Simulation Study of Phase Demodulation in Resonance Band for Gear Tooth Cracking Fault Detection

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Abstract. Due to the phase modulation is more sensible to early local gear faults than the amplitude modulation. This paper proposed a new method using phase demodulation based on resonance band for early detection of gear tooth cracking fault. The proposed scheme based on the fact that gear tooth crack can excite impacts which will stimulate the structural/transducer resonances when the cracked tooth is engaged. In this scheme, the synchronous averaging methods are used to eliminate the regular signal first. Then, frequency shift method is used to pick out the phase modulation harmonics in resonant band. At last, the features of gear fault can be indicated by demodulation the phase modulation harmonics. Simulation studies indicated that this method is an effective tool for early detection of local gear faults.

Introduction

Resonance demodulation method has effectively utilized on gear local faults detection. But it is usually utilized for the amplitude demodulation. This paper proposed a resonance phase demodulation technique, which supplements the current gear faults detection techniques. Due to the amplification of the interesting components by the resonance, it is good at early fault detection of local gear faults. Phase modulation can give earlier warning of a local defect such as a fatigue crack in a gear than that of the amplitude modulation does [1]. Hence, this paper is mainly discussing the implementation of resonance phase demodulation technique.

For a healthy gear, the meshing frequency components and it’s harmonics are called the regular signal [2]. The regular signal often modulated by transmission errors [3]. The synchronous average is a useful tool to eliminate random components or the periodic components didn’t connect to the shaft rotation frequency. The vibration signals of gears are first synchronous averaged with the corresponding shaft speed, and then the regular signal will be removed. And the resulting signal which we called residual signal [2] is estimation of the gear meshing vibration signal without noise.

When a local gear defect (such as tooth crack) is occurred, it will induce impacts when the cracked tooth is engaged in each revolution. The impacts will produce additional modulation effects to the normal meshing vibration signal [4]. Due to the short time nature of the impact, that will generate broad band modulation harmonics, then the high-order harmonics will stimulate structural/transducer resonance vibration. In a resonance band, the interesting modulation harmonic components will be amplified and these components usually are high-order modulation harmonics which do not affected by low frequency interferences. So when a suitable resonance band is determined, the phase demodulation can be effectively utilized to indicate the gear fault.

Mathematic Model of Gear Meshing Vibration

Under ideal conditions, the vibration signal of a gear is consist of fundamental frequency components. In general, even a healthy gear meshing harmonics will be accompanied by some low-order amplitude and phase modulation effects [3,5]. The gear vibration signal without noise can be given by [1,4]
\[
x(t) = \sum_{m=0}^{M} A_m \left[ 1 + a_m(t) \right] \cos(2\pi mZf_s t + \theta_m + b_m(t))
\]

where \(A_m\) is the amplitude, \(Z\) is gear teeth number, \(\theta_m\) is the number meshing frequency harmonics, \(f_s\) is rotational frequency of the gear and \(a_m(t), b_m(t)\) are the amplitude and phase modulation functions which are given by [1]

\[
a_m(t) = \sum_{n=0}^{N} A_{mn} \cos(2\pi nf_s t + \alpha_{mn}),
\quad b_m(t) = \sum_{n=0}^{N} B_{mn} \cos(2\pi nf_s t + \beta_{mn})
\]

When a local gear fault occurs, an impact will be produced. The impact produced additional amplitude and phase modulation. The impact vibration \(z(t)\) can be given as \(z(t) = d(t)\cos(2\pi f_i t + \theta_i)\), and the new amplitude and phase modulation function \(a'_m(t)\) and \(b'_m(t)\) can be expressed by

\[
a'_m(t) = \sum_{n=0}^{N} A_{mn} \cos(2\pi nf_s t + \alpha_{mn}) + d(t),
\quad b'_m(t) = \sum_{n=0}^{N} B_{mn} \cos(2\pi nf_s t + \beta_{mn}) - d(t)
\]

where \(d(t)\) is the envelope of the impact vibration, \(f_i\) is the oscillation frequency (carrier frequency) of the impact. The meshing vibration signal model of local faulty gear can be written as

\[
x(t) = \sum_{m=0}^{M} A_m \left[ 1 + a'_m(t) \right] \cos(2\pi mZf_s t + \theta_m + b'_m(t)) + d(t)\cos(2\pi f_i t + \theta_i)
\]

We set the rotation speed \(rpm = 1200 \text{ r/min}\), i.e. the rotation frequency \(f_s = 20 \text{ Hz}\), gear teeth \(Z = 32\), and simulate \(6\times\) meshing vibration signal. And assume the oscillation frequency \(f_i = 2500 \text{ Hz}\), the simulated modulation functions will be showing in Fig. 1 (a) and (b). We can see that there is a sudden change in phase modulation functions occurs at about 185° shaft angles in a complete revolution which is caused by the impact. It will demodulate the gear meshing vibration signal and generate broad band harmonics due its shot time nature.

