Analytic geometry method proved by finite difference on beam angular displacement

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Abstract: Based on the analytical geometry method, small-plane bending deformation theory and approximate deflection curve equation, a simplified calculation method of section angular displacement will be proposed in this paper, which have been certified by the finite difference method. Then, the influence of number and space of measuring points on the experiment result had been detailed analyzed. Apparently, the method has good precision on premise of sufficient points, which could be an algorithms and experimental norm for beam angular displacement test on bridge loading test and damage identification.

Introduction

Generally, strain and displacement of the key cross-section are the dominating monitoring index in bridge structure analysis and experiment. Strain is more sensitive to the degree of damage of the current cross-section than the neighboring. Displacement is closely related to the stiffness of all structure sections within the whole affected line (plane) range. Damage of few sections have little difference to the displacement experiment results of key sections, which embarrassed bridge experiment, particularly , in old bridge. Strain calibration coefficient of some crack-sections is higher than norm while the displacement is lower. Therefore, making a reasonably assessment of damage of every experimental section and deducing the damage property of the concerned section, combined with mechanical analysis of the bridge structure, have become an issue to most bridge engineers[1]-[7].

Angular displacement calculation method based on analytic geometry

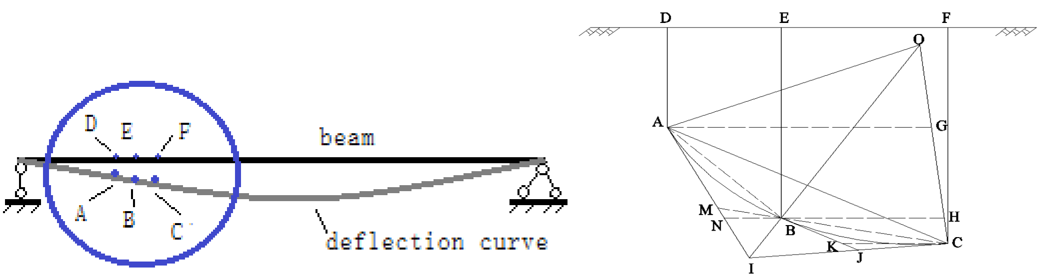

Fig.1 Deflection measurement point

Analytic Geometry, generally, as one of the most simple and scientific methods to interpret natural phenomena is used widely. As show in fig.1, a simplified beam bridge model deflected under load(force/moment), target unit DEF on the initial position while the centroid node E. A, B, C on the corresponding deflection curve for the centroid O of ABC with vertical displacement value f_A, f_B, f_C respectively. Assuming that AI BJ CI tangent to deflection curve at A, B, C and DF // AG // NH // CK.

Above all, we can derived that

f_A is length of DA, f_B is length of EB, f_C is length of FC.

$$\text{So} \quad \angle GAB \approx \frac{BE-AD}{L} = \frac{f_B - f_A}{L} \tag{1}$$

$$\angle HBC \approx \frac{CF-EB}{L} = \frac{f_C - f_B}{L}$$

Where L is length of DE and EF.

$$\text{And so} \qquad \theta_A = \angle GAB + \angle BAI = \frac{f_B - f_A}{L} + \frac{\angle AOC}{4} \tag{2}$$

$$\theta_B = \angle HBJ = \angle HBC + \angle CBJ = \angle HBC + \frac{\angle AOC}{4} = \frac{f_C - f_B}{L} + \frac{\angle AOC}{4} \tag{3}$$

Where θ_A, θ_B is the angular displacement at node A and B of unit ABC.

$\because \angle HBC = \angle MBN = \angle ABN - \angle ABM$

And $\quad \because AG//NH$

$\therefore \angle ABN = \angle GAB = \dfrac{f_B - f_A}{L}$

$\because \angle ABM = \angle BAC + \angle BCA$

And $\quad \because \angle BAC = \angle BCA = \dfrac{\angle AOC}{4}$

$$\therefore \angle ABM = 2 \times \frac{\angle AOC}{4} = \frac{\angle AOC}{2} \tag{4}$$

$$\therefore \angle HBC = \angle ABN - \angle ABM = \frac{f_B - f_A}{L} - \frac{\angle AOC}{2} \tag{5}$$

From the Eq. (3) and (5), we have

$$\theta_B = \angle HBC + \angle CBJ = \angle HBC + \frac{\angle AOC}{4} = \frac{f_B - f_A}{L} - \frac{\angle AOC}{4} \tag{6}$$

The average of the Eq. (3) and (6) can be obtained:

$$\theta_B = \frac{1}{2}\left(\frac{f_C - f_B}{L} + \frac{\angle AOC}{4} + \frac{f_B - f_A}{L} - \frac{\angle AOC}{4}\right) = \frac{f_C - f_A}{2L}$$

$$\text{In the same way:} \quad \theta_C = \angle KCI = \angle BCI - \angle BCK = \frac{\angle AOC}{4} - \frac{f_C - f_B}{L} \tag{7}$$

Where θ_C is the angular displacement at node C of unit ABC.

