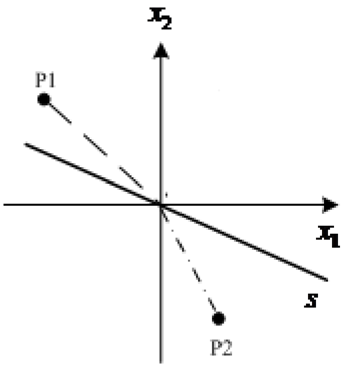

Fig.1 P1,P2

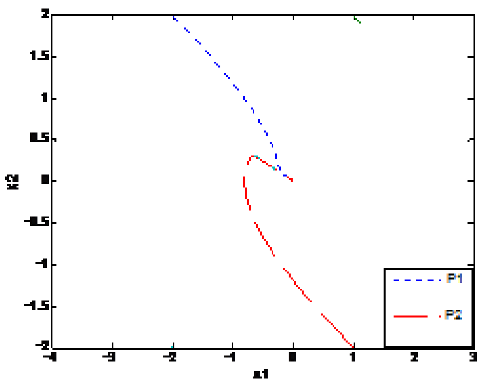

Fig.2 the phase trajectory

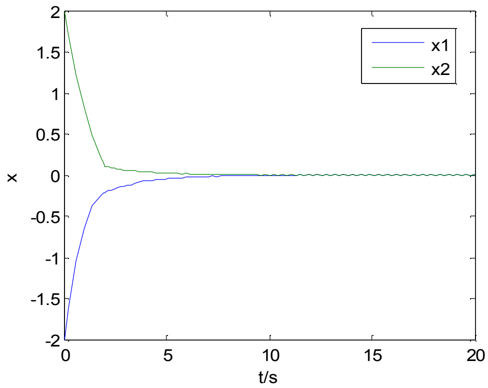

Fig.3 the changing procedure of P1

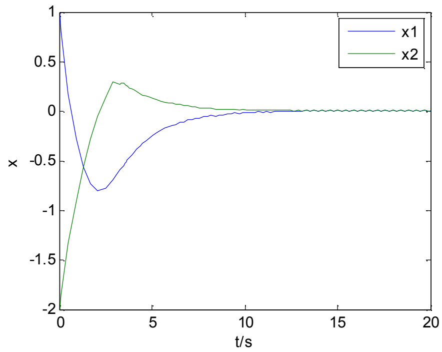

Fig.4 the changing procedure of P2

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