To simulate resonance vibration, we let the signal \(x(t)\) through a linear system simulated by simulink. This linear system is modeled and used to emulate the mechanical system of gear box with
a natural frequency equals the resonance frequency. We assume the resonance frequency \( f_r = 4500 \, \text{Hz} \) and the spectrum of gear vibration signal with excited resonance vibration \( y(t) \) is showing in Fig. 1 (d). For the purpose of fault detection, we should pay attention to the information embedded in the modulation functions \( a_m(t), b_m(t) \) and \( d(t) \). This paper is mainly discusses resonance phase demodulation, hence, we should concentrate on \( d(t) \).

**Resonance Phase Demodulation Technique and Its Application**

In the last section, we have established the meshing vibration signal mathematic model of a local faulty gear. Next, we will discuss the resonance phase demodulation technique and its application to the simulated gear vibration signal.

![Fig. 2 Resonance phase demodulation based on frequency shift method](image)

Fig. 2 indicates the calculating flow for resonance phase demodulation method. The simulated gear meshing vibration signal \( y(t) \) is firstly synchronous averaged to eliminate the regular signal, and the resulting signal is the so-called residual signal \( y(\theta) \). In order to get the modulation functions from \( y(\theta) \), the most effective method is the Hilbert transform. Take \( y(\theta) \) as the real component and its Hilbert transform \( y_H(\theta) \) as the imaginary component constitute the analytical signal \( y_A(\theta) = y(\theta) + jy_H(\theta) \). And the phase function of this analytical signal can be calculated by following equation

\[
\text{Arg}[y_A(\theta)] = \arctan(y_H(\theta)/y(\theta)) = 2\pi O\theta + P(\theta).
\]  

(5)

The phase function minus \( 2\pi O\theta \) equals \( P(\theta) \) which is the phase modulation function we want to get. Another effective way to calculate the phase modulation function is frequency shift method [6]. The implementation of frequency shift algorithm can be given as

\[
y^{A\text{-shift}}(\theta) = y^A(\theta)\exp(-j2\pi O\theta),
\]

(6)

where \( y^{A\text{-shift}}(\theta) \) is the shifted signal, \( O \) is the central resonance order.

In Fig. 3 (a), we can see that the central resonance frequency is about 225\( \pi \). Fig. 3 (b) shows the spectrum of frequency shifted signal (i.e. \( y^{A\text{-shift}}(\theta) \)) that the resonance band has been shifted to origin coordinates. And Fig. 3 (c) clearly indicates the spectrum what we wanted (phase modulation harmonics in resonance band), it is band-pass filtered from picture Fig. 3 (b) with a bandwidth about the length of resonance band. Then, implement the IFFT to the shifted resonance band signal and the phase modulation function is the phase of the complex signal.

Fig. 3 (e) shows the demodulated phase modulation function which is similar with the simulated phase modulation function showing in Fig. 3 (d). And it is clearly indicates the location of the gear fault is happened at about 185° shaft angles. This is conformity with the simulated gear fault location. Fig. 3 (f) is gotten by the same algorithm with Fig. 3 (e) utilized. But the simulated signal \( y(t) \) didn’t with a resonance band. Compact Fig. 3 (e) and (f), it indicates that the resonance vibration amplified the high-order and low-energy modulation harmonics which will help on demodulation work. Also, the resonance phase demodulation didn’t affected by low-order frequency components.
Figure 3. The implementation schematic of frequency shift (a) spectrum of $y(t)$, (b) spectrum of $y^{A\text{-shift}}(\theta)$, (c) spectrum of the modulation signal in resonance band and demodulation results (d) simulated-, (e) demodulated with resonance-, (f) demodulated without resonance- phase modulation function.

Conclusion

A scheme of using the resonance phase demodulation technique has been proposed for detection of gear tooth crack faults and the method is applied to numerically simulated data. Simulated studies indicate that the resonance phase demodulation technique is a powerful technique in gear diagnosis field. It will be extensively utilized in early detection of local gear fault.

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References