For the average deflect angle calculation, we can use Eq. θ_A and θ_C for both endpoints, and use Eq. θ_B for the points among both endpoints. Because of the direction of the geometric method can not be considered generally, so it deserves much more attention in the calculation.

Angular displacement proved by finite difference method

The beam in the case of symmetrical bending deformation rear beam axis with a smooth curve in the plane, which is the deflection curve [8] - [14]. Assuming that the beam deflection curve equation for deformation can be described as: $y = f(x)$, apparently, the displacement value near by the measuring point x can be expressed as: $y = f(x + \Delta x)$. The equation can be deployed with Taylor series as

$$y = f(x+\Delta x) = f(x) + \Delta x \frac{df}{x} + \frac{\Delta x^2}{2!}\frac{d^2 f}{x^2} + \frac{\Delta x^3}{3!}\frac{d^3 f}{x^3} + \cdots \tag{8}$$

$\therefore \theta = \dfrac{df}{x} = \dfrac{f(x+\Delta x)+f(x)}{\Delta x} + o(\Delta x)$, where $o(\Delta x)$ is a higher-order infinitesimal or truncation error for θ. When $o(\Delta x)$ is sufficiently small, the angular displacement of the section is:

$$\theta = \frac{df}{x} \approx \frac{f(x+\Delta x)-f(x)}{\Delta x} \qquad \text{First-order forward difference}$$

$$\theta = \frac{df}{x} \approx \frac{f(x)-f(x-\Delta x)}{\Delta x} \qquad \text{First-order backward difference}$$

$$\theta = \frac{df}{x} \approx \frac{f(x+\Delta x)-f(x-\Delta x)}{2\Delta x} \qquad \text{First-order central difference} \tag{9}$$

If the distance of point can be seen as increasing step length of point position, the finite difference equation can be achieved by analytic geometry:

$$\theta_B = \frac{f_C - f_A}{2L} = \frac{f(x+\Delta L)-f(x-\Delta L)}{2\Delta L} = \frac{f(x+\Delta x)-f(x-\Delta x)}{2\Delta x} \tag{10}$$

Curvature of the cross-section is

$$\frac{1}{\rho} = \frac{d^2 f}{x^2} \approx \frac{f(x+\Delta x)-2f(x)+f(x-\Delta x)}{\Delta x^2} \qquad \text{Second-order difference} \tag{11}$$

It can be obtained from equation (11) by using the finite difference method for solving unit center angle:

$$\theta_o = \frac{\Delta x}{\rho} = \frac{d^2 f}{x^2} \times \Delta x \approx \frac{f(x+\Delta x)-2f(x)+f(x-\Delta x)}{\Delta x},$$

Where in the Equation:

$f(x)$ -The beam displacements were measured at the point x, unit: mm;

$f(x+\Delta x)$ - The beam displacement at the point which is Δx far before the point x, unit: mm;

$f(x-\Delta x)$ -The beam displacement at the point which is Δx far after the point x, unit: mm;

Δx -The level distance bettween two adjacent measuring point, unit: mm;

θ_o - unit center angle, unit: rad.

$$\text{So } \angle \text{AOC} = \frac{2L}{\rho} = \frac{2(f(x+\Delta x)-2f(x)+f(x-\Delta x))}{\Delta x} \tag{12}$$

Where ρ is the curvature radius of deflection curve of unit ABC.

So the angular displacement of unit ABC can be established by analytic geometry below

$$\theta_A = \frac{f_B - f_A}{L} + \frac{\angle AOC}{4} = \frac{f_B - f_A}{L} + \frac{f_C - 2f_B + f_A}{2L}$$
$$\theta_C = \frac{f_B - f_A}{L} - \frac{\angle AOC}{4} = \frac{f_B - f_A}{L} - \frac{f_C - 2f_B + f_A}{2L} \qquad (13)$$

Example

To verifying the influence and precision of the difference step-length on endpoint computation results, a pre-stressed concrete hollow-slab will exemplify the issue with concrete grade C50, elastic modulus $E_c = 3.45 \times 10^4$MPa , which span L=16m. In this paper, structure displacement value on dead load as the comparative index allowed for the dead load account for more than 60% in small and medium-span bridge. The division of units is 4, 8 and 16 universally. Finally, exact, finite difference and analytic geometry solution of slab displacement will be deployed to analyze the error margin of precision for angular displacement in different divisions respectively. As showing in the figure 2, 3 and table1:

Fig.2 The division of displacement point

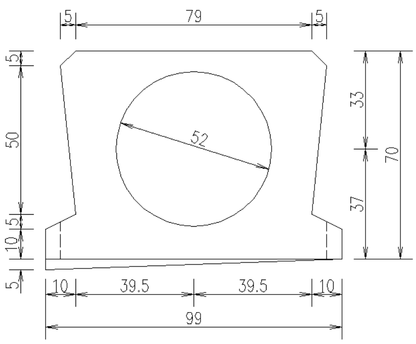

Fig.3 Hollow slab section

Conclusion

1. A simplified calculation method of bridge structural angular displacement have been presented and verified by finite difference method. The error-margin do meet the requirement of the accuracy of beam bridge loading experimental results.

2. The more measured point, the higher of the calculation accuracy of the beam angular displacement for each algorithm. As mentioned above, the maximum error margin is 19 ‰ when divided into 8 whereas 4 ‰ when 16 divisions.

3. Generally, analytic geometry and finite difference method applied universally in bridge structural field, but the former shared in both endpoints with a higher precision.

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Table1. Angular displacement calculation results and margin of error for different methods

Number	Calculation solution	Support point	L/16	L/8	3L/16	4L/16	4L/16	5L/16	7L/16	8L/16	9L/16	10L/16	11L/16	12L/16	13L/16	14L/16	15L/16	Support point
4 equal portions	Displacement[mm]	0.00				-7.99				-11.21				-7.99				0.00
	Exact Solution[rad /10^3]	2.222				1.528				0				-1.528				-2.222
	Finite difference solution[rad/10^3]	1.998				1.402				0.000				-1.402				-1.998
	Error[%]	-10.1				-8.3				/				-8.3				-10.1
	Analytic geometry Solution[rad /10^3]	1.848				1.402				0.000				-1.402				-1.848
	Error[%]	-16.8				-8.3				/				-8.3				-16.8
8 equal portions	Displacement[mm]	0.00		-4.36		-7.99		-10.38		-11.21		-10.38		-7.99		-4.36		0.00
	Exact Solution[rad /10^3]	2.222		2.031		1.528		0.816		0		-0.816		-1.528		-2.031		-2.222
	Finite difference solution[rad/10^3]	2.180		1.998		1.506		0.805		0		-0.805		-1.506		-1.998		-2.180
	Error[‰]	-19		-16		-15		-13		/		-13		-15		-16		-19
	Analytic geometry Solution[rad /10^3]	2.134		1.998		1.506		0.805		0.000		-0.805		-1.506		-1.998		-2.134
	Error[‰]	-39		-16		-15		-13		/		-13		-15		-16		-39
16 equal portions	Displacement[mm]	0	-2.23	-4.36	-6.30	-7.99	-9.37	-10.38	-11.00	-11.21	-11.00	-10.38	-9.37	-7.99	-6.30	-4.36	-2.23	0.00
	Exact Solution[rad /10^3]	2.222	2.172	2.031	1.812	1.528	1.191	0.816	0.414	0	-0.414	-0.816	-1.191	-1.528	-1.812	-2.031	-2.172	-2.222
	Finite difference solution[rad/10^3]	2.229	2.180	2.038	1.817	1.532	1.195	0.818	0.416	0.000	-0.415	-0.818	-1.195	-1.532	-1.817	-2.038	-2.180	-2.229
	Error[‰]	3	3	3	3	3	3	2	4	/	4	2	3	3	3	3	3	3
	Analytic geometry Solution[rad /10^3]	2.218	2.180	2.038	1.817	1.532	1.195	0.818	0.416	0.000	-0.415	-0.818	-1.195	-1.532	-1.817	-2.038	-2.180	-2.218
	Error[‰]	-2	3	3	3	3	3	2	4	/	4	2	3	3	3	3	3	-2